Damage Identification in Electrical Network for Structural Health Monitoring

Marek Kokot

Ph.D. Dissertation

supervised by
prof. Jan Holnicki-Szulc

Contents

Chapter 1

Introduction

1.1 Motivation

Problems considered in herein dissertation are closely related with development of a concept of structural health monitoring system, named in short as the ELGRID system, which is based on the idea of embedded electrical sensing network. According to the concept, a structural element being the object of inspection is equipped with a sensing circuitry in a form of spatially-distributed, interconnected electrical sensors/components which are expected to sustain damages along with the appearance or growth of mechanical defects. Problem of health monitoring is simply reduced to localization of damage-induced modifications of electrical parameters within the structure of the sensing network. In the simplest form, the ELGRID system may be considered as an array of crack wires monitored for the occurrence of breaks resulting from crack propagation. In a more general picture, branches of the network may contain various electrical sensors/transducers and mechanical defects (cracks, delamination, debonding) are indicated by changes of different electrical parameters (resistance, capacitance). Diagnosis of the network is performed by the accompanying signal processing unit which invokes predefined test signals in the network, collects the responses and runs a model-based inverse analysis to locate the occurred faults. The system is intended to be a cheap and special-purpose alternative to the main-stream sensing technologies based on piezoelectric transducers or optical fibers, aimed at detection and localization of damages in composite materials. Economical aspect is apparent as the cost of electric/electronic materials, components and equipment should be substantial lower than in the case of piezoceramic and optical technologies. However, the main strength of the proposed approach lies in the fact that no reference to mechanical properties of the monitored material or a structure is made at any stage of monitoring process. Taking into account that composites are usually non-homogeneous and anisotropic materials, whose characteristics and parameters frequently can only be approximated, identification and validation of mechanical model is often the most difficult task, having the crucial impact on accuracy and reliability of health assessment procedure. In this sense, model of electrical sensing network can be much accurately determined, and moreover, topology and parameters of the network can be chosen at the design stage in order to facilitate the procedure of damage identification. As a trade-off, natural deficiency of the proposed system is its lack of prediction – as a stand-alone unit, it will assumedly be able to detect and localize damages, and to some extent specify their type and size, but without assessing their influence on the overall condition of the monitored structure. The most prospective area of system application is detection of defects in crucial components or joints of a structure. Also, it may possibly work as an auxiliary module enhancing performance of other monitoring systems (e.g. to confirm or verify locations of defects).

An essential issue in the concept of the ELGRID system is a manner of interaction between mechanical and electrical defects, namely the type of applied sensors, their integration with monitored structure and how particular kind of damages will translate into modifications of electrical properties. However, technological issues will not be discussed in detail – at the time of this writing, they are still under development and are intended to be an object of future patent claims, hence only vague hints about possible hardware implementation will be delivered. The aim of the work is to investigate the problem of defect identification for some general class of electrical circuits, simulating construction and performance of the ELGRID system, and to propose, develop and verify a numerical technique which will provide an effective and reliable solution. The ultimate goal is to formulate an algorithm of self-diagnosis which could be implemented in the signal processing unit of the system, as well as to specify guidelines concerning topology design and selection of parameters of the sensing network. The main assumption regarding the sensing network is that it can be built-in within the structural material (for example inside a block of concrete or between plies of laminated composite panel), hence particular sensors/components may not be directly accessible for reading/testing. Thus, the idea was to design the sensing layer not as an array of individual sensors but as a compact network of some predefined topology, where damage identification problem can be solved through inverse analysis. An approach based on the adapted algorithms of the Virtual Distortion Method has been chosen as the core of proposed methodology.

VDM has been successfully applied in structural mechanics to solve similar problems, particularly in the case of truss structures, which from the point of view of structural analogies, are equivalents of resistive circuits. Taking into account that VDM can handle multiple, simultaneous damages and may be formulated both in steady-state and dynamic case, it was expected that the method would provide a viable solution to the posed problem.

The title of dissertation, enigmatic as it may seem at the first sight, precisely defines the scope and purpose of the work. The aim is to develop the methodology of solving the inverse problem (*Damage Identification*) for a model of certain sensing device (*Electrical Network*), which is intended to be applied in specific engineering area (*Structural Health Monitoring*).

1.2 Structural Health Monitoring

Motivation behind the work, techniques which are going to be utilized and possible resulting applications all refer or belong to the scope of problems considered within the art of Structural Health Monitoring [1, 2, 3]. It is rapidly developing branch of engineering which combine various measuring and signal processing techniques in the aim to provide technologies capable to detect, localize and identify damages, assess the condition and predict a failure of structures, machines or crucial structural components. A motivation is not only to increase safety and life cycle of the infrastructure but also to reduce time and cost of maintenance and periodical inspections. Most of the utilized methods and technologies origin from vibroacoustics and NDT/E (Non-Destructive Testing/Evaluation), but the ultimate goal is to develop autonomous systems which could be integrated with the structure and perform the diagnosis automatically (without human attendance), preferably in global scale and during normal operation of a structure (monitoring on-line). Monitoring of structural health is a complex and multidisciplinary problem involving issues from the areas of mechanics, electronics, signal processing and computer science. After [4], the process of health monitoring may be described as "(...) the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of the system's health". Taking into account all the relevant issues, any SHM system may be considered from the perspective of:

- the monitored structure itself (e.g. size, shape, material and mechanical properties, load and environmental conditions),

- type of expected damages (e.g. cracks, corrosion, delamination, fatigue) or external dangers (overload, impact),
- physical phenomenon being the basis of identification method (e.g. vibrations, elastic wave propagation, acoustic emission),
- type of sensors (e.g. piezoelectric, optical fibers, MEMS) and measured quantities (e.g. strains, accelerations, electromechanical impedance)
- data collection and signal processing (e.g. modal or wavelet analysis)
- damage identification method (e.g. pattern recognition, gradient-based model updating)

Main areas of SHM applications are mechanical, civil and aerospace engineering. In mechanical engineering [5, 6], SHM concepts are mostly applied to diagnosis of rotating machinery (turbines, power generators). Standard methods are based on observation of vibro-acoustical characteristics of operating machines, which are usually self-excited systems. In civil engineering [7, 8], the objects of inspection are usually large, massive, concrete and metallic structures (bridges, buildings, dams, storage tanks). Vibration-based modal analysis is a standard tool for system identification and health assessment. In aerospace applications [9], usually thin-walled, metallic and composite aircraft components (fuselage, wings, rotor blades) are the objects of diagnosis. Most common methods are based on guided waves and acoustic emission.

