Effective interactions between colloidal particles at the surface of a liquid drop

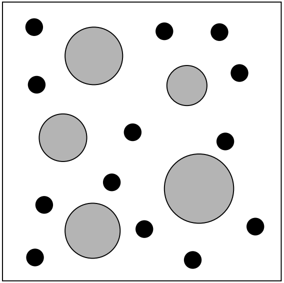
Jan Guzowski

Instytut Chemii Fizycznej PAN, Warszawa

IPPT, Warszawa, 09.11.2011

Pickering Emulsions

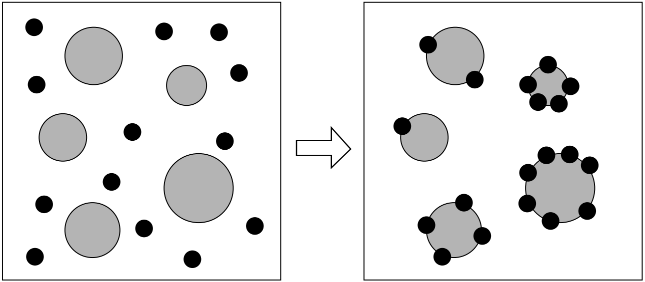
- emulsion + colloidal particles
- particles get trapped at the surface of droplets



- applications: stabilization of emulsions, engineering of functional particles

Pickering Emulsions

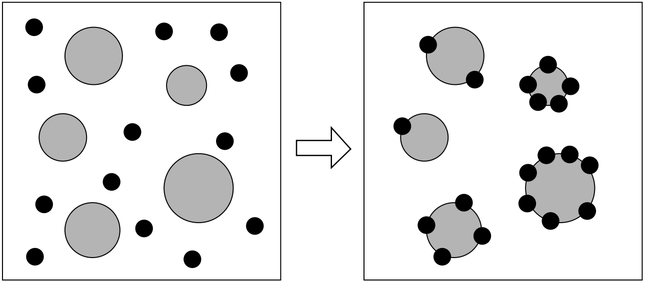
- emulsion + colloidal particles
- particles get trapped at the surface of droplets



- applications: stabilization of emulsions, engineering of functional particles

Pickering Emulsions

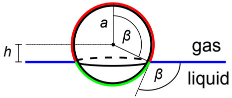
- emulsion + colloidal particles
- particles get trapped at the surface of droplets



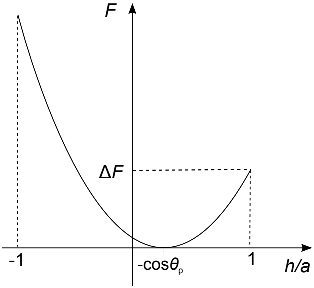
- applications: stabilization of emulsions, engineering of functional particles

Stability of a colloidal particle at the interface

- macroscopic picture: interplay of surface energies
- contributions from three possible interfaces: $F = \gamma_{pl} S_{pl} + \gamma_{pg} S_{pg} + \gamma_{lg} S_{lg}$
- rough estimate: undeformable flat interface $\Rightarrow$ $F(h) = \pi\gamma a^2 (h/a + \cos\theta_p)^2$, where $\cos\theta_p = (\gamma_{pg} - \gamma_{pl})/\gamma_{lg}$

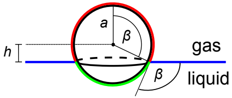


a	ΔF [k_BT]
10nm	10^3
100nm	10^5
1μm	10^7

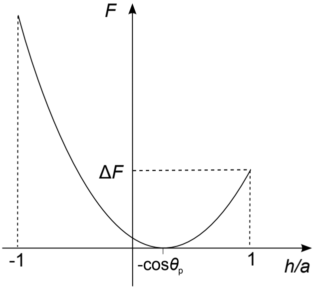


Stability of a colloidal particle at the interface

- macroscopic picture: interplay of surface energies
- contributions from three possible interfaces: $F = \gamma_{pl} S_{pl} + \gamma_{pg} S_{pg} + \gamma_{lg} S_{lg}$
- rough estimate: undeformable flat interface $\Rightarrow$ $F(h) = \pi\gamma a^2(h/a + \cos\theta_p)^2$, where $\cos\theta_p = (\gamma_{pg} - \gamma_{pl})/\gamma_{lg}$

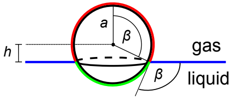


a	ΔF [$k_B T$]
10nm	10^3
100nm	10^5
1μm	10^7

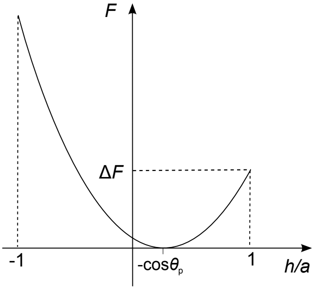


Stability of a colloidal particle at the interface

- macroscopic picture: interplay of surface energies
- contributions from three possible interfaces: $F = \gamma_{pl} S_{pl} + \gamma_{pg} S_{pg} + \gamma_{lg} S_{lg}$
- rough estimate: undeformable flat interface $\Rightarrow$ $F(h) = \pi\gamma a^2(h/a + \cos\theta_p)^2$, where $\cos\theta_p = (\gamma_{pg} - \gamma_{pl})/\gamma_{lg}$

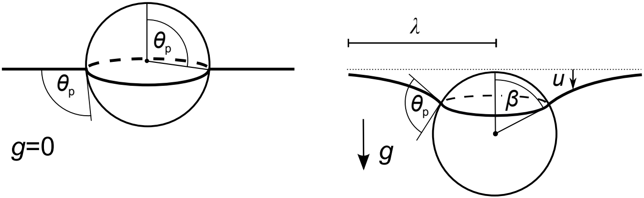


a	ΔF [$k_B T$]
10nm	10^3
100nm	10^5
1μm	10^7



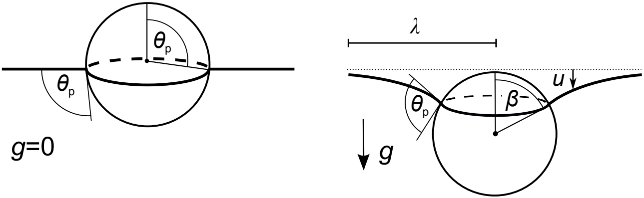
Generic case: single particle at a flat interface

- particle pulled by the force f = weight - buoyancy
- interface effectively pinned by gravity at the distance $\lambda = \sqrt{\gamma/\Delta\rho g}$ (capillary length)



Generic case: single particle at a flat interface

- particle pulled by the force f = weight - buoyancy
- interface effectively pinned by gravity at the distance $\lambda = \sqrt{\gamma/\Delta\rho g}$ (capillary length)



Generic case: single particle at a flat interface

- the capillary equation for $|\nabla_{\parallel} u| \ll 1$ (balance of capillary and hydrostatic pressures across the interface)

$$-\gamma \nabla_{\parallel}^2 u + \frac{\gamma}{\lambda^2} u = 0$$

- the corresponding Green's function $G(\mathbf{x}, \mathbf{x}') = G(|\mathbf{x}, \mathbf{x}'|)$ obeying the condition $G(r \to \infty) = 0$ reads

$$G(r) = (1/2\pi) K_0(r/\lambda) \sim \begin{cases} \ln(\lambda/r) & \text{for} \quad r \ll \lambda \\ r^{-1/2} e^{-r/\lambda} & \text{for} \quad r \gg \lambda \end{cases}$$

Generic case: single particle at a flat interface

- the capillary equation for $|\nabla_{\|} u| \ll 1$ (balance of capillary and hydrostatic pressures across the interface)

$$-\gamma \nabla_{\|}^2 u + \frac{\gamma}{\lambda^2} u = 0$$

- the corresponding Green's function $G(\mathbf{x}, \mathbf{x'}) = G(|\mathbf{x}, \mathbf{x'}|)$ obeying the condition $G(r \to \infty) = 0$ reads

$$G(r) = (1/2\pi) K_0(r/\lambda) \sim \begin{cases} \ln(\lambda/r) & \text{for} \quad r \ll \lambda \\ r^{-1/2} e^{-r/\lambda} & \text{for} \quad r \gg \lambda \end{cases}$$

