Comparison of selected methods of modelling of heat transfer in perfused biological tissue

M. Stańczyk

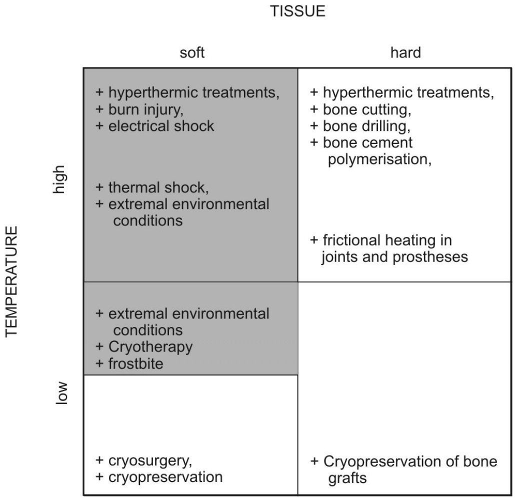
Institute of Fundamental Technological Research
Polish Academy of Sciences
Warsaw

March 1, 2005

Overview

1. heat transfer in perfused tissues—where it is important?
2. existing modelling techniques,
3. the DIscrete VAsculature modelling program (DIVA),
4. alternate method to model counter-current vessels (GRID),
5. DIVA↔GRID comparison: assumptions, advantages, limitations,
6. test simulation results,
7. conclusions.

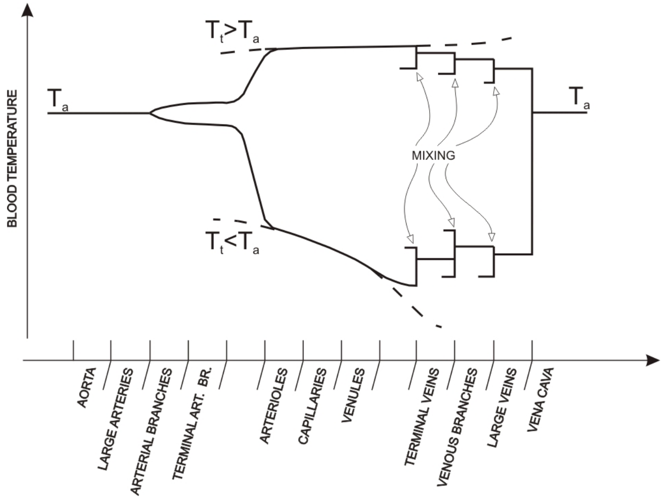
Heat Transport in Perfused Tissues



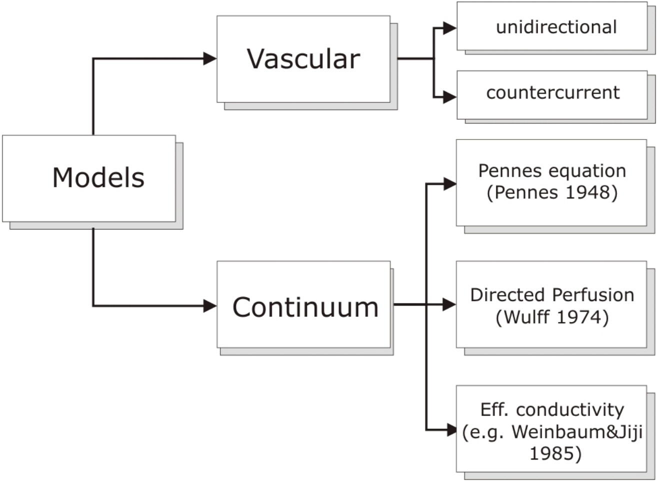
Reasons for interest:

1. evaluation of effectiveness of hyperthermic treatments and the assessment of the danger involved,
2. assessment of the heat dissipation rate in soft tissues,
3. investigations of thermoregulatory mechanisms,
4. others.