Low-frequency vibration-based methods, which origin from machine diagnostics, and high-frequency wave propagation methods, which origin from ultrasonic non-destructive testing, may be distinguished as main-stream SHM methodologies. Vibration-based methods are suited for civil engineering applications where they provide global monitoring of a structure. Wave propagation methods, which include guided Lamb waves, acoustic emission and impedance-based approach, are suited for aerospace applications for detection of local defects (cracks, delamination). Other techniques include electrical resistance measurements, vibro-thermography, electro-magnetic tomography and optical holography.

The most commonly used measuring devices are piezoelectric transducers and optical fibers. Piezoelectric sensors [10], based on natural crystals (quartz), ceramic (PZT) or thin films (PVDF) can directly measure strains, forces and accelerations. Piezo-Wafer Active Sensor (PWAS) [11] is a concept of a network of piezo-elements which operate both as sensors and actuators, finding application in guided waves methods or electromechanical impedance approach. Examples of commercially available SHM systems based on the PWAS concept are SMART Layer by Accelent Technologies and SWISS by EADS/Siemens NDT. Fiber optic sensors [12, 13] include intensity-based, phase-modulated (interferometric) and Fiber Bragg Grating (FBG) sensors for measurement of strains, pressure and environmental conditions (temperature, humidity). Other emerging sensing technologies are micro electromechanical systems (MEMS) and laser vibrometers. Along with the sensing technologies, there is extensive development of hardware and software devoted to acquisition, conditioning, filtering, reduction and pre-processing of measured data. Wireless transmission between the network of sensors and signal processing unit becomes a standard in modern SHM systems.

1.3 Methods of damage identification

SHM system should provide an answer to at least one of the three main problems: detection, localization and quantification of the occurring damages. The next step – estimation of the system's remaining useful life, belongs to the new developing art of Damage Prognosis [14, 15]. Measurements registered on real structures consists of local strains or accelerations, as well as environmental conditions. The next step is extraction and selection of features which are sensitive to structural modifications (e.g. eigen frequencies, modal shapes). In low-frequency methods, system identification is usually required to fit the numerical model with experimental data. The last step is analytical or statistical analysis providing a solution to identification problem. Generally, two main approaches can be distinguished: sensitivity-based inverse analysis, which utilize analytical formulation of the problem, and methods of artificial intelligence: statistical pattern recognition, artificial neural networks and soft computing (genetic algorithms, simulated annealing, swarm intelligence).

From the point of view of electrical engineering, problem considered in this work belongs to the area of analog fault diagnosis [16, 17, 18, 19], which encapsulates techniques of detection, location and prediction of electrical faults in analog and mixed-signal circuits. Fault diagnosis is usually aimed at identification of hard faults (open and short circuits) using DC, single- and multi-frequency AC, transient time domain or noise measurements. In Fault Dictionary approach, a collection of nominal circuit behaviors for various scenarios of fault locations is compiled and post-test algorithm compares the measured responses with dictionary signatures to isolate a fault. Fault Dictionary is usually applied to diagnose single, hard faults and is considered incapable of detection of parameter drifts. In Parameter Identification approach, the inverse problem is solved based on model of a circuit and responses registered for known test signals. Fault Verification approach uses the same methods as parameter identification but assumes some restrictions on the number and location of faults. Artificial neural networks and statistical methods are frequently used in all the approaches. A common problem in fault diagnosis is limited accessibility of internal nodes (e.g. in integrated circuits) for signal measurements. *Testability* studies [20, 21], based on topological analysis, provides theoretical conditions of circuit diagnosability and also give specifications for test signal selection.

1.4 Objectives and scope

The main goal of the work is to solve the inverse problem of defect identification within the electrical circuit being the model of electrical sensing network. Generally, solution to any numerical problem, the inverse one in particular, depends not only on the applied algorithm but on the conditioning of the problem itself. It is assumed that most or all of the internal nodes of the network will be inaccessible, only selected terminals on external edges will be available for test signal supply and response measurements. Also, system functionality dictates that sensors within the network should be preferably distributed in some regular fashion (e.g. in a grid pattern) and be of similar electrical characteristics. Under such conditions, problem of damage identification is expected to be not only severely ill-conditioned, but even ill-posed (in a sense that there is no unique solution). In answer to this challenge, **the main thesis** of the work is stated as follows:

> *If configuration of the electrical sensing network complies with some pre-defined design rules, then a problem of damage identification can be efficiently solved through dynamic inverse analysis.*

In fact, it is claimed that the overall goal of the work can be reached in two major steps. First one is to propose the method of solving the damage identification problem for a generally formulated model of electrical sensing network. This will be accomplished by providing the algorithm of dynamic, gradient-based model updating. Second task is to optimize/design the sensing network in order to make it diagnosable. This will be accomplished by selection of proper topological pattern defining locations and interconnections of sensing elements within the network. Overall diagnosability of a circuit depends on its configuration (topology and parameters of components), shape of test signals and measured responses (number, kind and locations), hence the task of network design should be considered as a multi-criteria optimization problem. However, issues of optimal topology or optimal sensor placement are by themselves very difficult inverse problems which cannot be formulated analytically and require combinatronics or heuristic approach. That's why the task of system optimization will not be solved *sensu stricto*. Rather, based on topological and inverse analysis of several network configurations, some general conclusions will be derived and a set of heuristic rules/guidelines will be prescribed. Combination of these rules along with the proposed damage identification procedure will enable to define a class of diagnosable network configurations, satisfying conceptual and functional requirements of the ELGRID system.

