Poincaré section analysis of an experimental frequency intermittency in an open cavity flow

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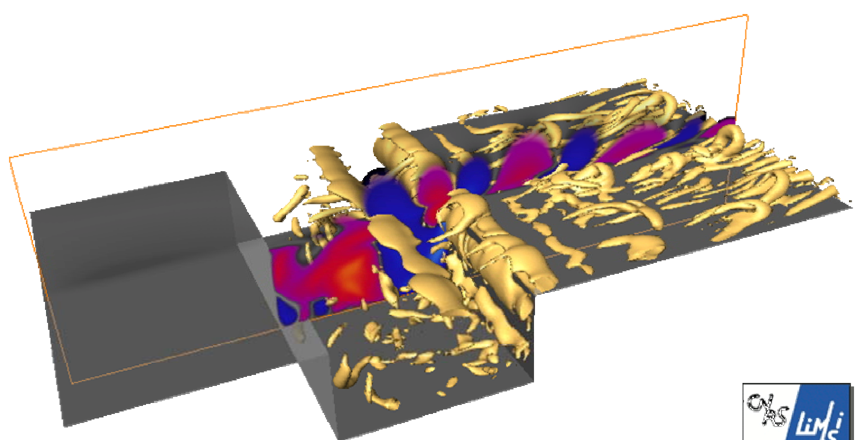
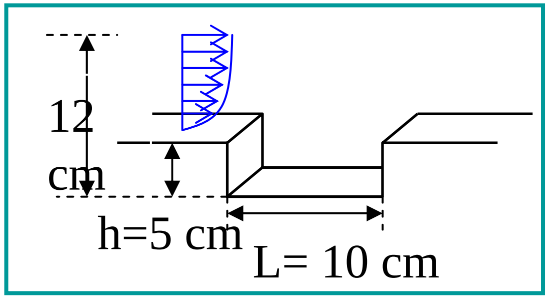
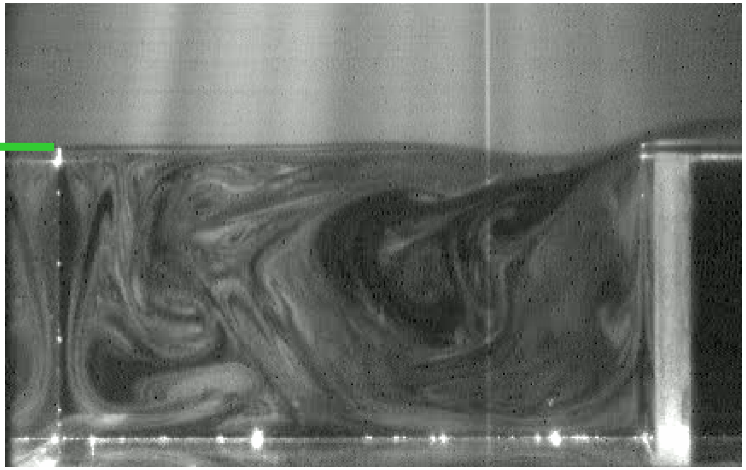


Short flow description

Open cavity



U

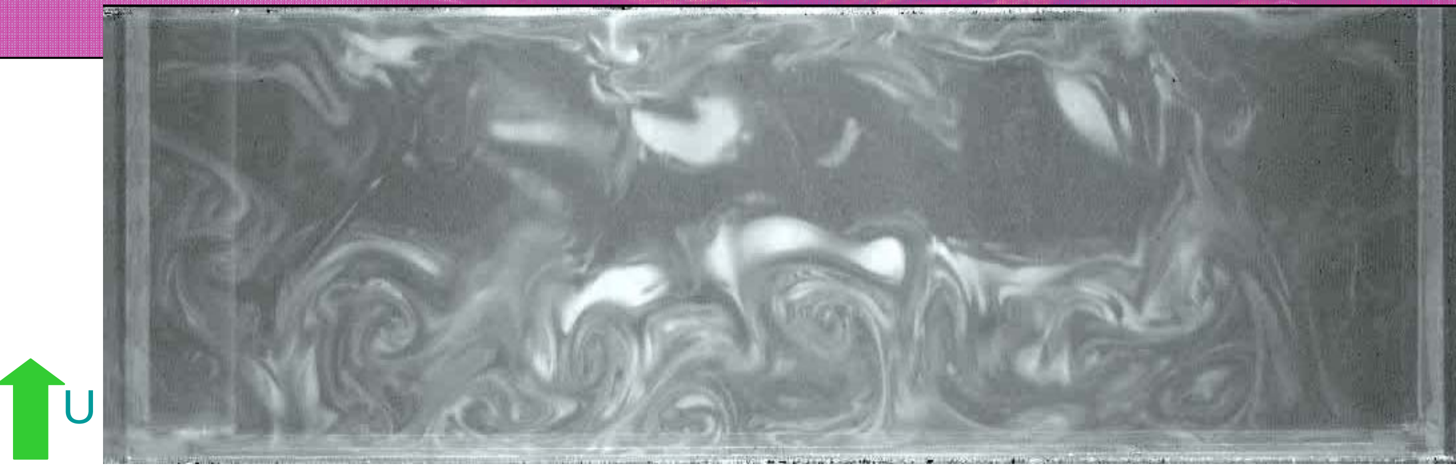


U=1.27 m/s, R=2, → Re=8500

U

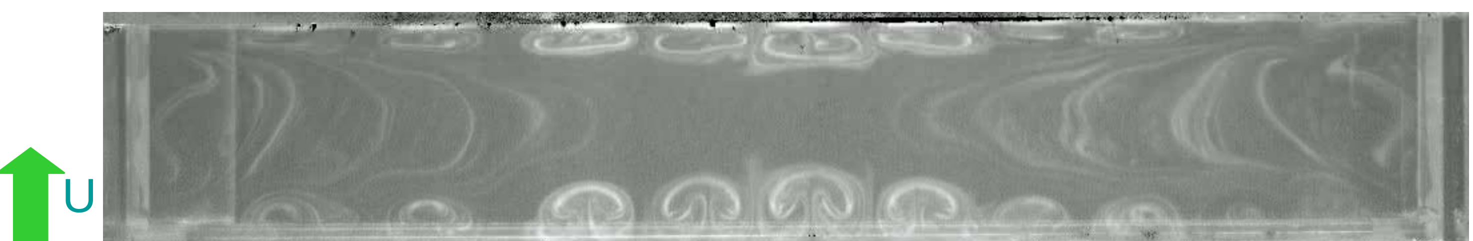
[Exp. Fluids, vol. 42, n°2, pp. 169-184 (2007)]

Description qualitative de l’écoulement en cavité (Exp.)