Effective description, limit $\lambda \to \infty$

- particle replaced by an effective pressure distribution $\Pi(\mathbf{x})$
- for $\lambda \to \infty$ Poisson equation $-\gamma \nabla_{\parallel}^2 u = \Pi(\mathbf{x})$
- in terms of complex variables $u(\mathbf{x}) = \mathrm{Re} V(z)$ with $V(z) = (2\pi\gamma)^{-1} \int d^2\mathbf{x}' \, \Pi(z') \ln[\lambda/(z - z')]$
- for $\Pi(z')$ localized around the origin one can use the Taylor expansion

$$2\pi\gamma V(z) = \tilde{Q}_0 \ln(\lambda/z) + \sum_{n=1}^{\infty} \tilde{Q}_n n^{-1} z^{-n}$$

- with the multipoles $\tilde{Q}_n := \int d^2\mathbf{x}' \, \Pi(z') z'^{\,n} = Q_n e^{i\phi_n}$ so that Q_0 = total external force, Q_1 = total external torque; $Q_{n\geq 2}$ correspond to free particles
- residue theorem $\Rightarrow$ all multipoles fully determined by the deformation around an arbitrary contour C enclosing the origin: $\tilde{Q}_n = i\gamma \oint_C dz \, z^n (dV/dz)$

Effective description, limit $\lambda \to \infty$

- particle replaced by an effective pressure distribution $\Pi(\mathbf{x})$
- for $\lambda \to \infty$ Poisson equation $-\gamma \nabla_{\parallel}^2 u = \Pi(\mathbf{x})$
- in terms of complex variables $u(\mathbf{x}) = \mathrm{Re} V(z)$ with $V(z) = (2\pi\gamma)^{-1} \int d^2\mathbf{x}' \, \Pi(z') \ln[\lambda/(z - z')]$
- for $\Pi(z')$ localized around the origin one can use the Taylor expansion

$$2\pi\gamma V(z) = \tilde{Q}_0 \ln(\lambda/z) + \sum_{n=1}^{\infty} \tilde{Q}_n n^{-1} z^{-n}$$

- with the multipoles $\tilde{Q}_n := \int d^2\mathbf{x}' \, \Pi(z') z'^{\,n} = Q_n e^{i\phi_n}$ so that $Q_0 =$ total external force, $Q_1 =$ total external torque; $Q_{n \geq 2}$ correspond to free particles
- residue theorem $\Rightarrow$ all multipoles fully determined by the deformation around an arbitrary contour C enclosing the origin: $\tilde{Q}_n = i\gamma \oint_C dz \, z^n (dV/dz)$

Effective description, limit $\lambda \to \infty$

- particle replaced by an effective pressure distribution $\Pi(\mathbf{x})$
- for $\lambda \to \infty$ Poisson equation $-\gamma \nabla_{\|}^2 u = \Pi(\mathbf{x})$
- in terms of complex variables $u(\mathbf{x}) = \mathrm{Re} V(z)$ with $V(z) = (2\pi\gamma)^{-1} \int d^2\mathbf{x'}\, \Pi(z') \ln[\lambda/(z-z')]$
- for $\Pi(z')$ localized around the origin one can use the Taylor expansion

$$2\pi\gamma V(z) = \tilde{Q}_0 \ln(\lambda/z) + \sum_{n=1}^{\infty} \tilde{Q}_n n^{-1} z^{-n}$$

- with the multipoles $\tilde{Q}_n := \int d^2\mathbf{x'}\, \Pi(z') z'^{\,n} = Q_n e^{i\phi_n}$ so that Q_0 = total external force, Q_1 = total external torque; $Q_{n\geq 2}$ correspond to free particles
- residue theorem $\Rightarrow$ all multipoles fully determined by the deformation around an arbitrary contour C enclosing the origin: $\tilde{Q}_n = i\gamma \oint_C dz\, z^n (dV/dz)$

Effective description, limit $\lambda \to \infty$

- particle replaced by an effective pressure distribution $\Pi(\mathbf{x})$
- for $\lambda \to \infty$ Poisson equation $-\gamma \nabla_{\parallel}^2 u = \Pi(\mathbf{x})$
- in terms of complex variables $u(\mathbf{x}) = \mathrm{Re} V(z)$ with $V(z) = (2\pi\gamma)^{-1} \int d^2\mathbf{x'}\, \Pi(z') \ln[\lambda/(z-z')]$
- for $\Pi(z')$ localized around the origin one can use the Taylor expansion

$$2\pi\gamma V(z) = \tilde{Q}_0 \ln(\lambda/z) + \sum_{n=1}^{\infty} \tilde{Q}_n n^{-1} z^{-n}$$

- with the multipoles $\tilde{Q}_n := \int d^2\mathbf{x'}\, \Pi(z') z'^{\,n} = Q_n e^{i\phi_n}$ so that Q_0 = total external force, Q_1 = total external torque; $Q_{n\geq 2}$ correspond to free particles
- residue theorem $\Rightarrow$ all multipoles fully determined by the deformation around an arbitrary contour C enclosing the origin: $\tilde{Q}_n = i\gamma \oint_C dz\, z^n (dV/dz)$

Effective description, limit $\lambda \to \infty$

- particle replaced by an effective pressure distribution $\Pi(\mathbf{x})$
- for $\lambda \to \infty$ Poisson equation $-\gamma\nabla_{\parallel}^2 u = \Pi(\mathbf{x})$
- in terms of complex variables $u(\mathbf{x}) = \mathrm{Re}V(z)$ with $V(z) = (2\pi\gamma)^{-1}\int d^2\mathbf{x'}\,\Pi(z')\ln[\lambda/(z-z')]$
- for $\Pi(z')$ localized around the origin one can use the Taylor expansion

$$2\pi\gamma V(z) = \tilde{Q}_0\ln(\lambda/z) + \sum_{n=1}^{\infty}\tilde{Q}_n n^{-1} z^{-n}$$

- with the multipoles $\tilde{Q}_n := \int d^2\mathbf{x'}\,\Pi(z')z'^{\,n} = Q_n e^{i\phi_n}$ so that Q_0 = total external force, Q_1 = total external torque; $Q_{n\geq 2}$ correspond to free particles
- residue theorem $\Rightarrow$ all multipoles fully determined by the deformation around an arbitrary contour C enclosing the origin: $\tilde{Q}_n = i\gamma\oint_C dz\, z^n(dV/dz)$

Effective description, limit $\lambda \to \infty$

- particle replaced by an effective pressure distribution $\Pi(\mathbf{x})$
- for $\lambda \to \infty$ Poisson equation $-\gamma \nabla_{\parallel}^2 u = \Pi(\mathbf{x})$
- in terms of complex variables $u(\mathbf{x}) = \mathrm{Re} V(z)$ with $V(z) = (2\pi\gamma)^{-1} \int d^2\mathbf{x'}\, \Pi(z') \ln[\lambda/(z - z')]$
- for $\Pi(z')$ localized around the origin one can use the Taylor expansion

$$2\pi\gamma V(z) = \tilde{Q}_0 \ln(\lambda/z) + \sum_{n=1}^{\infty} \tilde{Q}_n n^{-1} z^{-n}$$

- with the multipoles $\tilde{Q}_n := \int d^2\mathbf{x'}\, \Pi(z') z'^{\,n} = Q_n e^{i\phi_n}$ so that Q_0 = total external force, Q_1 = total external torque; $Q_{n\geq 2}$ correspond to free particles
- residue theorem $\Rightarrow$ all multipoles fully determined by the deformation around an arbitrary contour C enclosing the origin: $\tilde{Q}_n = i\gamma \oint_C dz\, z^n (dV/dz)$