Validation of the thesis will be achieved by accomplishing the following objectives:

- A damage identification problem in electrical sensing networks complying with the concept of the ELGRID system will be formulated.
- Virtual Distortion Method will be adapted to electrical circuit analysis, including DC, AC and transient, time-domain case.
- An original methodology of damage identification in electrical sensing networks, based on the adapted VDM algorithms, will be developed.
- Numerical and experimental verification of the method will be performed.
- Topological and inverse analysis of different network configurations will be performed and a set of rules concerning network design will be derived.
- Diagnosable configurations of sensing networks will be identified and possible implementations in monitoring system will be proposed.

Problems discussed in the work concern mainly the issues from the area of electrical circuit analysis and optimization. Thus, to ensure the logical order of presented problems, the dissertation is organized as follows:

Chapter 2 provides some notions from circuit theory necessary from the perspective of considered problems. Physical quantities, parameters, laws and relations which enable to describe and analyze linear models of RLC circuits are provided. Procedures for steady-state DC, AC and transient, discrete-time domain analysis, based on the modified nodal approach, are presented. In the last section, the concept of electro-mechanical analogies is explored, with the emphasis on structural similarities between electrical circuits and plane truss structures.

Chapter 3 introduces the basic notions of the Virtual Distortion Method. Electrical equivalents of virtual distortions and influence matrices are defined and implemented as efficient numerical tools which enable to simulate modifications of circuit parameters. VDM-based algorithms of quick re-analysis and sensitivity analysis are consequently adapted for the case of DC, AC and dynamic circuits.

Chapter 4 is devoted to the inverse problem of damage identification in electrical circuits. At the beginning, a general concept of the ELGRID system is presented, along with considerations regarding possible hardware implementation. Next, the problem of damage identification is formulated. Steady-state approach, being of less practical importance, enables to explain the overall concept of damage identification algorithm and the related issues and problems (conditioning, uniqueness of solution). Dynamic approach, based on gradient optimization, is projected to be the core of the algorithm applied in signal-processing unit of the monitoring system. The method is illustrated on numerical examples and verified on experimental setup. Rules of topological design are specified and several different network configurations are examined. Possible implementations of electrical sensing networks are presented.

Chapter 2

Excerpts from circuit theory

In Chapter 4, the concept of monitoring system utilizing embedded electrical sensing network will be presented. In brief, the problem of health monitoring will be formulated as a search for damage-induced modifications of parameters within the sensing network considered as a purely electrical system. The assumed model of the network will have a form of time-invariant linear circuit consisting of ideal, two-terminal, discrete elements: resistors, capacitors, coils and independent sources. Conceptual and technological arguments which support this assumtion will be discussed in Chapter 4. In the following sections, all notions and relations necessary to describe and analize the said model of the network will be provided. Also, a system of structural analogies between electrical circuits and truss structures, which establishes a basis for adaptation of the Virtual Distortion Method, will be discussed.

2.1 Basic notions

Circuit description

Electrical circuit/network is a collection of interconnected components and devices which are capable to conduct, store, enforce or control the movement of electric charges. A graphical representation of a circuit is schematic diagram, where various types of components are depicted by predefined symbols. An exemplary circuit diagram shown in Figure 2.1 includes at least one representative from the assumed set of considered components. Namely, there are six resistors $R_1 \div R_6$, two capacitors C_1,C_2, a coil L, a source of voltage E and a source of current J. All the elements are two-terminal models of ideal components, which means they are characterized entirely by a single lumped parameters. Connecting wires are considered to be ideal conductors. *Node*

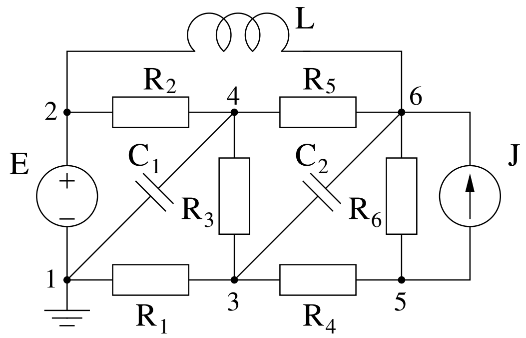


Figure 2.1: Exemplary circuit diagram

is a point of connection of at least two terminals which belong to different components. There are six distinct nodes in the exemplary circuit and node number 1 has been chosen as a common ground. *Loop* is a set of elements comprising a closed path. It is assumed that there are no self-loops, i.e. terminals of every component are associated with a pair of different nodes.

Electrical quantities

The scope of considerations is limited to macroscopic quantities describing causes and effects of electron movement in a circuit of discrete structure.

Electric charge (symbol Q, unit coulomb [C]) is an amount of uncompensated particles (electrons, protons, ions) within the considered component or space, being a multiple of elementary charge $e = 1.602 \times 10^{-19}$C.

Intensity of current (symbol I, unit ampere [A]) is a rate of flow of electric charge through a cross-section of a conductor or component: $I = \frac{\mathrm{d}Q}{\mathrm{dt}}$

Electric potential (symbol V, unit volt [V]) at a given point of electric field is a measure of potential energy per unit charge, defined with regard to some reference level. Electric potentials are associated with circuit nodes and zero reference level is assumed for grounded nodes.

Voltage (symbol U, unit volt [V]) is a difference of electric potentials between two points of electric field. Voltage across circuit component is a measure of energy required to move a unit charge through it.