Blood Temperatures throughout the Vascular Tree



Modelling Techniques



Continuum Models

Pennes Equation (Pennes 1948):

$$\rho_\mathrm{t} c_\mathrm{t} \frac{\partial T_\mathrm{t}}{\partial t} = \lambda_\mathrm{t} \nabla^2 T_\mathrm{t} + w_\mathrm{bl} c_\mathrm{bl} (T_\mathrm{a} - T_\mathrm{t}) + q_\mathrm{m}$$

Directed Perfusion Model (Wulff 1974):

$$\rho_\mathrm{t} c_\mathrm{t} \frac{\partial T_\mathrm{t}}{\partial t} = \lambda_\mathrm{t} \nabla^2 T_\mathrm{t} - \rho_\mathrm{bl} c_\mathrm{bl} \mathbf{U} \cdot \nabla T_\mathrm{t} + q_\mathrm{m}$$

Effective Conductivity Models (e.g. Weinbaum-Jiji 1985 here):

$$\overline{\rho c} \frac{\partial T_\mathrm{t}}{\partial t} = \nabla \cdot (\boldsymbol{\lambda}_\mathrm{eff} \nabla T_\mathrm{t}) - \frac{n \lambda_\mathrm{t}}{\sigma} \left(\frac{\pi r \lambda_\mathrm{bl} \mathrm{Pe}}{2 \lambda_\mathrm{t}} \right)^2 (\nabla \cdot \mathbf{l})(\mathbf{l} \cdot \nabla T_\mathrm{t}) + q_m$$

Features of Continuum Models

They:

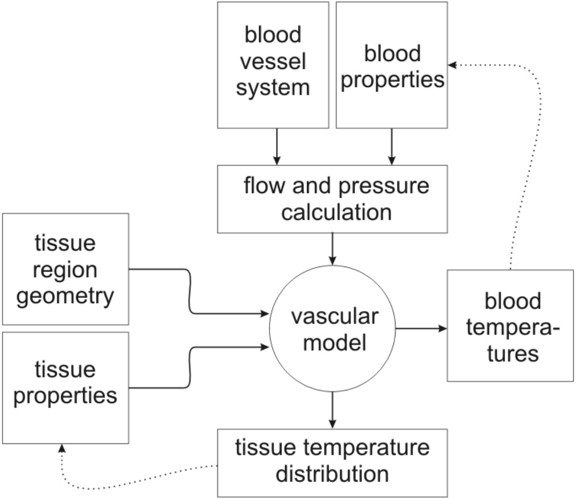
1. describe the heat flow in terms of a single temperature T_{t},
2. rest on strong assumptions concerning the blood-tissue thermal equilibration,
3. will often yield erroneous results whenever thermal loading deviates significantly from normothermia,
4. require a number of model-specific parameters to be fitted to experimental results,
5. are unable to describe the local effects of large blood vessels.

Features of Vascular Models

They:

1. describe the heat flow in terms of a blood temperature defined in the vascular system domain and tissue temperature defined in the tissue region,
2. blood-tissue thermal equilibration results from the model,
3. describe the local effects of large blood vessels.

Vascular Modelling Concept



DIVA Overview

- created in 1994–1998 by team of 4–8 people in Utrecht University, works on adapting DIVA to mass transfer problems are still going on today,
- capable of 3D simulations in arbitrary regions,
- several extensions exist: simulation of ferromagnetic seeds and electrodes used for hyperthermic treatment,
- numerous applications,
- implemented in C++ (around 1 MB of OO code),
- makes use of Hierarchical Document Format library developed at NCSA (USA),

Bibliography:

A. Kotte, *Design of a numerical model for describing the heat transfer due to vascular trees*, Ph.D. thesis, Utrecht University, 1998.

G. van Leeuven, *Numerical modelling of heat transfer in hyperthermia*, Ph.D. thesis, Utrecht University, 1998.

J. deBree, *A 3-D anatomy based treatment planning system for interstitial hyperthermia*, Ph.D. thesis, Utrecht University, 1998.