Capillary interactions

- two particles at distance d, effective pressure $\Pi = \Pi_1 + \Pi_2$
- free energy

$$F = \int d^2\mathbf{x} \left[\frac{\gamma}{2}(\nabla_{\parallel} u)^2 - \Pi(\mathbf{x})u(\mathbf{x})\right] = -\frac{1}{2\gamma}\int d^2\mathbf{x} \int d^2\mathbf{x}'\, \Pi(\mathbf{x})G(\mathbf{x},\mathbf{x}')\Pi(\mathbf{x}')$$

- $F = F_{1,self} + F_{2,self} + \Delta F(d)$
- multipole expansion yields

$$\Delta F(d) = -\frac{1}{\gamma}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} Q_{1,n}Q_{2,n'}g_{nn'}\cos(n\varphi_{1n} + n'\varphi_{2n'}) \times \begin{cases} \ln(\lambda/d) & n = n' = 0, \\ d^{-n-n'} & \text{otherwise} \end{cases}$$

- in general $Q_{i,n} = Q_{i,n}(d)$ (feedback $u \to \Pi$), many-body interactions!
- but Q_0 and Q_1 can be fixed by external forces and torques

Capillary interactions

- two particles at distance d, effective pressure $\Pi = \Pi_1 + \Pi_2$
- free energy

$$F = \int d^2\mathbf{x} \left[\frac{\gamma}{2}(\nabla_{\parallel} u)^2 - \Pi(\mathbf{x})u(\mathbf{x})\right] = -\frac{1}{2\gamma}\int d^2\mathbf{x} \int d^2\mathbf{x}' \, \Pi(\mathbf{x})G(\mathbf{x},\mathbf{x}')\Pi(\mathbf{x}')$$

- $F = F_{1,self} + F_{2,self} + \Delta F(d)$
- multipole expansion yields

$$\Delta F(d) = -\frac{1}{\gamma}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} Q_{1,n}Q_{2,n'}g_{nn'}\cos(n\varphi_{1n} + n'\varphi_{2n'}) \times \begin{cases} \ln(\lambda/d) & n = n' = 0, \\ d^{-n-n'} & \text{otherwise} \end{cases}$$

- in general $Q_{i,n} = Q_{i,n}(d)$ (feedback $u \to \Pi$), many-body interactions!
- but Q_0 and Q_1 can be fixed by external forces and torques

Capillary interactions

- two particles at distance d, effective pressure $\Pi = \Pi_1 + \Pi_2$
- free energy

$$F = \int d^2\mathbf{x} \left[\frac{\gamma}{2}(\nabla_{\parallel} u)^2 - \Pi(\mathbf{x})u(\mathbf{x})\right] = -\frac{1}{2\gamma}\int d^2\mathbf{x} \int d^2\mathbf{x}' \, \Pi(\mathbf{x})G(\mathbf{x},\mathbf{x}')\Pi(\mathbf{x}')$$

- $F = F_{1,self} + F_{2,self} + \Delta F(d)$
- multipole expansion yields

$$\Delta F(d) = -\frac{1}{\gamma}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} Q_{1,n}Q_{2,n'}g_{nn'}\cos(n\varphi_{1n} + n'\varphi_{2n'}) \times \begin{cases} \ln(\lambda/d) & n = n' = 0, \\ d^{-n-n'} & \text{otherwise} \end{cases}$$

- in general $Q_{i,n} = Q_{i,n}(d)$ (feedback $u \to \Pi$), many-body interactions!
- but Q_0 and Q_1 can be fixed by external forces and torques

Capillary interactions

- two particles at distance d, effective pressure $\Pi = \Pi_1 + \Pi_2$
- free energy

$$F = \int d^2\mathbf{x} \left[\frac{\gamma}{2}(\nabla_{\parallel} u)^2 - \Pi(\mathbf{x})u(\mathbf{x})\right] = -\frac{1}{2\gamma}\int d^2\mathbf{x} \int d^2\mathbf{x}' \, \Pi(\mathbf{x})G(\mathbf{x},\mathbf{x}')\Pi(\mathbf{x}')$$

- $F = F_{1,self} + F_{2,self} + \Delta F(d)$
- multipole expansion yields

$$\Delta F(d) = -\frac{1}{\gamma}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} Q_{1,n}Q_{2,n'}g_{nn'}\cos(n\varphi_{1n} + n'\varphi_{2n'}) \times \begin{cases} \ln(\lambda/d) & n = n' = 0, \\ d^{-n-n'} & \text{otherwise} \end{cases}$$

- in general $Q_{i,n} = Q_{i,n}(d)$ (feedback $u \to \Pi$), many-body interactions!
- but Q_0 and Q_1 can be fixed by external forces and torques

Capillary interactions

- two particles at distance d, effective pressure $\Pi = \Pi_1 + \Pi_2$
- free energy

$$F = \int d^2\mathbf{x} \left[\frac{\gamma}{2}(\nabla_{\parallel} u)^2 - \Pi(\mathbf{x})u(\mathbf{x})\right] = -\frac{1}{2\gamma}\int d^2\mathbf{x} \int d^2\mathbf{x'}\, \Pi(\mathbf{x})G(\mathbf{x},\mathbf{x'})\Pi(\mathbf{x'})$$

- $F = F_{1,self} + F_{2,self} + \Delta F(d)$
- multipole expansion yields

$$\Delta F(d) = -\frac{1}{\gamma}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} Q_{1,n}Q_{2,n'}g_{nn'}\cos(n\varphi_{1n} + n'\varphi_{2n'}) \times \begin{cases} \ln(\lambda/d) & n = n' = 0, \\ d^{-n-n'} & \text{otherwise} \end{cases}$$

- in general $Q_{i,n} = Q_{i,n}(d)$ (feedback $u \to \Pi$), many-body interactions!
- but Q_0 and Q_1 can be fixed by external forces and torques

Capillary interactions

- two particles at distance d, effective pressure $\Pi = \Pi_1 + \Pi_2$
- free energy

$$F = \int d^2\mathbf{x} \left[\frac{\gamma}{2}(\nabla_{\parallel} u)^2 - \Pi(\mathbf{x})u(\mathbf{x})\right] = -\frac{1}{2\gamma}\int d^2\mathbf{x} \int d^2\mathbf{x'}\, \Pi(\mathbf{x})G(\mathbf{x},\mathbf{x'})\Pi(\mathbf{x'})$$

- $F = F_{1,self} + F_{2,self} + \Delta F(d)$
- multipole expansion yields

$$\Delta F(d) = -\frac{1}{\gamma}\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} Q_{1,n}Q_{2,n'}g_{nn'}\cos(n\varphi_{1n} + n'\varphi_{2n'}) \times \begin{cases} \ln(\lambda/d) & n = n' = 0, \\ d^{-n-n'} & \text{otherwise} \end{cases}$$

- in general $Q_{i,n} = Q_{i,n}(d)$ (feedback $u \to \Pi$), many-body interactions!
- but Q_0 and Q_1 can be fixed by external forces and torques

Spherical interfaces

- assume small radial deformations $v(\Omega) = (r(\Omega) - R_0)/R_0$ and incompressibility of liquid $\Rightarrow$ free energy functional:

$$\mathcal{F}[\{v(\Omega)\}] = \gamma R_0^2 \int_{\Omega_0} d\Omega \left[\frac{1}{2}(\nabla_a v)^2 - v^2 - (\pi(\Omega) + \mu) v\right] + O(v^3, (\nabla_a v)^3)$$

- with $\int d\Omega \, v(\Omega) = 0$; condition $\delta\mathcal{F} \overset{!}{=} 0$ yields $-\nabla_a^2 v - 2v = \pi(\Omega) + \mu$
- free energy $F = min_{\{v(\Omega)\}}\mathcal{F}$ in terms of the corresponding Green's function G reads

$$F = -\frac{\gamma R_0^2}{2} \int d\Omega \int d\Omega' \, \pi(\Omega) G(\Omega, \Omega') \pi(\Omega').$$