Magnetic flux (symbol Φ, unit weber [W]) is a measure of magnetic field per unit area. According to Faraday's law of induction, alternating magnetic flux acting on conductive element produces voltage: $U = \frac{\mathrm{d}\Phi}{\mathrm{dt}}$

Components and parameters

The object of consideration are stationary linear circuits where parameters of components are assumed to be time-invariant and independent of the level of current or voltage, type of excitation signal or ambient conditions (temperature, humidity, electric/magnetic fields, etc.).

Resistors are components able to conduct electric charge which is accompanied by the dissipation of energy in a form of a heat. In the simplest form, resistor is a piece of a wire made of resistive material. *Resistance* (symbol R, unit ohm [Ω]) determines the level of voltage which needs to applied across resistor terminals in order to obtain a given level of current ($U = RI$). Resistance depends mainly on geometrical dimensions of resistor (length and cross-section area) and conductivity of the material. A reciprocal of resistance is *conductance* (symbol G, unit siemens [S]).

Capacitors are components able to store electric charge. In the simplest form, capacitor consists of two over-lapping conductive plates (electrodes), separated by a thin dielectric layer. Voltage applied across capacitor results in polarization of the insulating layer and occurence of opposite electrical charges on electrodes. *Capacitance* (symbol C, unit farad [F]) determines the amount of charge stored in capacitor for the given level of applied voltage ($Q = CU$). Capacitance depends mainly on the area of overlapping plates, thickness of insulating layer and permittivity of dielectric material.

Coils (inductors) are components able to produce magnetic field. In the simplest form, inductor is a piece of spiral-shaped wire, with or without an inside core. Current flowing through a coil wire results in magnetic field produced inside the core. *Inductance* (symbol L, unit henry [H]) determine the intensity of magnetic flux produced in coil for a given level of current ($\Phi = LI$). Inductance depends mainly on geometrical dimensions of a coil (length, area of a single turn), number of turns and permeability of a core material.

Voltage sources are active components, characterized by *electromotive force* (symbol E, unit volt [V]), which defines the difference of electric potentials enforced on source terminals.

Current source are active components, characterized by *source intensity* (symbol J, unit amper [A]), which describes the intensity of current enforced through source terminals.

Types of analysis

Type of signals generated in a circuit by active sources determines the mathematical form of equations describing circuit behaviour and consequently, a numerical technique which need to be applied to obtain a solution. The following types of circuits will be considered:

Direct Current (DC) circuit is supplied by sources of constant values of electromotive forces or current intensities. Responses of the circuit are also constant. More precisely, it is assumed that analysis is conducted long after any switching operation, when the unsteady component of responses can be neglected. DC circuit is described by a system of algebraic equations and solved using algorithms of linear algebra (e.g. Gaussian elimination or LU decomposition).

Alternating Current (AC) circuit is supplied by harmonic sources of fixed frequency and constant amplitudes and phases. Responses of the circuit are also harmonic of the same frequency. The standard method of analysis is the symbolic method where signals and parameters are expressed using complex numbers. Single-frequency AC circuit is described by a system of algebraic equations and solved using algorithms of linear algebra in the complex number domain.

Dynamic circuit is supplied by sources of arbitrary shape of input signals. Functions of responses are generally different then input functions. Dynamic circuit is described by a system of ordinary differential equations, which can be solved using various method, either in time domain (e.g. direct numerical integration or in the state-space representation) or in frequency domain (using Laplace or Fourier transform). In this work, transient analysis in discrete-time domain will be performed with solution obtained through direct numerical integration.

Different notation of electrical quantities will be used for every type of analysis (Table 2.1). In DC case, a single electrical quantity will be denoted by a capital letter. In AC case, underlined capital letters will be used to denote complex representation of signals. In dynamic case, small letters with direct relation of time will be used. Interrelations between various quantities will be described using matrix notation. Quantities will be gathered in column vectors denoted by small bold letters, underlined in AC case and with relation of time in dynamic case. Vectors of voltages and currents will also have additional subscript indicating the type of component which they refer to. Matrices of parameters will be donoted by bold capital letters.

	DC	AC	dynamic
Single quantity	V, U, I, E, J	$\underline{V}, \underline{U}, \underline{I}, \underline{E}, \underline{J}$	$v(t), u(t), i(t), e(t), j(t)$
Matrix form	$\mathbf{v}, \mathbf{u}_\mathrm{R}, \mathbf{i}_\mathrm{C}, \mathbf{e}, \mathbf{j}$	$\underline{\mathbf{v}}, \underline{\mathbf{u}}_\mathrm{R}, \underline{\mathbf{i}}_\mathrm{C}, \underline{\mathbf{e}}, \underline{\mathbf{j}}$	$\mathbf{v}(t), \mathbf{u}_\mathrm{R}(t), \mathbf{i}_\mathrm{C}(t), \mathbf{e}(t), \mathbf{j}(t)$

Table 2.1: Notations for different types of analysis

Flow notation

Historically, Benjamin Franklin, one of the pioneers in studies of electricity, postulated that electrical phenomena are related with movement of some hypotethical carriers – an excess of these carriers makes an object positively charged while a deficiency makes it negatively charged. Later studies demonstrated that, according to this convention, electrons have to be denoted as carriers of negative charge. This introduced a confusion concerning the direction of current flow. In the *electron flow notation*, the direction of current is in accordance with actual motion of electrons, that is from the negative toward positive potential. In the *conventional flow notation*, which complies with Franklin's assumption, current flows from positive toward negative potential. Regardless of convention, the arrow of voltage which denotes polarity, points toward positive or higher potential (Figure 2.2). Since the conventional

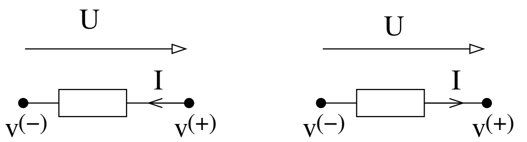


Figure 2.2: Conventional (left) vs electron (right) flow notation

flow notation is more popular and widely used (e.g. unified graphical symbols of electrical components are in accordance with it) and the fact that actual direction of electron flow is of no importance to the considered problems, the conventional notation will be used throughout the work.