R=2., U=1.27 m/s
Re=8500

R=1.5 , U=1.27 m/s
Re=6350



R=1., U=1.27 m/s
Re=4200

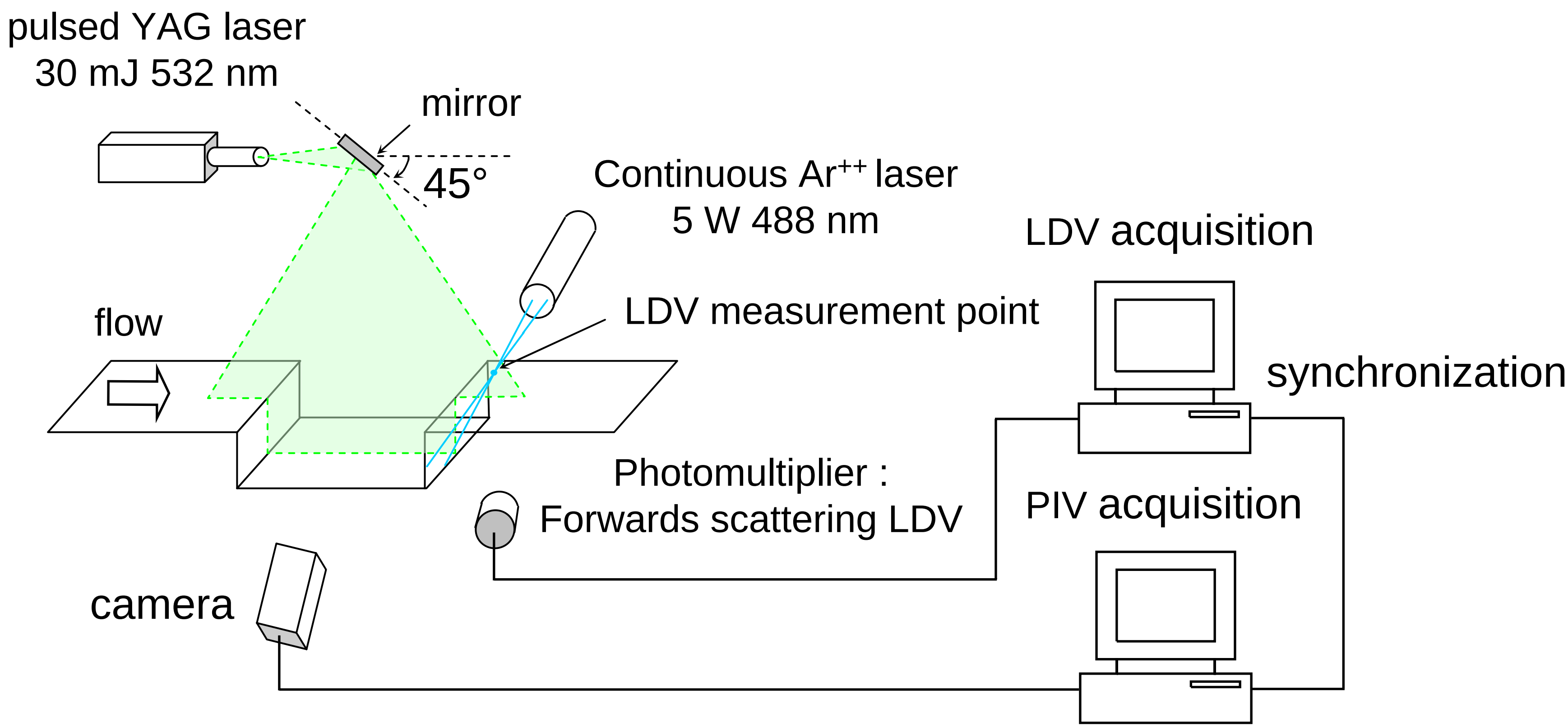
R=0.5 , U=1.27 m/s
Re=2100

Gö

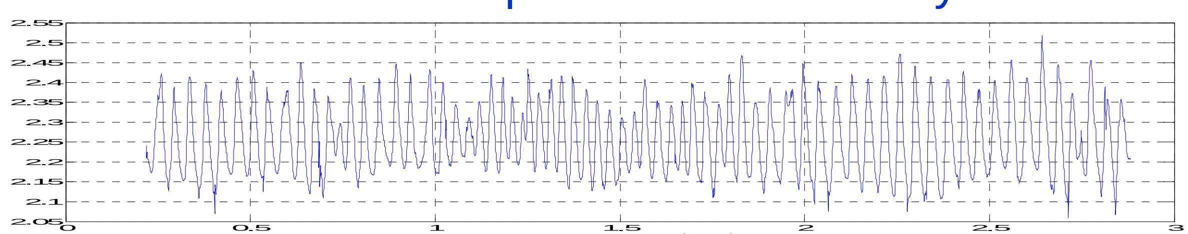
Outline

- Open cavity flow qualitative description
- Measurement setup
- Spectral characterization of the flow dynamics + phases averaging
- **Non-linear Phase portrait characterization of the flow dynamics**
  - Dynamics reduction ← deterministic approach
  - Embedding method, Poincaré section, 1st return maps
  - Symbolic sequences analysis
  - Typical trajectories extraction
- Conclusion