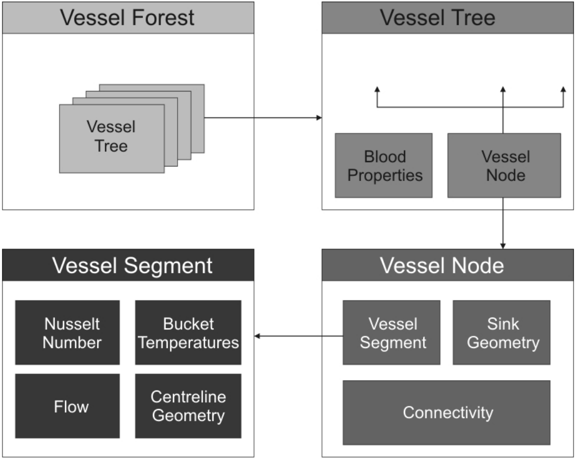
Basic Data Structures

Type	Source	Storage
Tissue Anatomy	CT/MRI/prescribed	HDF file
Material Data	prescribed	GOF file
Vasculature Structure	CT/MRI/computer generation/prescribed	GOF file
Blood Flow	prescribed	GOF file

HDF: Hierarchical Data Format (NCSA, USA), standarized binary file, access from C/C++/Fortran via HDF library function calls (freely available, documentation exists)

GOF: Generic Object Format (Utrecht University, the Netherlands), textual file, documentation available

Vascular Tree Description



Blood Temperature Determination Method

Radial heat balance around infinite circular vessel:

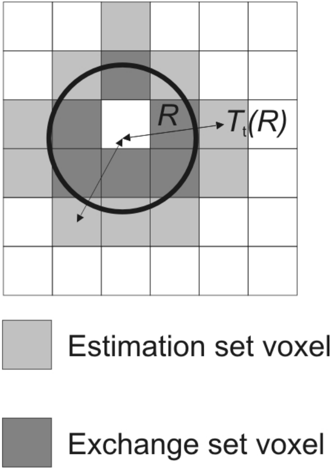
$$\rho_t c_t \frac{\partial T_t}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r \frac{\partial T_t}{\partial r}\right) + P$$

Convective heat flux from blood to the vessel wall:

$$q\Big|_{r=r_{ves}} = -\frac{\mathrm{Nu}\lambda_{bl}}{2r_{ves}}(T_t(r_{ves}) - T_{blood})$$

Therefore, assuming $T_t(R) = \text{const.}$ for some R one can express $q\Big|_{r=r_{ves}}$ in terms of $T_t(R)$, Nu, λ, λ_{bl}, R, r_{ves}...

Estimation of Vessel Wall Flux



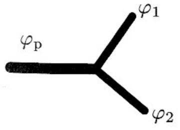
For each vascular bucket two tissue voxel sets are maintained:

- **exchange set**: where heat to/from the bucket is delivered,
- **estimation set**: whose temperatures serve as $T_\mathrm{t}(R)$ to calculate the vessel wall heat flux estimation,

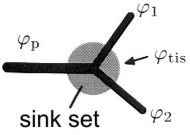
All sets are determined in run-time

Bleed-off

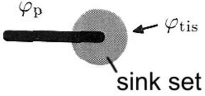
No mass balance at vessel junctions is enforced, any possible imbalance contributes to the perfusion in the appropriate voxel sink set (giving rise to the local Pennes-like term)



(a) balanced inflows and outflows from a junction



(b) flow imbalance leads to Pennes-like heating of the sink set voxels

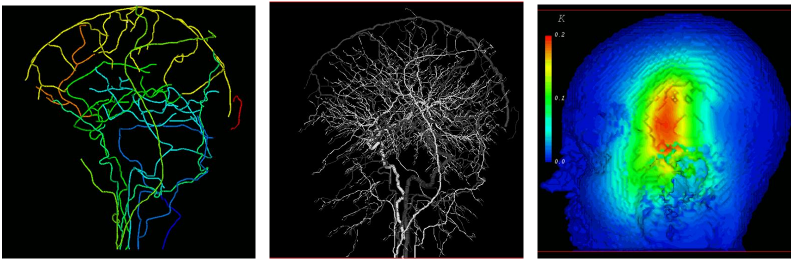


(c) the flow into the terminal vessel is entirely balanced by the bleed-off into the tissue

after A. Kotte, *Design of a numerical model for describing the heat transfer due to vascular trees*, Ph.D. thesis, 1998.