Topology description

To describe a scheme of connections between components, the notions of *directed graph* and *incidence matrix* will be applied. If every component of a circuit is represented as an edge of fixed orientation which links a pair of nodes, the directed graph is obtained (Figure 2.3). Orientation of edges defines a reference for assigning directions of currents and polarities of voltages: the positive instantenous value of the quantity means that it complies with

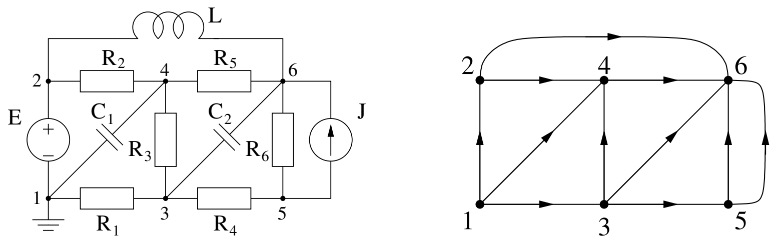
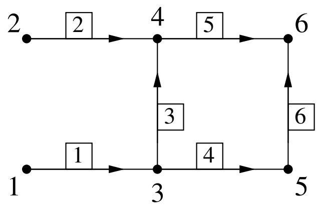


Figure 2.3: Scheme of a circuit and its oriented graph

the assumed orientation. In this and all the following examples, every edge of a graph is initialy oriented toward the node of a higher index. Since the conventional flow notation will be used, the sign of current will be always opposite to the sign of voltage.

The structure of a graph can be described in the form of *incidence matrix* $\mathbf{M}$. Rows of the matrix correspond to nodes and columns to edges. If k-th edge leaves i-th node and enters j-th node then $M_{ik} = -1$, $M_{jk} = 1$ and all other entries in the k-th column equal zero. For every kind of component, a separate incidence matrix will be used – a letter above the matrix will indicate the considered components ($\overset{\mathrm{R}}{\mathbf{M}}$ for resistors, $\overset{\mathrm{C}}{\mathbf{M}}$ for capacitors etc.). In Figure 2.4, a subgraph obtained for resistors from the exemplary circuit and a corresponding incidence matrix are presented.



$$\overset{\mathrm{R}}{\mathbf{M}} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Figure 2.4: Oriented graph and the corresponding incidence matrix

Incidence matrices enable to formulate relations between quantities associated with nodes and components. Namely, vector of voltages across a given set of components (for example resistors) and vector of nodal potentials can be related as follows:

$$\mathbf{u}_{\mathrm{R}}(t) = \overset{\mathrm{R}}{\mathbf{M}}^{\mathrm{T}} \mathbf{v}(t) \tag{2.1}$$

while the product of incidence matrix and vector of currents (e.g. $\overset{\mathrm{R}}{\mathbf{M}}\mathbf{i}_{\mathrm{R}}$) conveys a relation for algebraic sums of currents entering/exiting nodes from the given set of components.

2.2 Laws, relations and theorems

There are two sets of rules which determine the behavior of a circuit. First one results from mathematical modeling of circuit elements which leads to formulation of local current-voltage relationships. Second one results from topological constraints which govern global states of currents and voltages. A fundamental principle in analysis of linear circuits is superposition theorem which also constitutes a basis of the Virtual Distortion Method. From the perspective of later considerations, some principles regarding handling and transformation of sources are also worth mentioning.

Constitutive equations

Relations between currents and voltages for ideal passive components in different types of analysis have been gathered in Table 2.2. Ohm's law, presented here in the conductance form, states that for resistors current and voltage are proportional regardless of signals variation. Relations for capacitors and coils, in a general case, are described by integral or differential equations, but in the steady-state analysis they reduce to algebraic equations. In DC case, charged capacitor prevents current flow (it acts as a break) while in coil no voltage is induced (it acts as a short-circuit). In AC case, relations are formulated with respect to complex amplitudes of signals (see Section 2.3).

	Resistor	Capacitor	Coil
General case	$i(t) = G\,u(t)$	$i(t) = C\dfrac{du(t)}{dt}$	$i(t) = \dfrac{1}{L}\int u(t)dt$
DC case	$I = G\,U$	$I = 0$	$U = 0$
AC case	$\underline{I} = G\,\underline{U}$	$\underline{I} = j\omega C\,\underline{U}$	$\underline{I} = \dfrac{1}{j\omega L}\,\underline{U}$

Table 2.2: Constitutive relations

Capacitors and coils are components which store energy – capacitors in the form of electric field, coils in the form of magnetic field. Level of stored energy is determined by the voltage across capacitor and current in coil:

$$W_C = \frac{1}{2}CU^2; \qquad W_L = \frac{1}{2}LI^2; \tag{2.2}$$

Just as the changes of energy must be continuous, so do changes of voltage across capacitor and current in coil. In a case of a sudden modification in circuit structure (e.g. opening a switch) or non-continuity in supply signal (e.g. excitation by a step function), values of the mentioned quantities are preserved – they determine initial conditions after the switching operation. Both current in capacitor and voltage across coil may however change in any way possible. There are switching operations which result in non-linear behavior. Opening of saturated coil, short-circuit of charged capacitor, connection of a capacitor to DC voltage source or connection of a coil to DC current source will be accompanied by a sudden release of energy, which usually takes a form of electric arc (non-linear resistance). Such *pathological* switching operations will not be considered in this work.

Kirchhoff's laws

Global states of currents and voltages in a circuit are governed by the rules known as Kirchhoff's laws:

Kirchhoff's Current Law (KCL):

An algebraic sum of currents entering/exiting a node equals zero.

Kirchhoff's Voltage Law (KVL):

An algebraic sum of voltage drops in a loop equals zero.