- at small separations $G(\bar{\theta}) \xrightarrow[\bar{\theta}\to 0]{} -(2\pi)^{-1} \ln(\bar{\theta}) = -(2\pi)^{-1} \ln(r/R_0)$

Spherical interfaces

- assume small radial deformations $v(\Omega) = (r(\Omega) - R_0)/R_0$ and incompressibility of liquid $\Rightarrow$ free energy functional:

$$\mathcal{F}[\{v(\Omega)\}] = \gamma R_0^2 \int_{\Omega_0} d\Omega \left[\frac{1}{2}(\nabla_a v)^2 - v^2 - (\pi(\Omega) + \mu) v\right] + O(v^3, (\nabla_a v)^3)$$

- with $\int d\Omega\, v(\Omega) = 0$; condition $\delta\mathcal{F} \stackrel{!}{=} 0$ yields $-\nabla_a^2 v - 2v = \pi(\Omega) + \mu$
- free energy $F = min_{\{v(\Omega)\}}\mathcal{F}$ in terms of the corresponding Green's function G reads

$$F = -\frac{\gamma R_0^2}{2} \int d\Omega \int d\Omega'\, \pi(\Omega) G(\Omega, \Omega') \pi(\Omega').$$

- at small separations $G(\bar{\theta}) \xrightarrow[\bar{\theta}\to 0]{} -(2\pi)^{-1} \ln(\bar{\theta}) = -(2\pi)^{-1} \ln(r/R_0)$

Spherical interfaces

- assume small radial deformations $v(\Omega) = (r(\Omega) - R_0)/R_0$ and incompressibility of liquid $\Rightarrow$ free energy functional:

$$\mathcal{F}[\{v(\Omega)\}] = \gamma R_0^2 \int_{\Omega_0} d\Omega \left[\frac{1}{2}(\nabla_a v)^2 - v^2 - \left(\pi(\Omega) + \mu\right) v\right] + O(v^3, (\nabla_a v)^3)$$

- with $\int d\Omega\, v(\Omega) = 0$; condition $\delta\mathcal{F} \overset{!}{=} 0$ yields $-\nabla_a^2 v - 2v = \pi(\Omega) + \mu$
- free energy $F = min_{\{v(\Omega)\}}\mathcal{F}$ in terms of the corresponding Green's function G reads

$$F = -\frac{\gamma R_0^2}{2} \int d\Omega \int d\Omega'\, \pi(\Omega) G(\Omega, \Omega') \pi(\Omega').$$

- at small separations $G(\bar{\theta}) \xrightarrow[\bar{\theta}\to 0]{} -(2\pi)^{-1} \ln(\bar{\theta}) = -(2\pi)^{-1} \ln(r/R_0)$

Spherical interfaces

- assume small radial deformations $v(\Omega) = (r(\Omega) - R_0)/R_0$ and incompressibility of liquid $\Rightarrow$ free energy functional:

$$\mathcal{F}[\{v(\Omega)\}] = \gamma R_0^2 \int_{\Omega_0} d\Omega \left[\frac{1}{2}(\nabla_a v)^2 - v^2 - (\pi(\Omega) + \mu) v\right] + O(v^3, (\nabla_a v)^3)$$

- with $\int d\Omega\, v(\Omega) = 0$; condition $\delta\mathcal{F} \stackrel{!}{=} 0$ yields $-\nabla_a^2 v - 2v = \pi(\Omega) + \mu$
- free energy $F = min_{\{v(\Omega)\}}\mathcal{F}$ in terms of the corresponding Green's function G reads

$$F = -\frac{\gamma R_0^2}{2} \int d\Omega \int d\Omega'\, \pi(\Omega) G(\Omega, \Omega') \pi(\Omega').$$

- at small separations $G(\bar{\theta}) \xrightarrow[\bar{\theta}\to 0]{} -(2\pi)^{-1} \ln(\bar{\theta}) = -(2\pi)^{-1} \ln(r/R_0)$

Spherical interfaces

- assume small radial deformations $v(\Omega) = (r(\Omega) - R_0)/R_0$ and incompressibility of liquid $\Rightarrow$ free energy functional:

$$\mathcal{F}[\{v(\Omega)\}] = \gamma R_0^2 \int_{\Omega_0} d\Omega \left[\frac{1}{2}(\nabla_a v)^2 - v^2 - (\pi(\Omega) + \mu)\, v\right] + O(v^3, (\nabla_a v)^3)$$

- with $\int d\Omega\, v(\Omega) = 0$; condition $\delta\mathcal{F} \stackrel{!}{=} 0$ yields $-\nabla_a^2 v - 2v = \pi(\Omega) + \mu$
- free energy $F = min_{\{v(\Omega)\}}\mathcal{F}$ in terms of the corresponding Green's function G reads

$$F = -\frac{\gamma R_0^2}{2} \int d\Omega \int d\Omega'\, \pi(\Omega) G(\Omega, \Omega') \pi(\Omega').$$

- at small separations $G(\bar{\theta}) \xrightarrow[\bar{\theta}\to 0]{} -(2\pi)^{-1} \ln(\bar{\theta}) = -(2\pi)^{-1} \ln(r/R_0)$

Expansion in spherical harmonics

- capillary equation
  $[l(l+1)-2]v_{lm} = \pi_{lm} + \mu\delta_{l0}$ with
  $X_{lm} = \int d\Omega\, X(\Omega) Y_{lm}(\Omega)$
- $l=0$: incompressibility $v_{00}=0 \Rightarrow \mu = \pi_{00}$
- $l=1$: translations v_{1m} undefined, assume fixed center of mass $v_{1m}=0$
- free energy in terms of irreducible representation of rotation group

$$\Delta F = -\gamma R_0^2 \sum_{l\geq 2} \sum_{m=-l}^{l} \sum_{m'=-l}^{l} \pi_{1,lm}\, \pi_{2,lm'}\, \frac{(-1)^{m'}}{l(l+1)-1} d^{\,l}_{m,-m'}(\bar{\theta})\, e^{i(m\phi_1+m'\phi_2)}$$

Expansion in spherical harmonics

- capillary equation
  $[l(l+1)-2]v_{lm} = \pi_{lm} + \mu\delta_{l0}$ with
  $X_{lm} = \int d\Omega\, X(\Omega) Y_{lm}(\Omega)$
- $l = 0$: incompressibility $v_{00} = 0 \Rightarrow \mu = \pi_{00}$
- $l = 1$: translations v_{1m} undefined, assume fixed center of mass $v_{1m} = 0$
- free energy in terms of irreducible representation of rotation group

$$\Delta F = -\gamma R_0^2 \sum_{l\geq 2} \sum_{m=-l}^{l} \sum_{m'=-l}^{l} \pi_{1,lm}\, \pi_{2,lm'}\, \frac{(-1)^{m'}}{l(l+1)-1} d^{l}_{m,-m'}(\bar{\theta})\, e^{i(m\phi_1 + m'\phi_2)}$$

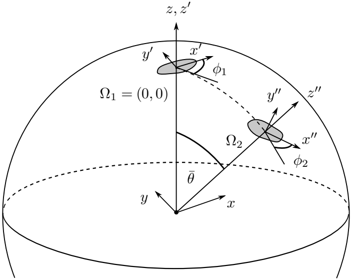
Expansion in spherical harmonics

- capillary equation
  $[l(l+1)-2]v_{lm} = \pi_{lm} + \mu\delta_{l0}$ with
  $X_{lm} = \int d\Omega\, X(\Omega) Y_{lm}(\Omega)$
- $l=0$: incompressibility $v_{00}=0 \Rightarrow \mu = \pi_{00}$
- $l=1$: translations v_{1m} undefined, assume fixed center of mass $v_{1m}=0$
- free energy in terms of irreducible representation of rotation group

$$\Delta F = -\gamma R_0^2 \sum_{l\geq 2} \sum_{m=-l}^{l} \sum_{m'=-l}^{l} \pi_{1,lm}\, \pi_{2,lm'}\, \frac{(-1)^{m'}}{l(l+1)-1} d^{\,l}_{m,-m'}(\bar{\theta})\, e^{i(m\phi_1+m'\phi_2)}$$