Example Applications

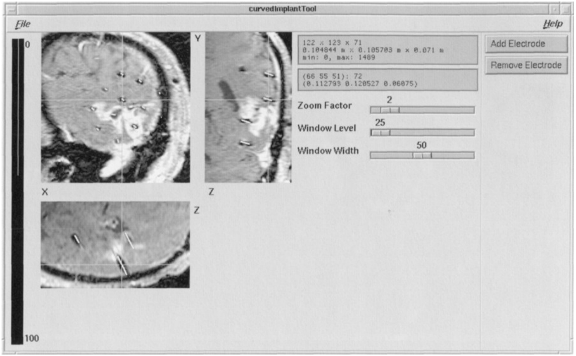
Analysis of the head temperature rise due to cellular phone

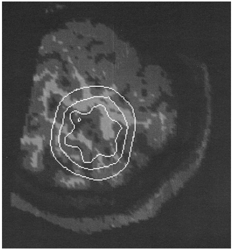


after G. van Leeuven, 1999

Example Applications

A 3-D anatomy based treatment planning system for interstitial hyperthermia





after Jacob de Bree, Ph.D. thesis 1998

GRID Overview

- created in 2005 in IFTR PAS by a single person,
- capable of limited range of simulations for a 2D region,
- implemented in C++ (around 100 kB of OO code),
- rudimentary GUI implemented with the use of Qt library.

Bibliography:

M. Stańczyk, *Vascular model of heat transfer in perfused tissue*, [in:] Blood Flow Modelling and Diagnostics, T.A. Kowalewski [ed.], Abiomed Lecture Notes, **6** : 451–487

Basic Data Structures

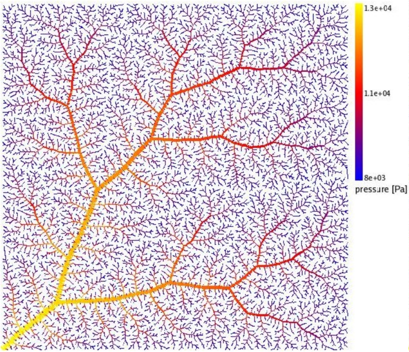
Type	**Source**	**Storage**
Tissue Anatomy	generic, fixed	—
Tissue Mesh	generic, computer generated	`grid` file
Material Data	prescribed at runtime	—
Vasculature Structure	computer generated	`tree` file
Blood Flow	computer generated	`tree` and `grid` files

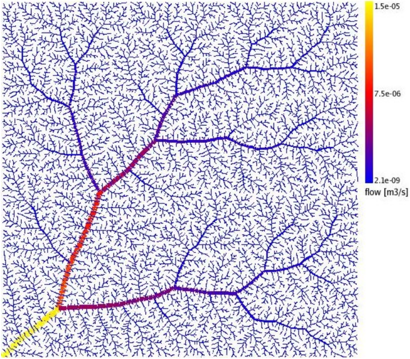
tree: dedicated textual file format for storage of vascular tree information,

grid: dedicated textual file format for storage of meshed vascular tree information, along with the finite element tissue mesh: node coordinates and connectivity.

Initial calculations

Blood flows and pressures are calculated once (assuming no deformation of vessels or temperature-dependence is present) at the stage of vascular tree generation





Estimation of Vessel-Tissue Heat Flow Rate

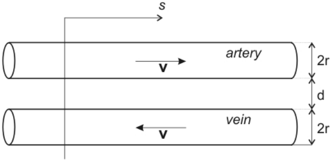
Energy balance equation for a single vessel:

$$\underbrace{\pi r^2 \frac{\partial E(s)}{\partial t}}_{\text{inertia}} = \underbrace{2\pi r q_{\mathrm{w}}(s)}_{\text{wall flux}} - \underbrace{\pi r^2 \frac{\partial q(s)}{\partial s}}_{\text{axial conduction}} - \underbrace{\pi r^2 \rho_{\mathrm{bl}} c_{\mathrm{bl}} v \frac{\partial T(s)}{\partial s}}_{\text{advection}}$$