KCL results from the principle of conservation of electric charge (nodes cannot store charge) while KVL is a consequence of the principle of conservation of energy (in potential field, sum of energy gained/lost by a charge moving in a closed path is zero). Kirchhoff's laws hold true regardless of the type of circuit and signals variation, and are fulfilled for every node and arbitrary path comprising a loop. However, in a circuit consisting of n nodes and b branches, there are at most *(n-1)* independent KCL equations and *(b-n+1)* independent KVL equations.

Superposition theorem

In linear systems, the outputs produced by individual inputs are additive. In regard to electrical circuits, superposition theorem can be stated as follows:

> The overall response of a circuit supplied by multiple sources equals to the algebraic sum of responses generated by individual sources acting alone (other voltage sources shpuld be replaced by short-circuits and current sources by breaks).

Superposition principle enables also to solve circuits supplied by mixed kinds of sources. For example, any periodical supply signal can be decomposed on a sum of harmonic functions (Fourier transform) hence the overall response of a circuit can be calculated as a sum of independent AC responses generated by individual harmonics.

Equivalent sources

Model of real source should at least include an inner resistance which enables to simulate losses of power and changes of nominal value due to the load. Model of real voltage source includes resistance R_e in series while real current source contains conductance G_j in parallel (Figure 2.5). Let suppose that R represents total resistance of the load. State of current/voltage in the load can be described by the following relations:

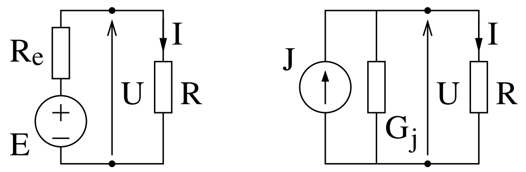


Figure 2.5: Models of real sources

$$E = I\,(R_e + R)\,; \qquad J = U\left(G_j + \frac{1}{R}\right); \qquad U = RI \tag{2.3}$$

Eliminating resistance of the load we obtain:

$$I + \frac{U}{R_e} = \frac{E}{R_e}; \qquad I + G_j U = J \tag{2.4}$$

Comparing coefficients standing by respective quantities, the following conditions can be obtained:

$$G_j = \frac{1}{R_e}; \qquad J = \frac{E}{R_e} \tag{2.5}$$

If parameters of sources fulfill the above conditions then, regardless of resistance of the load, they both produce the same state of current and voltage. The principle is valid only for models of real sources, ideal sources have no equivalents.

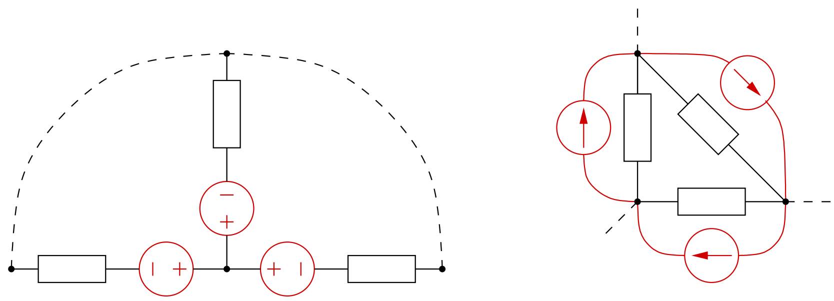


Figure 2.6: Impotent sets of sources

Impotent sources

There are certain configurations of sources which have a specific influence on circuit behavior. The so-called impotent set of sources don't generate certain responses at all:

- A set of ideal voltage sources inserted in series with all branches comprising a node cutset (Figure 2.6 – left) will not affect the state of currents in a circuit (sources are of equal instantenous value and the same polarity with respect to the node). The principle results from the fact that in every loop which contains the common node, there are two equal voltage sources of opposite polarity, hence they cancel each other in Kirchhoff's voltage law. The only affected responses are potential in the common node and voltages in the branches comprising the cutset.

- A set of ideal current sources inserted in parallel with all branches comprising a loop (Figure 2.6 – right) will not affect the state of voltages in a circuit (sources are of equal intensities and of the same directions with respect to the loop). The principle results from the fact that every node which belongs to the loop is connected with two equal current sources of opposite directions, hence they cancel each other in Kirchhoff's current law. The only affected responses are currents in the branches comprising the loop.

The above principles allow to explain the concept of impotent states of virtual distortions, which will be discussed in Chapter 4.

Reciprocity principles for sources

The following principles are simplified cases of general Lorentz reciprocity theorem:

- If a current source J inserted in parallel with i-th branch of a circuit invokes voltage U in j-th branch, then the same source J inserted with j-th branch will invoke the same voltage U in i-th branch.
- If a voltage source E inserted in series with i-th branch of a circuit invokes current I in j-th branch, then the same source E inserted with j-th branch will invoke the same current I in i-th branch.

2.3 Network analysis

To evaluate an overall state of currents and voltages in a circuit of known configuration (global direct problem), it is required to formulate and solve a system of simultaneous equations (algebraic or differential) resulting from Kirchhoff's laws and constitutive relations. From the practical perspective, an efficient procedure of analysis should enable to formulate circuit equations in a systematic way and to solve them at reduced numerical cost. In profesional literature [25, 26], two popular approaches are distinguished: *Sparse Tableau Analysis* (STA) and *Modified Nodal Analysis* (MNA). STA, formulated by Hachtel et al. [27], is based on sparse matrix techniques. System of equations (in a complete form) is formulated with respect to all unknown quantities – currents, voltages and nodal potentials. Although the resulting global matrices are of large dimension, the problem can be effectively solved using dedicated sparse algorithms. MNA, formulated by Ho et al.[28], is an extension of classical nodal analysis which aims at reducing the dimension of the system. Equations are formulated with respect only to nodal potentials and selected branch currents. The remaining unknown quantities are found in post-processing.