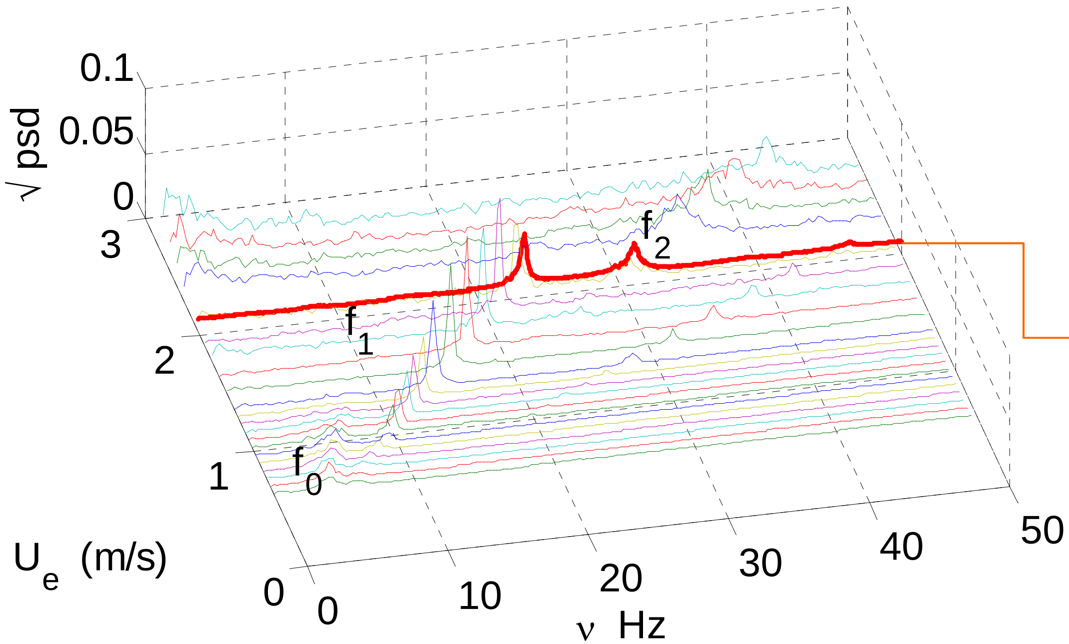
PIV-LDV measurement setup



Time series of the axial component of the velocity



Spectral components

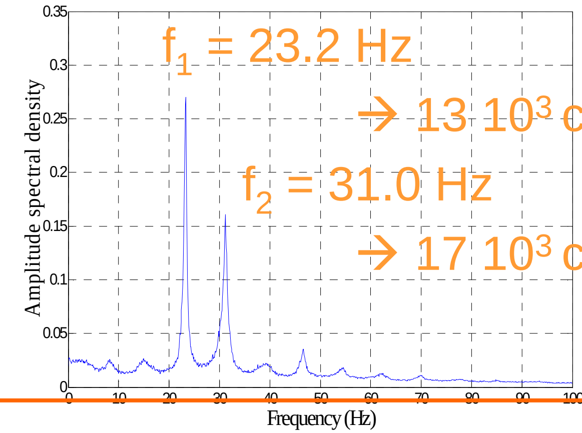


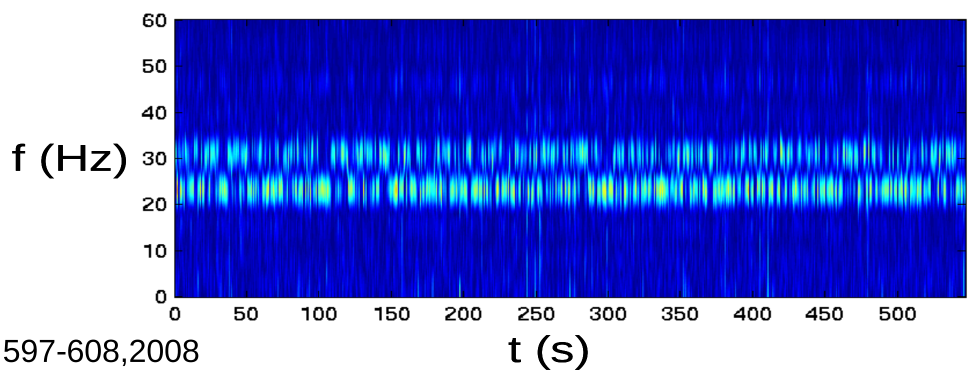
U_e = 2.09 m/s

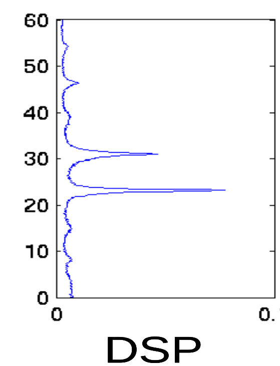
Re= U_eL/ν = 14000

Sampling: f_s=1530 Hz

Samples number : 840000 → ~9mn





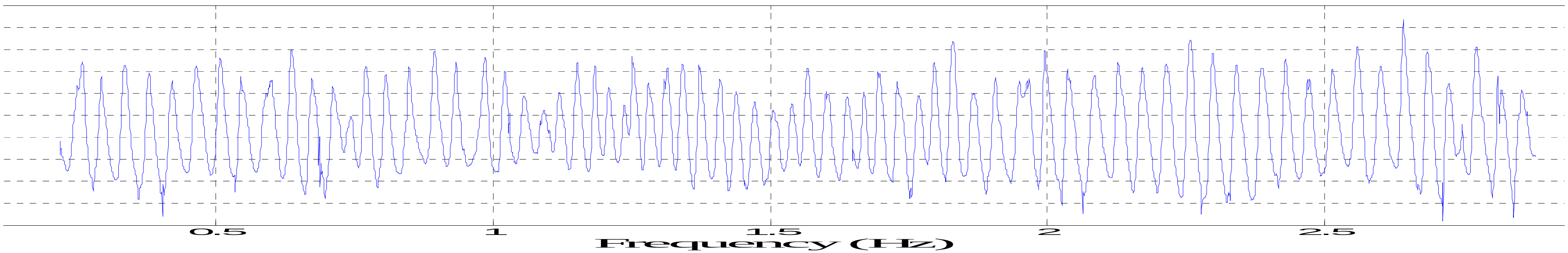


[Exp. Fluids, 44(4),597-608,2008

→ mode switching phenomenon : mode competition.

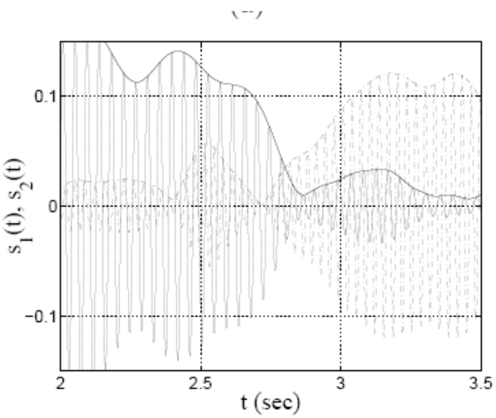
Complex demodulation

Time series of the axial component of the velocity

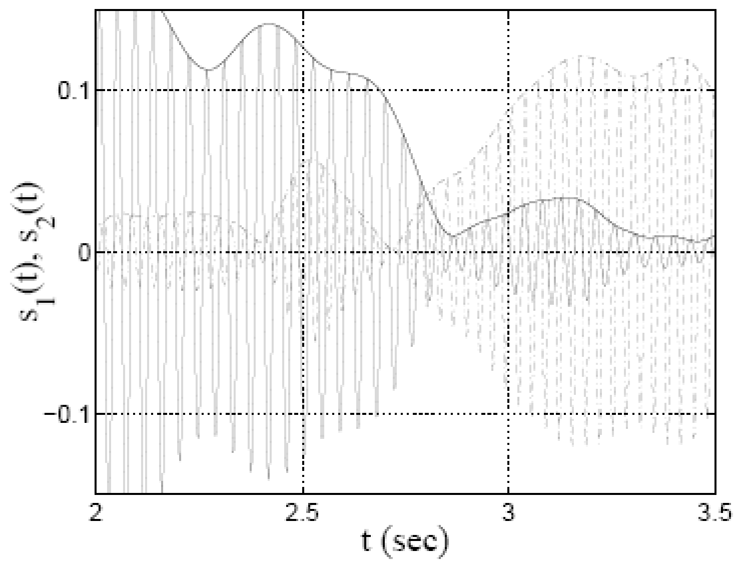


- Band-pass filtering of the signal around the spectral component under interest
- Hilbert Transform of each component :

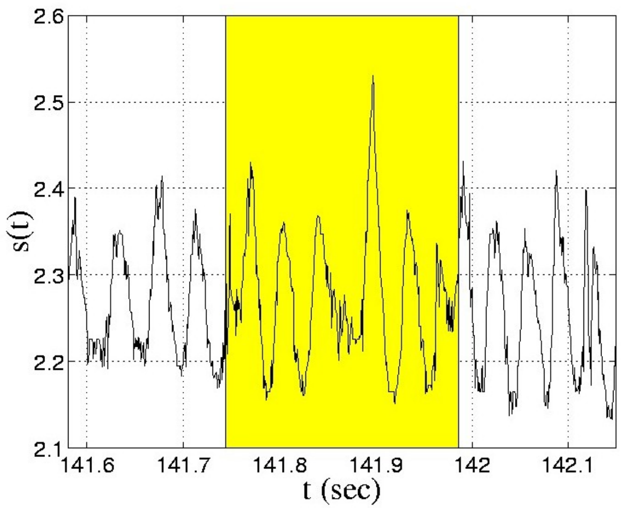
$$\mathcal{H}\{s(t)\} = \frac{1}{\pi t} * s(t) \longrightarrow w(t) = s(t) + i\,\mathcal{H}\{s\}(t) \equiv A(t)\cdot e^{i\phi(t)}$$