Wall flux is decomposed into vessel-vessel and vessel-tissue fluxes:

$$q_{\mathrm{w}} = \underbrace{\bar{\sigma}_{\mathrm{cc}} \lambda_{\mathrm{t}} (T_{\mathrm{cc}}(s) - T(s))}_{\text{vessel-vessel flux}} + \underbrace{\bar{\sigma}_{\mathrm{t}} \lambda_{\mathrm{t}} (T_{\mathrm{t}}(s) - T(s))}_{\text{vessel-tissue flux}}$$

Model Equations



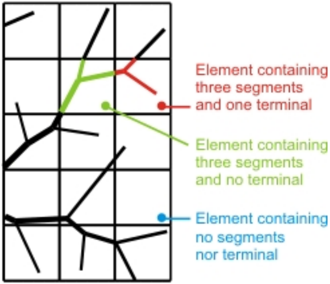
Artery Temperature:

$$\rho_{\mathrm{bl}} c_{\mathrm{bl}} \frac{\partial T_{\mathrm{a}}}{\partial t} = \sigma_{\mathrm{cc}} \lambda_{\mathrm{t}} (T_{\mathrm{v}} - T_{\mathrm{a}}) + \sigma_{\mathrm{t}} \lambda_{\mathrm{t}} (T_{\mathrm{t}} - T_{\mathrm{a}}) - \rho_{\mathrm{bl}} c_{\mathrm{bl}} v \frac{\partial T_{\mathrm{a}}}{\partial s}$$

Vein Temperature:

$$\rho_{\mathrm{bl}} c_{\mathrm{bl}} \frac{\partial T_{\mathrm{v}}}{\partial t} = \sigma_{\mathrm{cc}} \lambda_{\mathrm{t}} (T_{\mathrm{a}} - T_{\mathrm{v}}) + \sigma_{\mathrm{t}} \lambda_{\mathrm{t}} (T_{\mathrm{t}} - T_{\mathrm{v}}) + \rho_{\mathrm{bl}} c_{\mathrm{bl}} v \frac{\partial T_{\mathrm{v}}}{\partial s}$$

Tissue Temperature Estimation



The vessel pair-tissue heat flux per unit length of the segment:

$$\hat{q}_{\mathrm{l}} = 2\sigma_{\mathrm{t}}\lambda_{\mathrm{t}}\left(\frac{T_{\mathrm{a}} + T_{\mathrm{v}}}{2} - T_{\mathrm{t}}\right)$$

The vessel pair-tissue heat flux per single i'th terminal:

$$\hat{q}_{\mathrm{term}} = \pi\rho_{\mathrm{bl}}c_{\mathrm{bl}}r_i^2 v_i(T_{\mathrm{a}}^e - T_{\mathrm{v}}^e)$$

Tissue Temperature Estimation

Tissue temperature is described by the usual heat conduction equation:

$$\rho_t c_t \phi_t \frac{\partial T_t}{\partial t} = \lambda_t \nabla^2 T_t + q_m + q_{bl}$$

ϕ_t: volumetric tissue fraction,
q_{bl}: volumetric blood heat, calculated for each kth element of the mesh

$$q_{bl}(x)\big|_{x \in \text{element}_k} = \frac{1}{V^{(k)}} \sum_{\substack{i=\text{segment} \\ i \in \text{element}_k}} \left(\int_{\text{start}_i}^{\text{end}_i} \hat{q}_l^{(i)} dl + \hat{q}_{\text{term}}^{(i)} \chi_{\text{term}}(i) \right)$$

where $\chi_{\text{term}}(i) = 1$ if i'th segment is a terminal and 0 otherwise

Model similarities

- models include representation of specific vasculature of the tissue region,
- separate geometrical representation and numerical discretization of the vessels and the tissue,
- complete blood equilibration in terminal vessels (in GRID by design, in DIVA by necessity),
- thermal coupling of the blood vessels and the tissue via source terms present in designated voxels.