Expansion in spherical harmonics

- capillary equation
  $[l(l+1)-2]v_{lm} = \pi_{lm} + \mu\delta_{l0}$ with
  $X_{lm} = \int d\Omega\, X(\Omega) Y_{lm}(\Omega)$
- $l = 0$: incompressibility $v_{00} = 0 \Rightarrow \mu = \pi_{00}$
- $l = 1$: translations v_{1m} undefined, assume fixed center of mass $v_{1m} = 0$



- free energy in terms of irreducible representation of rotation group

$$\Delta F = -\gamma R_0^2 \sum_{l\geq 2} \sum_{m=-l}^{l} \sum_{m'=-l}^{l} \pi_{1,lm}\, \pi_{2,lm'}\, \frac{(-1)^{m'}}{l(l+1)-1} d^{\,l}_{m,-m'}(\bar{\theta})\, e^{i(m\phi_1 + m'\phi_2)}$$

Limit of small particles

- in the limit $a, a' \ll R_0$ one has $\Delta F = \sum_{n,n'=0}^{\infty} \Delta F_{nn'}$ with $n = |m|$ and

$$\Delta F_{nn'} = \gamma a^2 \frac{Q_{1,n} Q_{2,n'}}{(-2)^{n+n'+1} n! n'! \pi} \left(\frac{a}{R_0}\right)^{n+n'} \sum_{l \geq \max\{2,n,n'\}} \frac{(2l+1)}{(l+2)(l-1)}$$

$$\times \begin{cases} \frac{(l+n')!}{(l-n)!} \Big[(-1)^n \cos(n\phi_1 + n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'-n} \left(\sin\frac{\bar\theta}{2}\right)^{n'+n} P_{l-n'}^{(n'+n,n'-n)}(\cos\bar\theta) \\ \quad + \cos(n\phi_1 - n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'+n} \left(\sin\frac{\bar\theta}{2}\right)^{n'-n} P_{l-n'}^{(n'-n,n'+n)}(\cos\bar\theta) \Big], & n > 0, \quad n' > 0, \\ (-1)^n \cos(n\phi_1) P_l^n(\cos\bar\theta), & n > 0, \quad n' = 0, \\ 2^{-1} P_l(\cos\bar\theta), & n = 0, \quad n' = 0, \end{cases}$$

- $Q_{i,n}$ are capillary multipoles on the locally flat interface, i.e., defined on the plane tangent to the unit sphere at Ω_i
- R_0 sets both the spatial separation and the capillary length
- more complex dependence on orientations

Limit of small particles

- in the limit $a, a' \ll R_0$ one has $\Delta F = \sum_{n,n'=0}^{\infty} \Delta F_{nn'}$ with $n = |m|$ and

$$\Delta F_{nn'} = \gamma a^2 \frac{Q_{1,n} Q_{2,n'}}{(-2)^{n+n'+1} n! n'! \pi} \left(\frac{a}{R_0}\right)^{n+n'} \sum_{l \geq \max\{2,n,n'\}} \frac{(2l+1)}{(l+2)(l-1)}$$

$$\times \begin{cases} \frac{(l+n')!}{(l-n)!} \Bigg[(-1)^n \cos(n\phi_1 + n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'-n} \left(\sin\frac{\bar\theta}{2}\right)^{n'+n} P_{l-n'}^{(n'+n,n'-n)}(\cos\bar\theta) \\ \quad + \cos(n\phi_1 - n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'+n} \left(\sin\frac{\bar\theta}{2}\right)^{n'-n} P_{l-n'}^{(n'-n,n'+n)}(\cos\bar\theta) \Bigg], & n > 0, \quad n' > 0, \\ (-1)^n \cos(n\phi_1) P_l^n(\cos\bar\theta), & n > 0, \quad n' = 0, \\ 2^{-1} P_l(\cos\bar\theta), & n = 0, \quad n' = 0, \end{cases}$$

- $Q_{i,n}$ are capillary multipoles on the locally flat interface, i.e., defined on the plane tangent to the unit sphere at Ω_i
- R_0 sets both the spatial separation and the capillary length
- more complex dependence on orientations

Limit of small particles

- in the limit $a, a' \ll R_0$ one has $\Delta F = \sum_{n,n'=0}^{\infty} \Delta F_{nn'}$ with $n = |m|$ and

$$\Delta F_{nn'} = \gamma a^2 \frac{Q_{1,n} Q_{2,n'}}{(-2)^{n+n'+1} n! n'! \pi} \left(\frac{a}{R_0}\right)^{n+n'} \sum_{l \geq \max\{2,n,n'\}} \frac{(2l+1)}{(l+2)(l-1)}$$

$$\times \begin{cases} \frac{(l+n')!}{(l-n)!} \Bigg[(-1)^n \cos(n\phi_1 + n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'-n} \left(\sin\frac{\bar\theta}{2}\right)^{n'+n} P_{l-n'}^{(n'+n,n'-n)}(\cos\bar\theta) & \\ \quad + \cos(n\phi_1 - n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'+n} \left(\sin\frac{\bar\theta}{2}\right)^{n'-n} P_{l-n'}^{(n'-n,n'+n)}(\cos\bar\theta) \Bigg], & n > 0, \quad n' > 0, \\ (-1)^n \cos(n\phi_1) P_l^n(\cos\bar\theta), & n > 0, \quad n' = 0, \\ 2^{-1} P_l(\cos\bar\theta), & n = 0, \quad n' = 0, \end{cases}$$

- $Q_{i,n}$ are capillary multipoles on the locally flat interface, i.e., defined on the plane tangent to the unit sphere at Ω_i
- R_0 sets both the spatial separation and the capillary length
- more complex dependence on orientations

Limit of small particles

- in the limit $a, a' \ll R_0$ one has $\Delta F = \sum_{n,n'=0}^{\infty} \Delta F_{nn'}$ with $n = |m|$ and

$$\Delta F_{nn'} = \gamma a^2 \frac{Q_{1,n} Q_{2,n'}}{(-2)^{n+n'+1} n! n'! \pi} \left(\frac{a}{R_0}\right)^{n+n'} \sum_{l \geq \max\{2,n,n'\}} \frac{(2l+1)}{(l+2)(l-1)}$$

$$\times \begin{cases} \frac{(l+n')!}{(l-n)!} \Bigg[(-1)^n \cos(n\phi_1 + n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'-n} \left(\sin\frac{\bar\theta}{2}\right)^{n'+n} P_{l-n'}^{(n'+n,n'-n)}(\cos\bar\theta) \\ \quad + \cos(n\phi_1 - n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'+n} \left(\sin\frac{\bar\theta}{2}\right)^{n'-n} P_{l-n'}^{(n'-n,n'+n)}(\cos\bar\theta) \Bigg], & n > 0, \quad n' > 0, \\ (-1)^n \cos(n\phi_1) P_l^n(\cos\bar\theta), & n > 0, \quad n' = 0, \\ 2^{-1} P_l(\cos\bar\theta), & n = 0, \quad n' = 0, \end{cases}$$

- $Q_{i,n}$ are capillary multipoles on the locally flat interface, i.e., defined on the plane tangent to the unit sphere at Ω_i
- R_0 sets both the spatial separation and the capillary length
- more complex dependence on orientations