•Choice of the separation threshold :



Lost of short events :



Construction moyennes de phases

- rééchantillonnage du signal LDV à une fréquence multiple des champs PIV
- filtrage autour de la fréquence d'un mode (filtre passe-bande largeur 1 Hz)

$$U_x \rightarrow S$$

- construction de la matrice des retards B

$$S = \begin{pmatrix} s(t_1) \\ s(t_2) \\ s(t_3) \\ s(t_4) \\ s(t_5) \\ s(t_6) \\ s(t_7) \end{pmatrix} \rightarrow B = \begin{pmatrix} s(t_1) & s(t_2) & s(t_3) \\ s(t_2) & s(t_3) & s(t_4) \\ s(t_3) & s(t_4) & s(t_5) \\ s(t_4) & s(t_5) & s(t_6) \\ s(t_5) & s(t_6) & s(t_7) \end{pmatrix}$$

- décomposition aux valeurs singulières

$$B = U \cdot D \cdot V^T$$

- matrice de la dynamique propre du système X

$$X = U \cdot D = B \cdot V$$

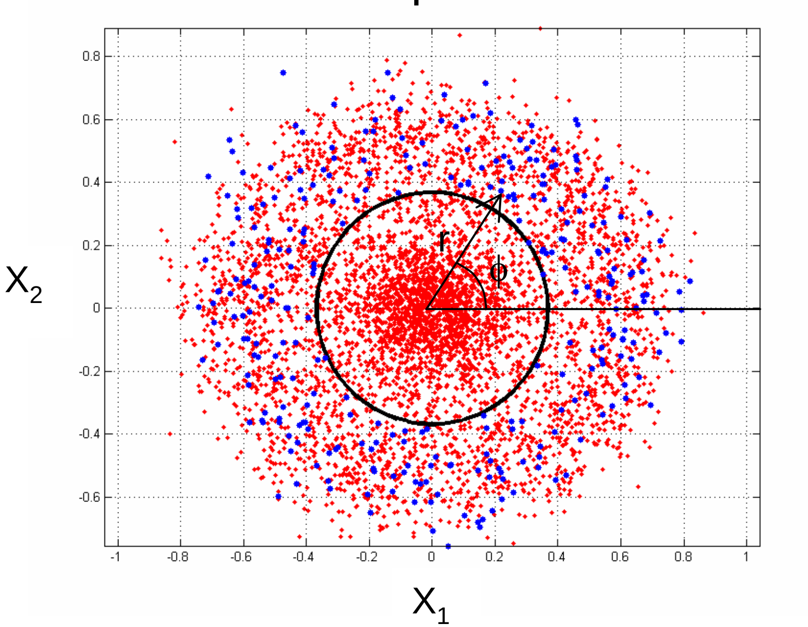
Phases averaging

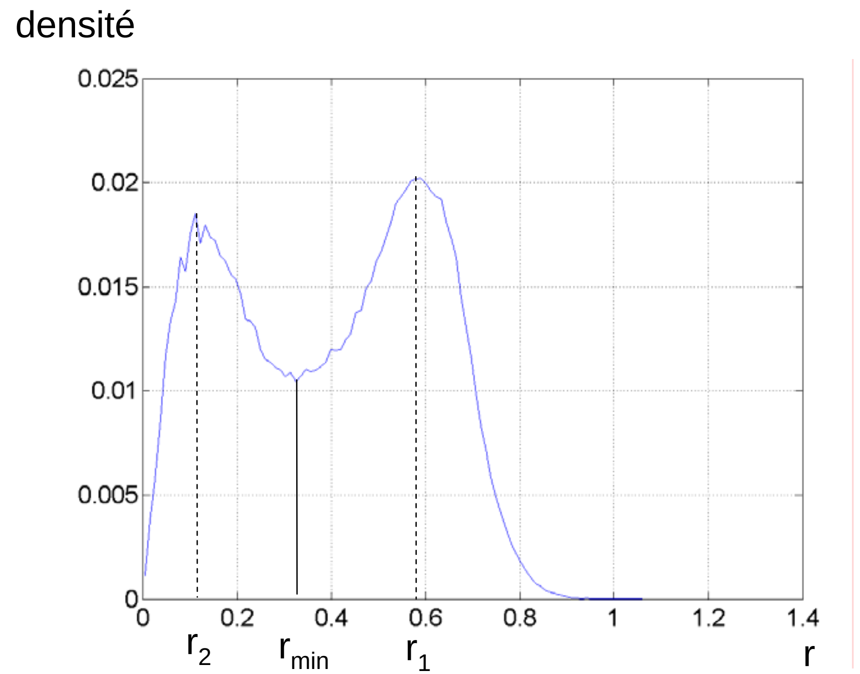
Mesures PIV $U_e = 2{,}09\ m.s^{-1}$

filtrage successif sur chacun des deux modes avant la moyenne par phase

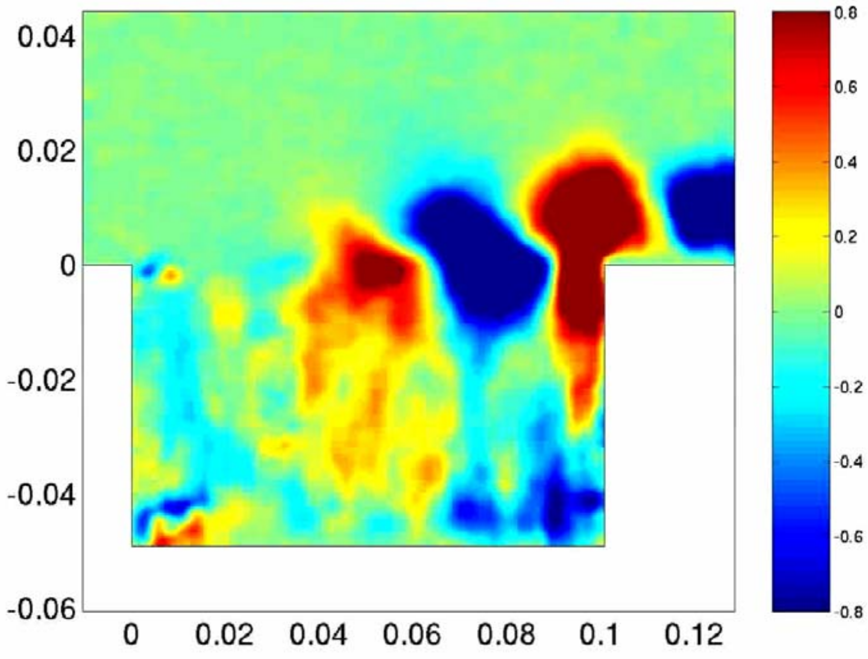
filtrage sur le mode 1 :