Differences between Models

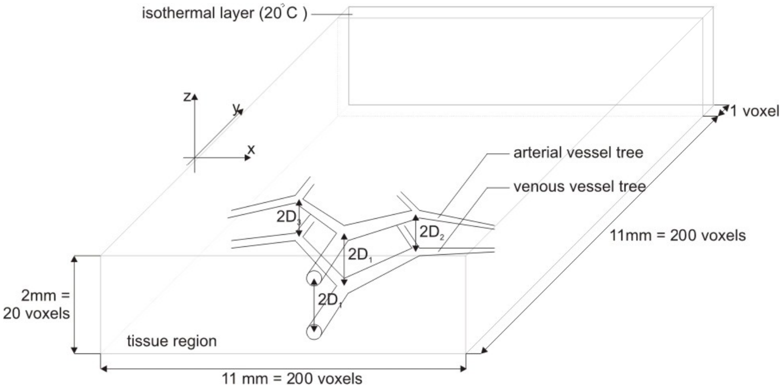
	GRID	DIVA
problem dimension	2D only	3D only
tissue temperature calculation method	FEM	FDM
basic unit of vascular tree representation	counter-current vessel pair segment	single vessel segment
vascular unit form geometry	straight line	parametric curve
bifurcations in vascular tree	only binary	arbitrary
flow conservation	automatically preserved	arbitrary bleed-off may be prescribed at any depth of the tree
unknown blood temperature	three per segment	arbitrary number of buckets per segment

Differences between Models—cont'd

	GRID	DIVA
blood-tissue heat flux	proportional to local (avg. blood temp.)-tissue temp. difference	based on analytical approximation and tissue temperature gradient sampled at multiple points (estimation set)
blood-tissue thermal coupling	source term in "host element" of each segment	source term in voxels of exchange voxel set of each bucket

Setup for Numerical Test

Perfusion of the tissue slice $11 \times 11 \times 2\,\mathrm{mm}$:



Vascular Connectivity

- in 3D simulation (DIVA) two symmetrical trees are used and the distance between them is equal to segment diameter at its endpoint,
- in 2D simulation (GRID) a single countercurrent tree is used.

Boundary and Initial Conditions

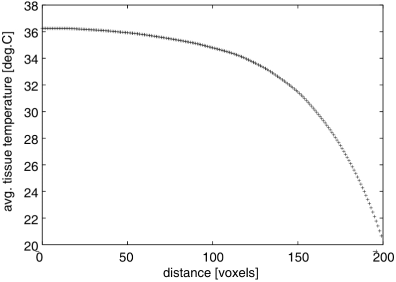
- initial tissue temperature $T_{t0} = 20^{\circ}$C,
- inlet arterial temperature $T_{a} = 37^{\circ}$C,
- "interior" wall: adiabatic,
- "exterior" wall: isothermal at 20°C,
- "side" walls in 3D problem (DIVA): adiabatic.

The thermal equilibration of the system is simulated and the steady-state is recorded.