Nodal approach has some useful features which are essential from the perspective of problems considered in this work. As it will be demonstrated in the next section, nodal method may be viewed as an equivalent of the Finite Element Method approach to truss structure analysis. This feature is a basis for adaptation of VDM concepts to circuit simulation. Also, nodal potentials are physical quantities which can be easily and practically measured in real circuits. Damage identification procedure, which is going to be developed and applied in the monitoring system, is intended to be based on measurements of electric potentials in selected nodes of a sensing network. From the point of view of numerical algorithms, MNA-based formulation of circuit equation is natural and most efficient.

Modified Nodal Analysis

Let suppose that the considered circuit is represented by a graph consisting of n nodes and m edges representing passive components. In general, there are $2m$ unknown quantities to be found: m currents and m voltages in all the edges. To this end:

- n-1 Kirchhoff's current laws, formulated for independent nodes,
- m-n+1 Kirchhoff's voltage laws, formulated for independent loops,
- m constitutive relations, formulated for every edge,

producing together a set of $2m$ independent equations, can be applied. In practice, the overall number of unknown variables and particular numbers of independent equations may be lower because of specific interconnections between components or supply conditions (for example, ideal sources enforce fixed currents or voltages, components in parallel share the same voltage, in DC analysis current in capacitor is zero, etc.). However, regardless of circuit configuration, there are always as many independent equations as there are unknown quantities and, barring a case of pathological switching operation, a well-posed system of equations may be formulated.

Let notice that constitutive relations for the considered set of passive components are decoupled. Kirchhoff's laws can be then formulated with respect to either currents or voltages and the problem reduces to m-dimensional system of equations (the omitted responses can be easily calculated in post-procesing using constitutive relations). Even further reduction of system dimension may be obtained by applying one of the two complimentary approaches: *mesh method* (up to m-n+1 equations) or *nodal method* (up to n-1 equations). In either case, only one of Kirchhoff's law is used and equations are formulated with respect to some intermediate variables. In mesh method, based on KVL, the unknown variables are mesh currents, which are virtual flows assumed in every independent loop of a circuit. Any branch current is an algebraic sum of mesh currents flowing through the component. In nodal method, based on KCL, the unknown variables are electric potentials in nodes, with grounded nodes having zero reference value. Any branch voltage is a difference of potentials on components' terminals.

However, classical nodal approach can only be applied if constitutive relations for all components have an admittance representation, i.e. it is possible to express branch current as a function of voltage. Moreover, voltage sources have to be transformed into equivalent currents sources or, in a case of ideal sources, special nodal conditions need to be applied. Nodal approach produces a system of linear algebraic or differential equations, with symmetric matrices of parameters (positive-definite in steady-state analysis), vector of unknown nodal potentials and input vector of node-reduced current excitations. The Modified Nodal Analysis [28, 29] is an extension of the nodal approach which enables to aggregate electrical components which don't have admittance representation or are described by coupled equations (e.g. controlled sources, coupled inductors, ideal transformers, transmission lines or operational amplifiers). It is accomplished by introducing additional variables and nodal conditions associated with a given kind of component (the so-called *stamps*) into the system of nodal equations. The main matrices lose the character of pure parameter matrix (they may contain numbers, resistances or gain factors) and the vector of unknowns includes variables other then nodal potentials (usually currents through components). Moreover, main matrices are no longer symmetrical and can have zero diagonal elements which complicates the solution from the point of view of numerics (e.g. pivoting is required in LU decomposition). However, because of general formulation and compactness, the MNA-based approach is a computational basis in most popular circuit simulators (e.g. SPICE [53], Gnucap [54] or Qucs [55] to name a few examples of freeware packages).

In the following subsections, the MNA-based approach will be applied to formulate circuit equations in the case of DC, AC and transient time-domain analysis. The starting point is always Kirchhoff's current law, which in general case can be formulated as follows:

$$\overset{\mathrm{R}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{R}}(t) + \overset{\mathrm{C}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{C}}(t) + \overset{\mathrm{L}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{L}}(t) + \overset{\mathrm{E}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{E}}(t) + \overset{\mathrm{J}}{\mathbf{M}}\,\mathbf{j}(t) = [\mathbf{0}] \tag{2.6}$$

The further procedure involves substitution of currents by voltages (based on constitutive relations) and expressing voltages in terms of nodal potentials (based on Equation 2.1). If some currents cannot be directly expressed in terms of nodal potentials, they will be treated as unknown variables and additional nodal conditions will be introduced.

DC case

In the steady-state DC analysis, ideal capacitor acts as a break (current equals zero) while ideal coil acts as a short-circuit (drop of voltage equals zero). Constitutive relations, described in the matrix form, can be expressed as follows:

$$\mathbf{i}_{\mathrm{R}} = -\mathbf{G}\,\mathbf{u}_{\mathrm{R}}; \qquad \mathbf{i}_{\mathrm{C}} = [\mathbf{0}]; \qquad \mathbf{u}_{\mathrm{L}} = [\mathbf{0}] \tag{2.7}$$

The negative sign in Ohm's law results from the assumed conventional flow notation and $\mathbf{G}$ is a diagonal matrix of conductances. Kirchhoff's current law has the following form:

$$\overset{\mathrm{R}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{R}} + \overset{\mathrm{L}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{L}} + \overset{\mathrm{E}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{E}} + \overset{\mathrm{J}}{\mathbf{M}}\,\mathbf{j} = [\mathbf{0}]\,. \tag{2.8}$$

Since currents in coils and in voltage sources cannot be directly related with nodal potentials, they will be treated as additional unknown variables and the following nodal conditions will be introduced:

$$\overset{\mathrm{L}}{\mathbf{M}}{}^{\mathrm{T}}\,\mathbf{v} = [\mathbf{0}]\,; \qquad \overset{\mathrm{E}}{\mathbf{M}}{}^{\mathrm{T}}\,\mathbf{v} = \mathbf{e} \tag{2.9}$$

Combining the above relations, the following system of equations can be formulated:

$$\begin{bmatrix} \tilde{\mathbf{G}} & -\overset{\mathrm{L}}{\mathbf{M}} & -\overset{\mathrm{E}}{\mathbf{M}} \\ \overset{\mathrm{L}}{\mathbf{M}}{}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \\ \overset{\mathrm{E}}{\mathbf{M}}{}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i}_{\mathrm{L}} \\ \mathbf{i}_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} \overset{\mathrm{J}}{\mathbf{M}}\,\mathbf{j} \\ \mathbf{0} \\ \mathbf{e} \end{bmatrix} \tag{2.10}$$

where the aggregated matrix of conductances $\tilde{\mathbf{G}} = \overset{\mathrm{R}}{\mathbf{M}}\,\mathbf{G}\,\overset{\mathrm{R}}{\mathbf{M}}{}^{\mathrm{T}}$.