- champ PIV

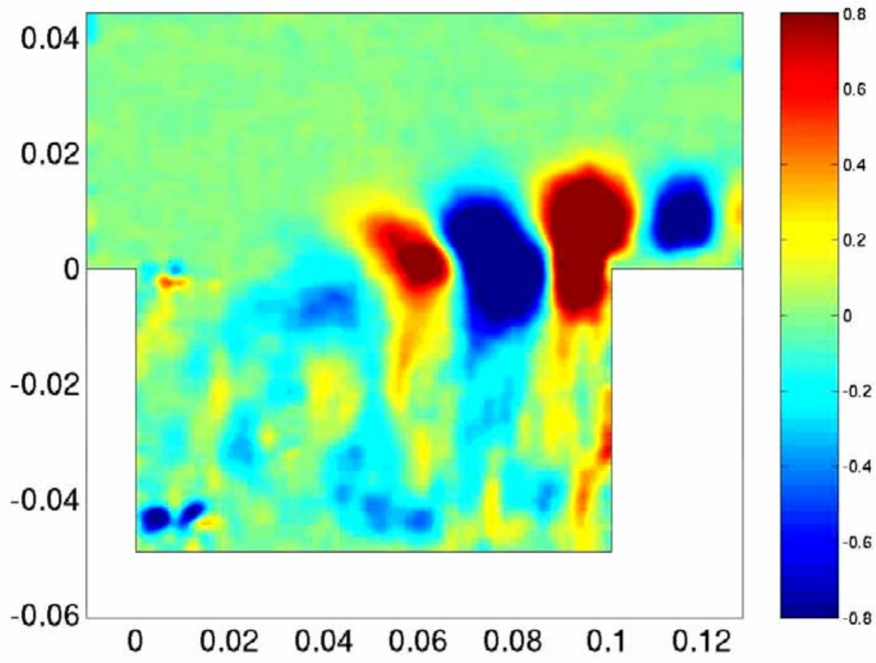




$f_1 = 23.2 Hz$



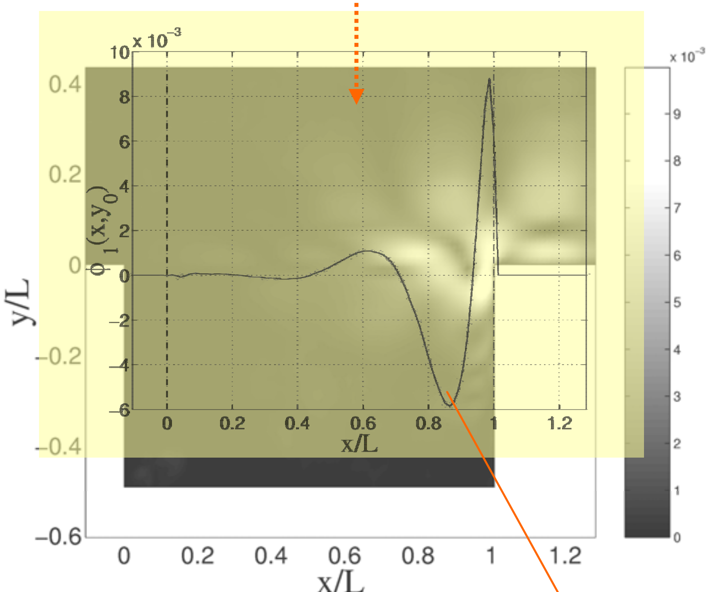
$f_2 = 31.0 Hz$

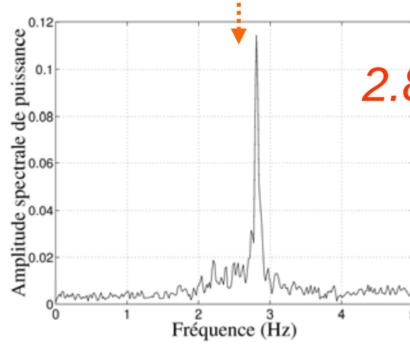


POD pour s1

POD :

$$\vec{u}(x,y,t_i)=\sum_{n=1}^{N} a_n(t_i)\,\vec{\phi}_n(x,y)$$





2.8+10=12.8 Hz

$$u_y(x)=A+Be^{\beta x}\cos\left(\frac{2\pi}{\lambda}x+\varphi\right)$$

Phase portrait characterization

Non-linear Phase portrait characterization of the flow dynamics

- Dynamics reduction ← deterministic approach
- Embedding method, Poincaré section, 1st return maps
- Symbolic sequences analysis
- Typical trajectories extraction

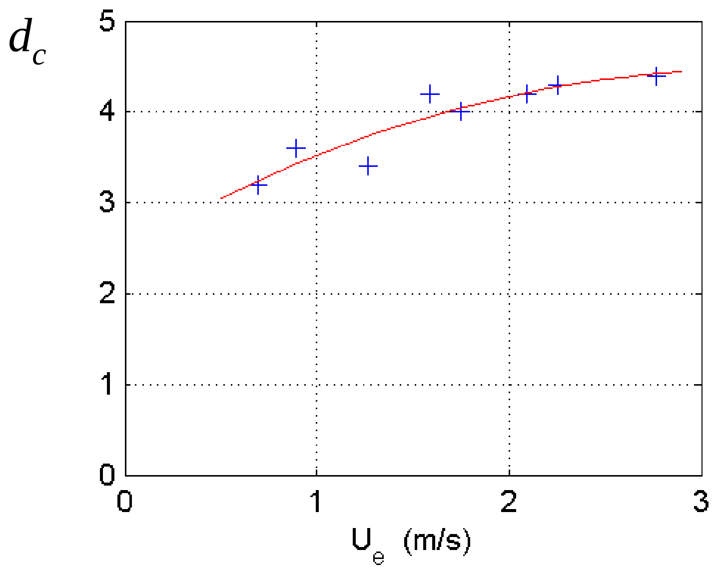
Dynamics reduction

Underlying dynamical system : $\dot{\vec{X}} = F(\vec{X})$

→ Measure of correlation dimension (Procaccia1988) :

$$d_c = \lim_{N\to\infty} \lim_{r\to 0} \frac{\log_2 C(r)}{\log_2 r} \qquad \text{with} \quad C(r) = \frac{1}{N_{ref}} \frac{1}{N} \sum_{i=1}^{N_{ref}} \sum_{j=1}^{N} H(r - \|\vec{x}_i - \vec{x}_j\|)$$

on LDV series, after non-linear filtering (T. Schreiber PRE 47, 1993).



Phases portrait dimension :

$$d_c = 4.2 \quad \text{at} \quad U = 2.09\ m/s$$

Embedding space dimension :

$$5 \le d_e \le 10$$

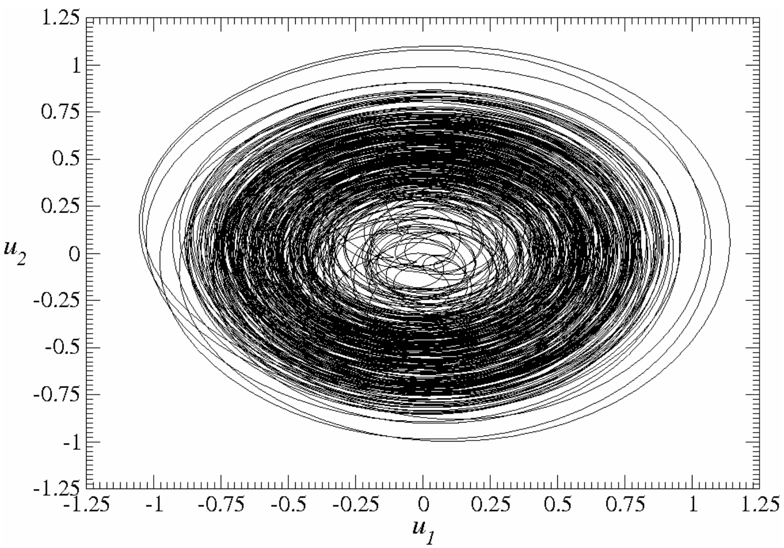
Embedding method

1 - delays matrix : $$S = \begin{pmatrix} s(t_1) & s(t_2) & \cdots & s(t_m) \\ s(t_2) & s(t_3) & \cdots & s(t_{m+1}) \\ \vdots & \vdots & \vdots & \vdots \\ s(t_{N-m+1}) & s(t_{N-m+2}) & \cdots & s(t_N) \end{pmatrix} \quad with \quad \begin{cases} N = 840000 \\ m = 70 \end{cases}$$