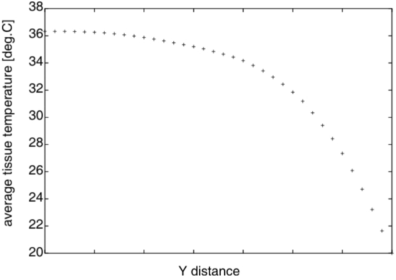
Test Results

Averaged Tissue Temperature

DIVA

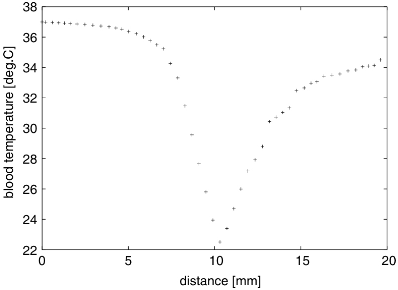


GRID

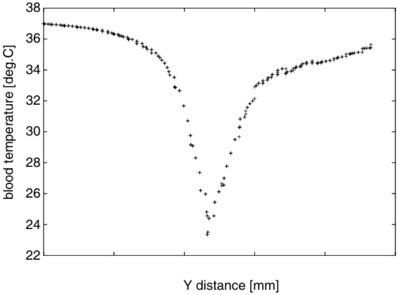


Test Results

Blood Loop Temperature

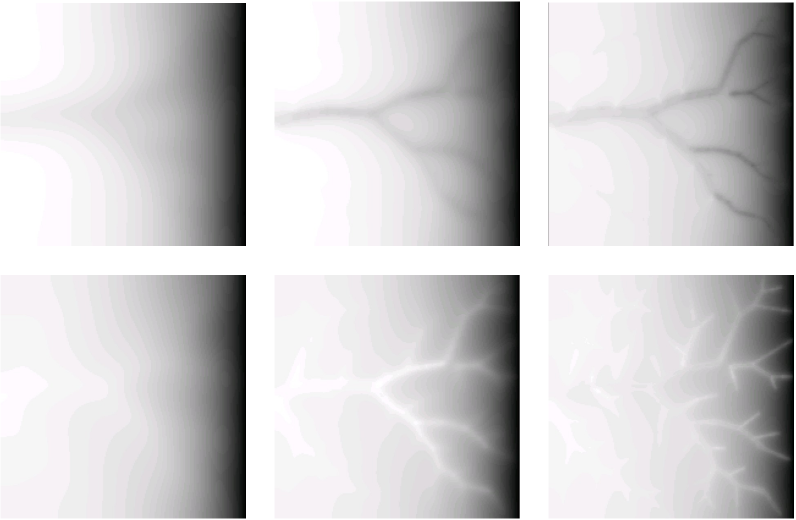


DIVA

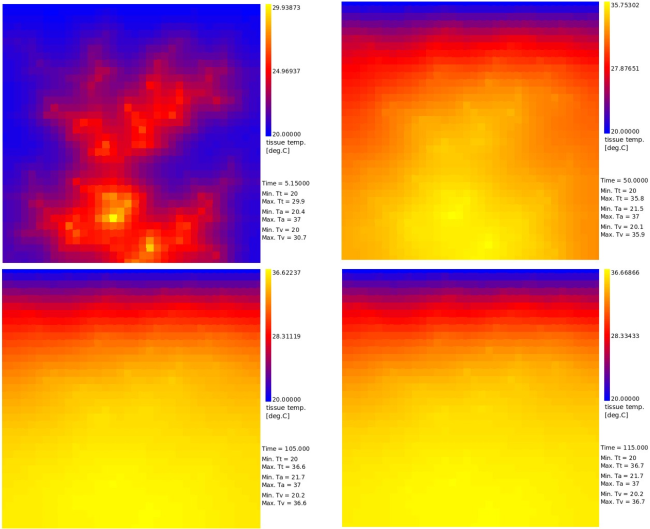


GRID

Test Results: Tissue Temperature (DIVA)

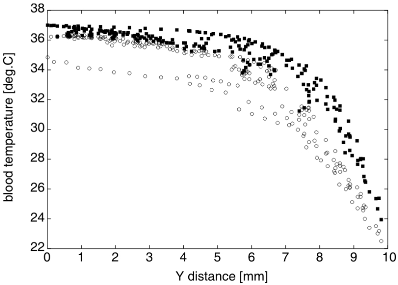


Test Results: Tissue Temperature (GRID)

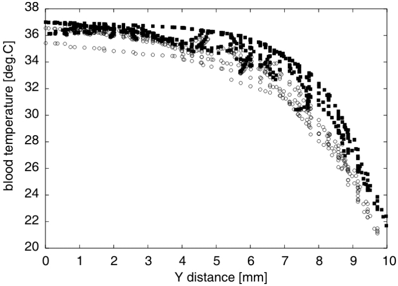


Test Results

Blood Temperature as a Function of Spatial Coordinate



DIVA



GRID

Conclusions

- DIVA system is a flexible and adaptable tool for calculation of heat transfer in vascularized tissues,
- the calculation method employed in GRID presents some potential for more efficient calculation of the counter-current arterio-venous trees when certain geometrical constraints are fulfilled,
- at the present stage of development the GRID program offers no alternative to the DIVA.