Limit of small particles

- in the limit $a, a' \ll R_0$ one has $\Delta F = \sum_{n,n'=0}^{\infty} \Delta F_{nn'}$ with $n = |m|$ and

$$\Delta F_{nn'} = \gamma a^2 \frac{Q_{1,n} Q_{2,n'}}{(-2)^{n+n'+1} n! n'! \pi} \left(\frac{a}{R_0}\right)^{n+n'} \sum_{l \geq \max\{2,n,n'\}} \frac{(2l+1)}{(l+2)(l-1)}$$

$$\times \begin{cases} \frac{(l+n')!}{(l-n)!} \Big[(-1)^n \cos(n\phi_1 + n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'-n} \left(\sin\frac{\bar\theta}{2}\right)^{n'+n} P_{l-n'}^{(n'+n,n'-n)}(\cos\bar\theta) \\ \quad + \cos(n\phi_1 - n'\phi_2) \left(\cos\frac{\bar\theta}{2}\right)^{n'+n} \left(\sin\frac{\bar\theta}{2}\right)^{n'-n} P_{l-n'}^{(n'-n,n'+n)}(\cos\bar\theta) \Big], & n > 0, \quad n' > 0, \\ (-1)^n \cos(n\phi_1) P_l^n(\cos\bar\theta), & n > 0, \quad n' = 0, \\ 2^{-1} P_l(\cos\bar\theta), & n = 0, \quad n' = 0, \end{cases}$$

- $Q_{i,n}$ are capillary multipoles on the locally flat interface, i.e., defined on the plane tangent to the unit sphere at Ω_i
- R_0 sets both the spatial separation and the capillary length
- more complex dependence on orientations

Numerical calculations

- surface free energy minimized by using software Surface Evolver based on the gradient descent method
- minimized expression:

$$\mathcal{F}[\{\mathbf{r}(\Omega)\}, h_i, \psi_i; \bar{\theta}, \phi_i, f_i, T_i, \theta_{p,i}, a_i, V_l, \lambda_0] =$$

$$= \gamma S_{lg} + \sum_{i=1,2} (-\gamma \cos\theta_{p,i} S_{pl,i} - f_i h_i - \mathbf{T}_i \cdot \boldsymbol{\psi}_i) - \lambda_0 (V - V_l).$$

Numerical calculations

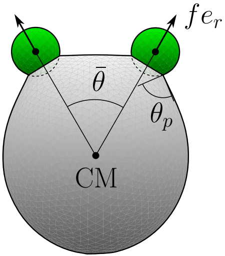
- surface free energy minimized by using software Surface Evolver based on the gradient descent method
- minimized expression:

$$\mathcal{F}[\{\mathbf{r}(\Omega)\}, h_i, \psi_i; \bar{\theta}, \phi_i, f_i, T_i, \theta_{p,i}, a_i, V_l, \lambda_0] =$$

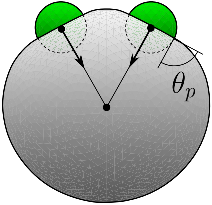
$$= \gamma S_{lg} + \sum_{i=1,2} (-\gamma \cos\theta_{p,i} S_{pl,i} - f_i h_i - \mathbf{T}_i \cdot \boldsymbol{\psi}_i) - \lambda_0 (V - V_l).$$

Results: monopoles

- smooth spherical particles, external radial forces $f = \gamma a Q_0$, fixed CM

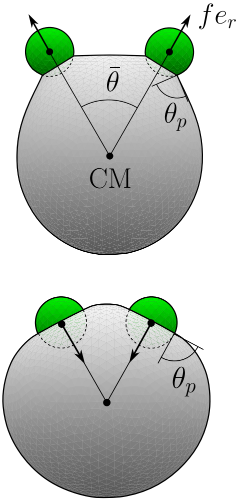


$$\frac{\Delta F_{00}(\bar{\theta})}{\gamma a^2} = -Q_0^2 G(\bar{\theta}) = \frac{Q_0^2}{4\pi}\left[\frac{1}{2} + \frac{4}{3}\cos\bar{\theta} + 2\cos\bar{\theta}\ln\left(\sin\frac{\bar{\theta}}{2}\right)\right]$$

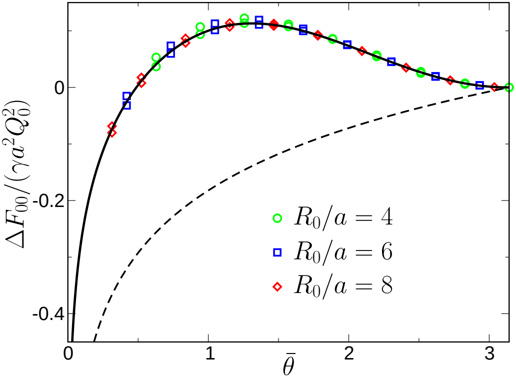


Results: monopoles

- smooth spherical particles, external radial forces $f = \gamma a Q_0$, fixed CM



$$\frac{\Delta F_{00}(\bar{\theta})}{\gamma a^2} = -Q_0^2 G(\bar{\theta}) = \frac{Q_0^2}{4\pi}\left[\frac{1}{2} + \frac{4}{3}\cos\bar{\theta} + 2\cos\bar{\theta}\ln\left(\sin\frac{\bar{\theta}}{2}\right)\right]$$



Results: dipoles

- three metastable branches for three different orientational configurations

$$\Delta F_{11}(\bar{\theta},\phi_1,\phi_2) = \gamma a^2 \frac{Q_1^2}{8\pi}\left(\frac{a}{R_0}\right)^2 \begin{cases} -f_+(\bar{\theta}) + f_-(\bar{\theta}), & \text{for} \quad \bar{\theta} < \bar{\theta}_0, & \uparrow\uparrow \\ -f_+(\bar{\theta}) - f_-(\bar{\theta}), & \text{for} \quad \bar{\theta}_0 < \bar{\theta} < \bar{\theta}_1, & \leftarrow\rightarrow \\ f_+(\bar{\theta}) - f_-(\bar{\theta}), & \text{for} \quad \bar{\theta} > \bar{\theta}_1, & \uparrow\downarrow \end{cases}$$

- where $f_-(\bar{\theta}_0) = 0$ and $f_+(\bar{\theta}_1) = 0$ and

$$f_+(\theta) := \frac{1}{\sin^2(\theta/2)} - 4\sin^2\frac{\theta}{2}\ln\left(\sin\frac{\theta}{2}\right) - \frac{20}{3}\sin^2\frac{\theta}{2} + 2,$$

$$f_-(\theta) := 4\left(\cos\frac{\theta}{2}\right)^2 \ln\left(\sin\frac{\theta}{2}\right) + \frac{20}{3}\cos^2\frac{\theta}{2}$$

Results: dipoles

- three metastable branches for three different orientational configurations

$$\Delta F_{11}(\bar{\theta},\phi_1,\phi_2)=\gamma a^2\frac{Q_1^2}{8\pi}\left(\frac{a}{R_0}\right)^2\begin{cases}-f_+(\bar{\theta})+f_-(\bar{\theta}), & \text{for}\quad \bar{\theta}<\bar{\theta}_0, \qquad \uparrow\uparrow\\ -f_+(\bar{\theta})-f_-(\bar{\theta}), & \text{for}\quad \bar{\theta}_0<\bar{\theta}<\bar{\theta}_1, \qquad \leftarrow\rightarrow\\ f_+(\bar{\theta})-f_-(\bar{\theta}), & \text{for}\quad \bar{\theta}>\bar{\theta}_1, \qquad \uparrow\downarrow\end{cases}$$