2 - singular value decomposition (SVD) : $S = U.\Sigma.V^t$ with $U = \{u_1, u_2, \cdots, u_m\}$

U is an orthonormal basis.

Phases portrait projection on the two first principal components

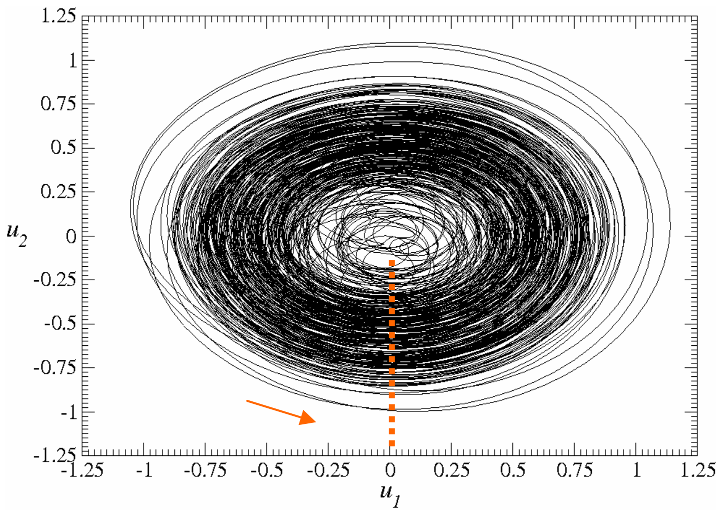


D. S. Broomhead & G. P. King, Extracting qualitative dynamics from experimental data, Physica D, 20, 1986.

1^{st} return map

Poincaré section :

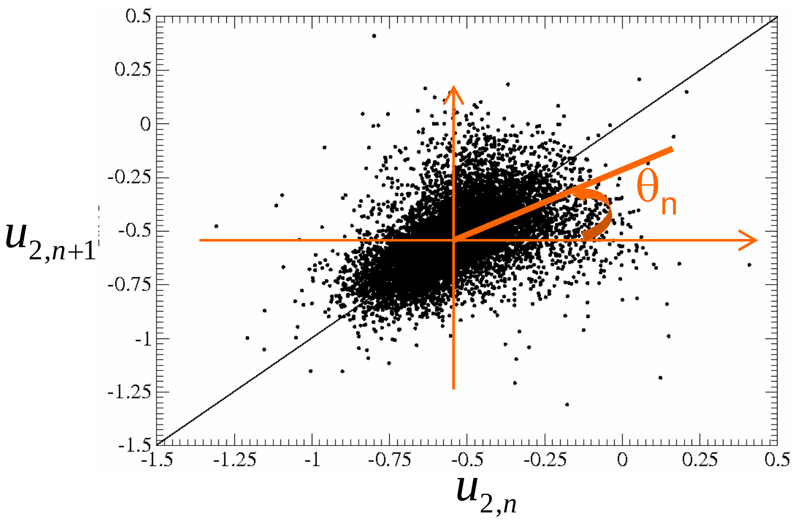
$$\Pi = \{ u_2 \in \mathbf{R}^2 \mid u_1 = 0, \dot{u}_1 > 0 \}$$



First return map :

$$u_{2,n+1} = f(u_{2,n})$$

$$\{u_{2,n}\}_{n=1,\ldots,K} \text{ with } K = 31000$$



1^{st} angular return map and symbolic dynamics

First angular return map :

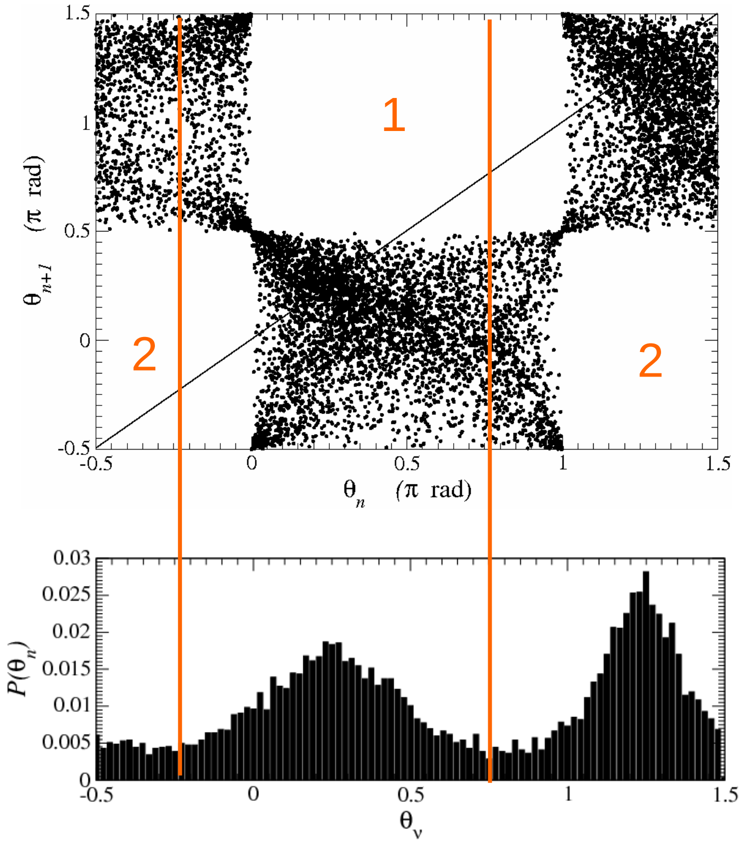
$$\theta_{n+1} = f\left(\theta_n\right)$$

Partition of first angular return map :

→ encoding in a sequence $\Sigma = \{\sigma_n\}$

$$\sigma_n \left| \begin{array}{l} 2 \quad \text{if} \quad \theta_n \in [-\pi/4; 3\pi/4] \\ 1 \quad \text{if} \quad \theta_n \in [-\pi/2; -\pi/4[\bigcup [3\pi/4; 3\pi/2[\end{array} \right.$$