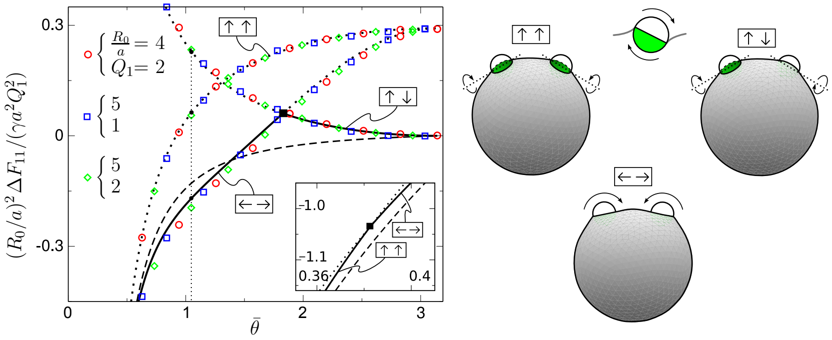
- where $f_-(\bar{\theta}_0)=0$ and $f_+(\bar{\theta}_1)=0$ and

$$f_+(\theta):=\frac{1}{\sin^2(\theta/2)}-4\sin^2\frac{\theta}{2}\ln\left(\sin\frac{\theta}{2}\right)-\frac{20}{3}\sin^2\frac{\theta}{2}+2,$$

$$f_-(\theta):=4\left(\cos\frac{\theta}{2}\right)^2\ln\left(\sin\frac{\theta}{2}\right)+\frac{20}{3}\cos^2\frac{\theta}{2}$$

Results: dipoles

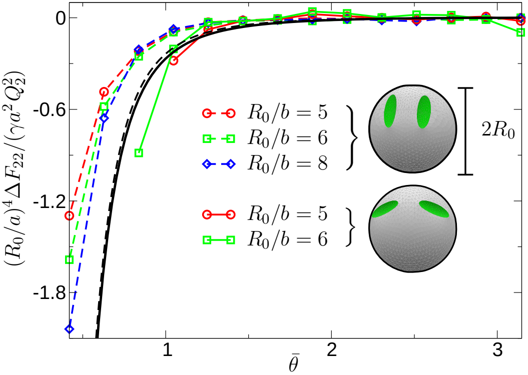
- pinned contact lines, external torques $T = \gamma a^2 Q_1$, fixed CM

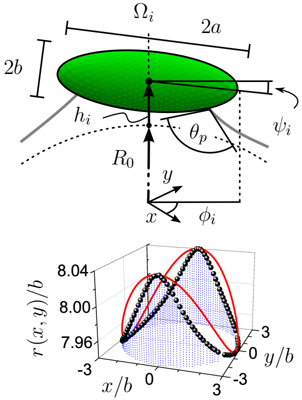


Results: free spheroidal particles

- free, smooth prolate spheroids; approximation: $Q_2 = Q_2(R_0) \simeq 2\pi \Delta r|_{\theta=a/R_0}/a$

$$\Delta F_{22}(\bar{\theta}, \phi_1, \phi_2) = -\gamma a^2 \frac{3Q_2^2}{64\pi} \left(\frac{a}{R_0}\right)^4 \frac{1}{\sin^4(\bar{\theta}/2)}.$$

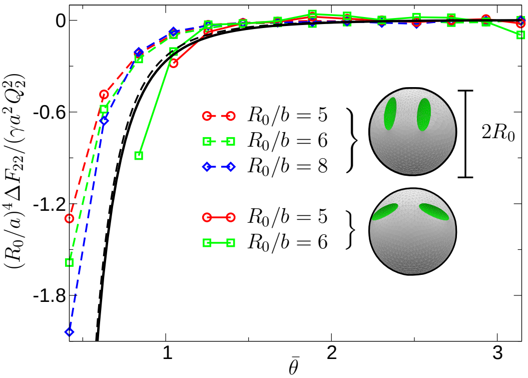


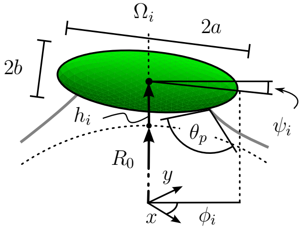


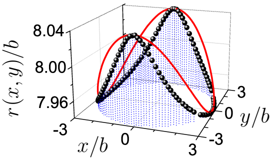
Results: free spheroidal particles

- free, smooth prolate spheroids; approximation: $Q_2 = Q_2(R_0) \simeq 2\pi \Delta r|_{\theta=a/R_0}/a$

$$\Delta F_{22}(\bar{\theta}, \phi_1, \phi_2) = -\gamma a^2 \frac{3Q_2^2}{64\pi}\left(\frac{a}{R_0}\right)^4 \frac{1}{\sin^4(\bar{\theta}/2)}.$$

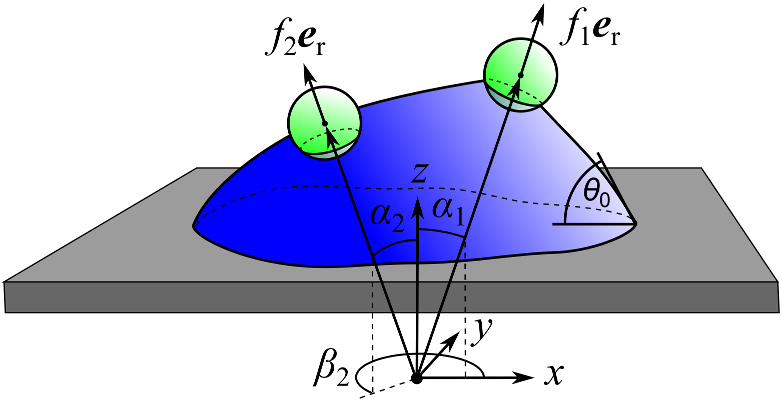






Sessile drops

- free energy depends on the contact angle θ_0 and boundary conditions of either a free ($\sigma = A$) or a pinned ($\sigma = B$) contact line at the substrate



Sessile drops: free energy

- after subtracting self-energies $F_{i,self}$ one gets

$$\Delta F_\sigma^{(N)} := F_\sigma^{(N)}(\Omega_1, \ldots, \Omega_N, \theta_0) - \sum_{i=1}^{N} F_{i,self} =$$

$$= \sum_{i=1}^{N} \Delta F_\sigma^{(1)}(\theta_i, \theta_0) + \sum_{i<j} V_\sigma(\Omega_i, \Omega_j, \theta_0)$$

- substrate potential $\Delta F_\sigma^{(1)}$ and pair-potential V_σ:

$$\Delta F_\sigma^{(1)} = -\frac{f_i^2}{2\gamma}[G_{\sigma,reg}(\Omega_i, \Omega_i) - G_{\sigma,reg}(0,0)]$$

$$V_\sigma = -\frac{f_i f_j}{2\gamma}[G_\sigma(\Omega_i, \Omega_j) + G_\sigma(\Omega_j, \Omega_i)],$$

Sessile drops: free energy

- after subtracting self-energies $F_{i,self}$ one gets

$$\Delta F_\sigma^{(N)} := F_\sigma^{(N)}(\Omega_1, \ldots, \Omega_N, \theta_0) - \sum_{i=1}^{N} F_{i,self} = \\ = \sum_{i=1}^{N} \Delta F_\sigma^{(1)}(\theta_i, \theta_0) + \sum_{i<j} V_\sigma(\Omega_i, \Omega_j, \theta_0)$$

- substrate potential $\Delta F_\sigma^{(1)}$ and pair-potential V_σ:

$$\Delta F_\sigma^{(1)} = -\frac{f_i^2}{2\gamma}[G_{\sigma,reg}(\Omega_i, \Omega_i) - G_{\sigma,reg}(0,0)]$$
$$V_\sigma = -\frac{f_i f_j}{2\gamma}[G_\sigma(\Omega_i, \Omega_j) + G_\sigma(\Omega_j, \Omega_i)],$$

Sessile drops: Green's functions

- for $\Omega \in \Omega_0$ Green's functions G_σ satisfy

$$-(\nabla_a^2 + 2)G_\sigma(\Omega, \Omega', \theta_0) = \delta(\Omega, \Omega') + \Delta_\sigma(\Omega, \Omega', \theta_0)$$

- functions $\Delta_\sigma(\Omega, \Omega', \theta_0)$ corresponding to μ and π_{CM} determined from the force balance and incompressibility condition $\int_{\Omega_0} d\Omega\, G_\sigma(\Omega, \Omega') = 0$
- with boundary conditions:

$$(\sin\theta_0 \partial_\theta G_A(\Omega, \Omega') - \cos\theta_0 G_A(\Omega, \Omega'))|_{\Omega \in \partial\Omega_0} = 0,$$
$$G_B(\Omega, \Omega')|_{\Omega \in \partial\Omega_0} = 0.$$

Sessile drops: Green's functions

- for $\Omega \in \Omega_0$ Green's functions G_σ satisfy

$$-(\nabla_a^2 + 2)G_\sigma(\Omega, \Omega', \theta_0) = \delta(\Omega, \Omega') + \Delta_\sigma(\Omega, \Omega', \theta_0)$$

- functions $\Delta_\sigma(\Omega, \Omega', \theta_0)$ corresponding to μ and π_{CM} determined from the force balance and incompressibility condition $\int_{\Omega_0} d\Omega\, G_\sigma(\Omega, \Omega') = 0$
- with boundary conditions:

$$(\sin\theta_0 \partial_\theta G_A(\Omega, \Omega') - \cos\theta_0 G_A(\Omega, \Omega'))|_{\Omega \in \partial\Omega_0} = 0,$$
$$G_B(\Omega, \Omega')|_{\Omega \in \partial\Omega_0} = 0.$$

Sessile drops: Green's functions

- for $\Omega \in \Omega_0$ Green's functions G_σ satisfy

$$-(\nabla_a^2 + 2)G_\sigma(\Omega, \Omega', \theta_0) = \delta(\Omega, \Omega') + \Delta_\sigma(\Omega, \Omega', \theta_0)$$

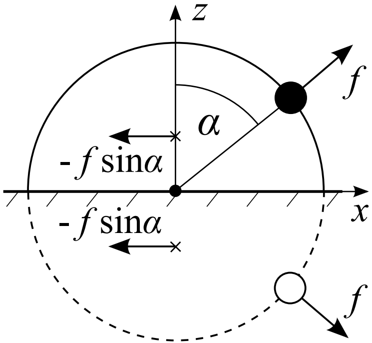
- functions $\Delta_\sigma(\Omega, \Omega', \theta_0)$ corresponding to μ and π_{CM} determined from the force balance and incompressibility condition $\int_{\Omega_0} d\Omega\, G_\sigma(\Omega, \Omega') = 0$
- with boundary conditions:

$$\begin{aligned}(\sin\theta_0 \partial_\theta G_A(\Omega, \Omega') - \cos\theta_0 G_A(\Omega, \Omega'))|_{\Omega \in \partial\Omega_0} &= 0,\\ G_B(\Omega, \Omega')|_{\Omega \in \partial\Omega_0} &= 0.\end{aligned}$$

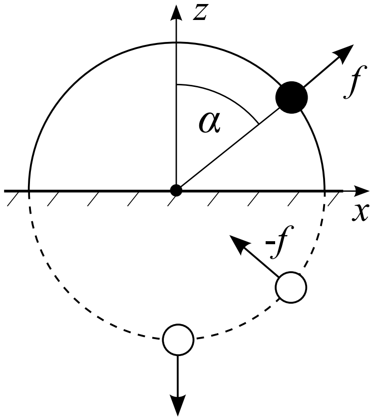
Sessile drops: special case $\theta_0 = \pi/2$

- f images

free c.l.

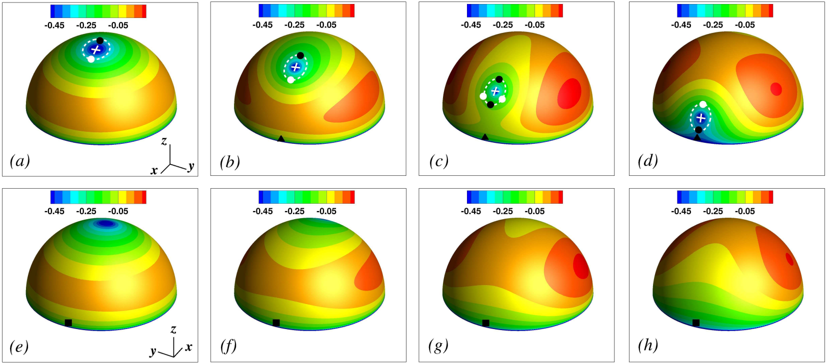


pinned c.l.



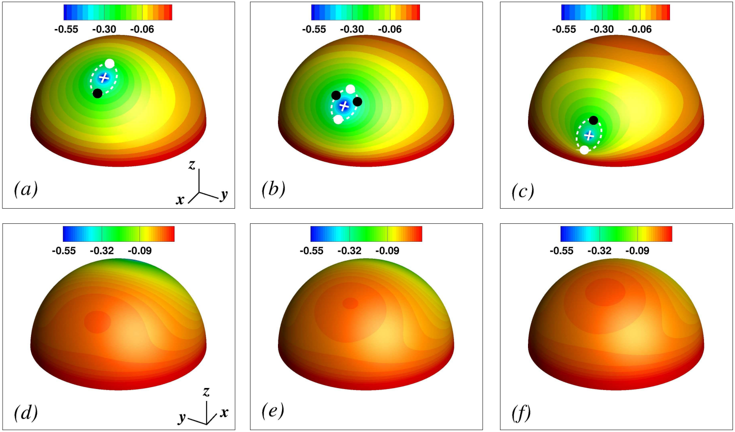
Sessile drops: special case $\theta_0 = \pi/2$

- $\gamma \Delta F_A^{(2)}/(f_1 f_2) =$



Sessile drops: special case $\theta_0 = \pi/2$

- $\gamma \Delta F_B^{(2)}/(f_1 f_2) =$



Summary

- interactions between monopoles and dipoles on spherical interface are non-monotonic and much different than on a flat interface
- interactions between spheroids are quite similar
- importance of curvature only in case of external fields
- the effects of boundary conditions on the substrate for monopoles are long-ranged and independent of R_0
- the effective confining potential depends qualitatively on the boundary conditions
- THANK YOU!

Summary

- interactions between monopoles and dipoles on spherical interface are non-monotonic and much different than on a flat interface
- interactions between spheroids are quite similar
- importance of curvature only in case of external fields
- the effects of boundary conditions on the substrate for monopoles are long-ranged and independent of R_0
- the effective confining potential depends qualitatively on the boundary conditions
- THANK YOU!

Summary

- interactions between monopoles and dipoles on spherical interface are non-monotonic and much different than on a flat interface
- interactions between spheroids are quite similar
- importance of curvature only in case of external fields
- the effects of boundary conditions on the substrate for monopoles are long-ranged and independent of R_0
- the effective confining potential depends qualitatively on the boundary conditions
- THANK YOU!

Summary

- interactions between monopoles and dipoles on spherical interface are non-monotonic and much different than on a flat interface
- interactions between spheroids are quite similar
- importance of curvature only in case of external fields
- the effects of boundary conditions on the substrate for monopoles are long-ranged and independent of R_0
- the effective confining potential depends qualitatively on the boundary conditions
- THANK YOU!

Summary

- interactions between monopoles and dipoles on spherical interface are non-monotonic and much different than on a flat interface
- interactions between spheroids are quite similar
- importance of curvature only in case of external fields
- the effects of boundary conditions on the substrate for monopoles are long-ranged and independent of R_0
- the effective confining potential depends qualitatively on the boundary conditions
- THANK YOU!

Summary

- interactions between monopoles and dipoles on spherical interface are non-monotonic and much different than on a flat interface
- interactions between spheroids are quite similar
- importance of curvature only in case of external fields
- the effects of boundary conditions on the substrate for monopoles are long-ranged and independent of R_0
- the effective confining potential depends qualitatively on the boundary conditions
- THANK YOU!