- locked dynamics : $\ldots 1111 \ldots$ or $\ldots 2222 \ldots$
- transitional dynamics : $\ldots 212112122 \ldots$



orbit time distribution of each modes 1/2

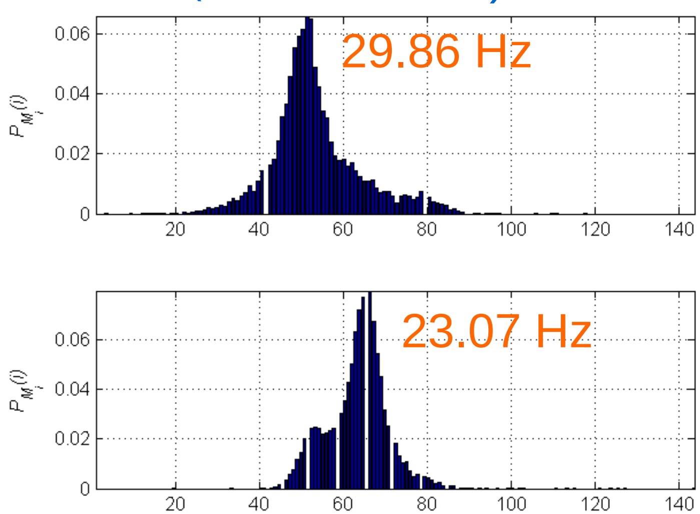
Correspondences between modes and frequencies

Orbit mean time:

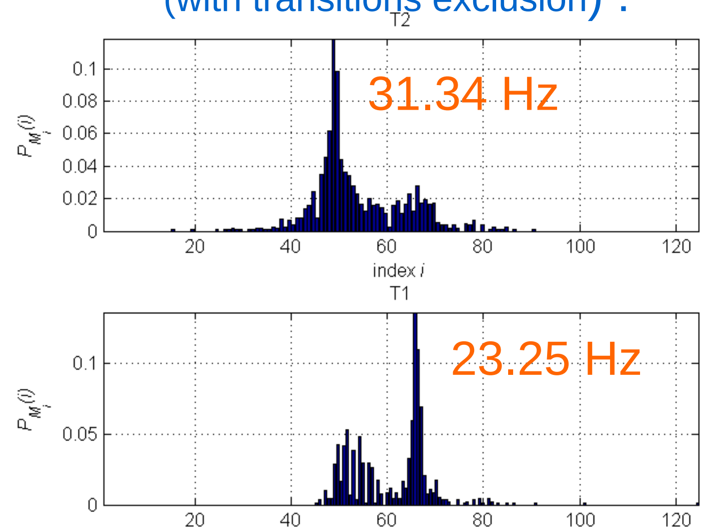
Orbits 11or 12 → 0.0423 s → f_1 = 24.13 Hz (PSD : 23.2 Hz)

Orbits 22 or 21 → 0.0335 s → f_2 = 28.77 Hz (PSD : 31.0 Hz)

Orbit time distribution (whole distribution) :

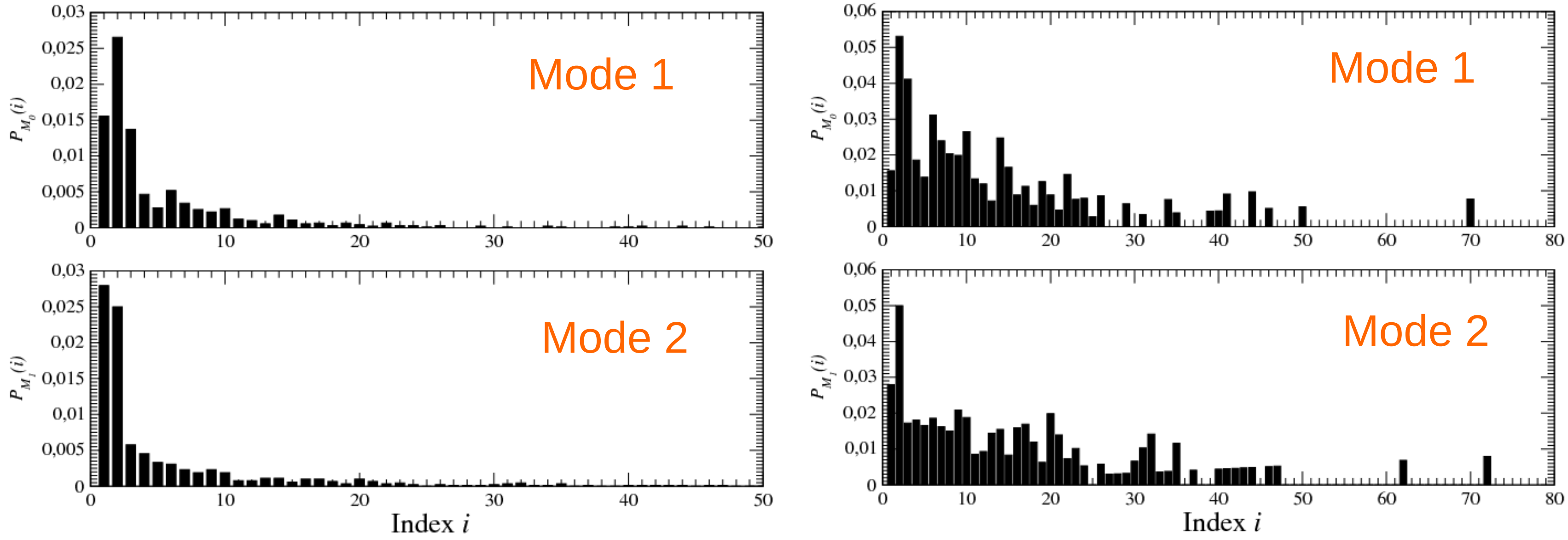


Orbit time distribution (with transitions exclusion) :



Probability in consecutive events numbers

Probability in time duration



→symbols (2 or 1) repeated most often 3 or 4 times

→rare long sequences give a significant temporal contribution

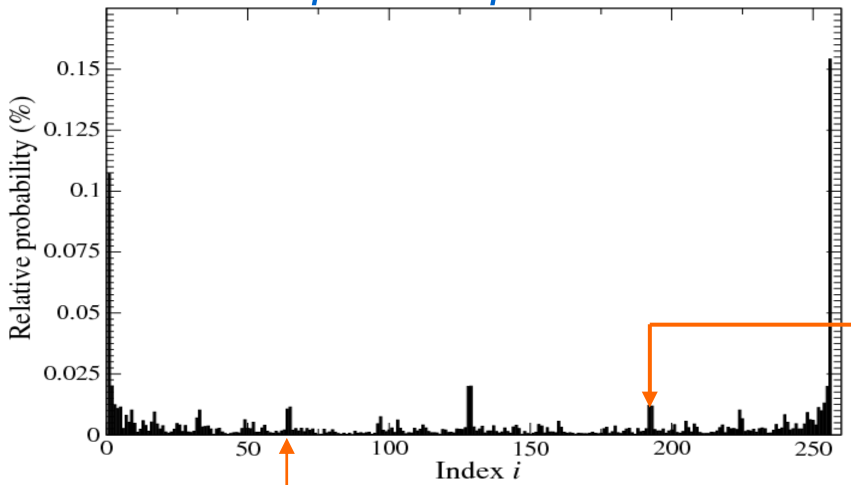
Decimal encoding of n symbols sequences :

211222222211211222111122 → 011000000011011000111100

Bin2dec of n=8 symbols → 96

→ the symbolic sequence Σ_i with i='encoding'+1

Sequences probabilities



•Main sequences:

$\Sigma_1 = 2222\ 2222$ and $\Sigma_{256} = 1111\ 1111$

→ preponderance for sustaining modes 1 & 2

•Isolated sequences from the back ground when $P > 0.017$:

$$
\begin{array}{ll}
\Sigma_{128} = 2111\ 1111 & \Sigma_{129} = 1222\ 2222 \\
\Sigma_{193} = 1211\ 1111 & \Sigma_{64} = 2122\ 2222 \\
\Sigma_{253} = 1111\ 1122 & \Sigma_{4} = 2222\ 2211 \\
\Sigma_{255} = 1111\ 1112 & \Sigma_{2} = 2222\ 2221
\end{array}
$$

$$
\begin{array}{ll}
P_{128} = 0.022 & P_{129} = 0.023 \\
P_{193} = 0.018 & P_{64} = 0.019 \\
P_{253} = 0.018 & P_{4} = 0.017 \\
P_{255} = 0.022 & P_{2} = 0.023
\end{array}
$$

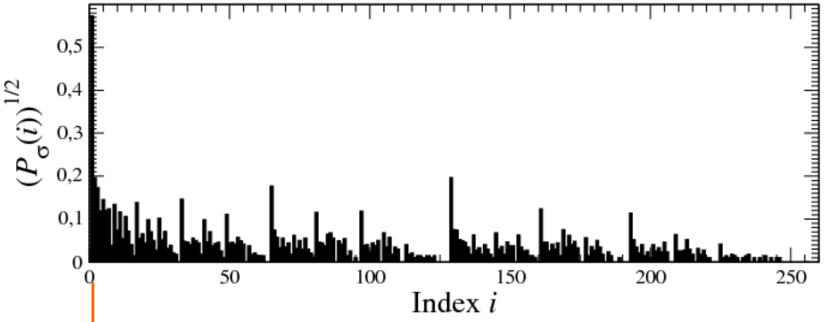
Transitional symbolic sequence Ξ_i

New encoding of the transition:

- *R* for repetition
- *T* transition

$$\xi_i \left| \begin{array}{lll} \mathrm{R} & \text{if} & \sigma_i\sigma_{i+1} = 22 \text{ or } \sigma_i\sigma_{i+1} = 11 \\ \mathrm{T} & \text{if} & \sigma_i\sigma_{i+1} = 12 \text{ or } \sigma_i\sigma_{i+1} = 21 \end{array} \right.$$

probabilities transitional sequences Ξ_i



Ξ_1 =RRRR RRRR

→ Repetition prevails

Repetition sequences longer than 8 :

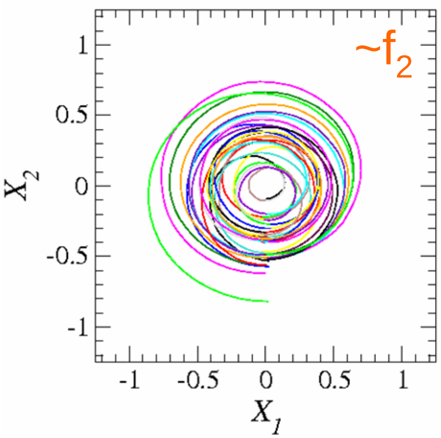
$$\Xi_{129} = \mathrm{TRRR\ RRRR}$$
$$\Xi_{65} = \mathrm{RTRR\ RRRR}$$
$$\Xi_{33} = \mathrm{RRTR\ RRRR}$$
$$\Xi_{17} = \mathrm{RRRT\ RRRR}$$
$$\Xi_{9} = \mathrm{RRRR\ TRRR}$$
$$\Xi_{5} = \mathrm{RRRR\ RTRR}$$
$$\Xi_{3} = \mathrm{RRRR\ RRTR}$$
$$\Xi_{2} = \mathrm{RRRR\ RRRT}$$

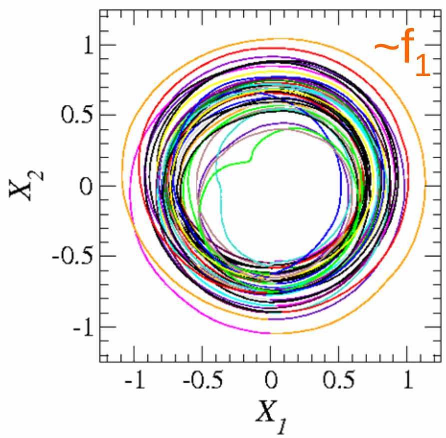
$$\vdots$$

$$\Xi_{193} = \mathrm{TTRR\ RRRR}$$
$$\Xi_{161} = \mathrm{TRTR\ RRRR}$$
$$\Xi_{97} = \mathrm{RTTR\ RRRR}$$
$$\Xi_{81} = \mathrm{RTRT\ RRRR}$$
$$\Xi_{49} = \mathrm{RRTT\ RRRR}$$

→ Transitions are mainly short exploration and coming back to the same mode.

Trajectories associated with mode 2 and 1:



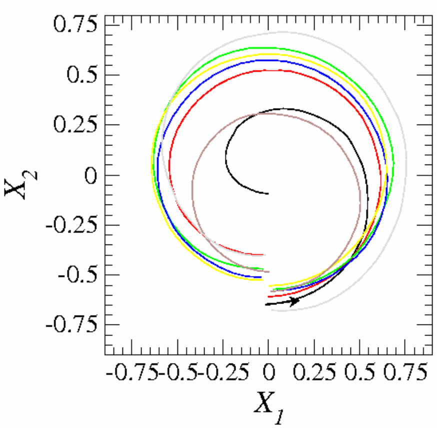


amplitude of f_1 > amplitude of f_2:

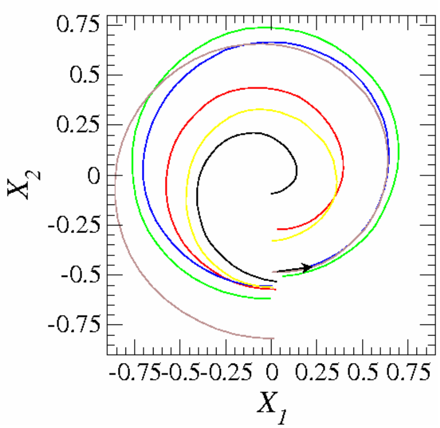
→ dynamics structured around a fixed point of the focus type.

Trajectories associated with a transition from :

1→2



2→1



→ Confirm that the transition mainly occur in a single oscillation (between two successive intersection with the Poincaré section).

Summary & Conclusion

- Investigation from temporal series of the dynamics underlying an open flow over a cavity,
- nonlinear competition between two modes is investigated using tools of to the nonlinear dynamical systems theory,
- After embedding of time series, an angular return map allows to define a symbolic dynamic with two symbols (distinguish the two modes in competition),
  - → The dynamics governing the mode switching is mainly deterministic,
  - → The dynamics behaves as structured by a focus type fixed point,
  - → The switching process is either 'long' reminding on one mode or short exploration of the other.

- Which flows are relevant for such a time analysis?
- What about the physics of intermittency in no compressible open cavity flow ?

[Physics of Fluids (2008), accepted, to be published]

END