Equation (2.10) is a system of linear algebraic equations, of dimension equal to the total number of nodes, coils and voltage sources. In a more compact form, it can be expressed as:

$$\mathbf{A}\,\mathbf{x} = \mathbf{z} \tag{2.11}$$

$\mathbf{A}$ will be called the main matrix, $\mathbf{z}$ – an input vector and $\mathbf{x}$ – a vector of unknowns or a vector of base solution (depending if it is considered before or after the solution). Main matrix $\mathbf{A}$, in a form presented in Equation (2.10), is singular. This results from the fact that the number of independent Kirchhoff's current laws is always lower then the number of nodes while Equation (2.8) has been formulated with respect to all nodes. At least one node in a circuit need to be chosen as a reference – usually it is a node connected to a common ground where the value of potential is assumed zero. There are two ways to implement the condition for the i^{th} grounded node:

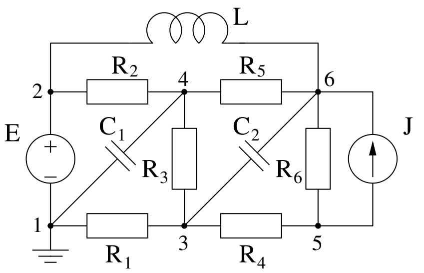
1. all entries in the i^{th} row and the i^{th} column of the main matrix, as well as the entry z_i in the input vector are set to zero, except the diagonal entry A_{ii} which is set to one.

2. i^{th} row and i^{th} column of the main matrix and the entry z_i in the input vector are removed.

In the latter case, the dimension of the system can be reduced (especially if there are multiple ground nodes) but in numerical implementation, re-indexing of nodes is required hence computational effort is similar. After imposing ground conditions, Equation (2.11) can be solved using standard routines and algorithms for linear systems (e.g. Gaussian elimination or LU decompozition). From the perspective of VDM-based algorithms, it is convenient to decompose the main matrix **A** or even to calculate its full inverse, as it is used for calculations of both linear responses and influence matrices (vide Chapter 3). The computed vector of base solution consist of nodal potentials, currents in coils and currents in voltage sources. Other circuit responses can be calculated using voltage-potential relation and constitutive equations.

Example 1

To find base DC solution for the circuit of configuration presented in Figure 2.7. Parameters of sources are: E = 5V, J = 2mA.



$\mathrm{R}_i = 1\,\mathrm{k}\Omega \qquad i = 1\ldots 6$
$\mathrm{C}_1 = \mathrm{C}_2 = 1\,\mu\mathrm{F}$
$\mathrm{L} = 1\,\mathrm{mH}$

Figure 2.7: Exemplary circuit

System of equations for the given set of parameters and ground conditions ($V_1 = 0$) has the following form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & -0.001 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0.003 & -0.001 & -0.001 & 0 & 0 & 0 \\ 0 & -0.001 & -0.001 & 0.003 & 0 & -0.001 & 0 & 0 \\ 0 & 0 & -0.001 & 0 & 0.002 & -0.001 & 0 & 0 \\ 0 & 0 & 0 & -0.001 & -0.001 & 0.002 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ I_L \\ I_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.002 \\ 0.002 \\ 0 \\ 5 \end{bmatrix}$$

The solution is:

$$\mathbf{v} = [0 \quad 5 \quad 2.23 \quad 4.08 \quad 2.62 \quad 5]^{\mathrm{T}} \qquad \mathbf{i}_{\mathrm{L}} = [1.31]\times 10^{-3} \qquad \mathbf{i}_{\mathrm{E}} = [2.23]\times 10^{-3}$$

Potentials are given in volts and currents in amperes. Additionally, let calculate the state of currents in resistors and voltages across capacitors:

$$\mathbf{i}_\mathrm{R} = -\mathbf{G}\overset{\mathrm{R}}{\mathbf{M}}^\mathrm{T}\,\mathbf{v} = \begin{bmatrix}-2.23 & 0.92 & -1.85 & -0.38 & -0.92 & -2.38\end{bmatrix}^\mathrm{T} \times 10^{-3}$$

$$\mathbf{u}_\mathrm{C} = \overset{\mathrm{C}}{\mathbf{M}}^\mathrm{T}\,\mathbf{v} = [4.08 \quad 2.77]^\mathrm{T}$$

The positive value of current/voltage means that its direction/polarity is in accordance with initial graph orientation (every edge of the graph is assumed to point toward the node of higher index).

AC case

In the steady-state AC analysis, all signals are harmonic of fixed frequency and constant amplitudes and phases. A standard approach is symbolic method based on complex numbers theory. Using the Euler's formula, harmonic signal of amplitude U, frequency f and initial phase φ can be expressed in the following form:

$$u(t) = U\sin(\omega t + \varphi) = \mathrm{Im}\left[U\,e^{j\varphi}\,e^{j\omega t}\right] \tag{2.12}$$

where $\omega = 2\pi f$ is angular frequency and j is an imaginary unit. A graphical interpretation of the formula is presented in Figure 2.8. On the right-hand

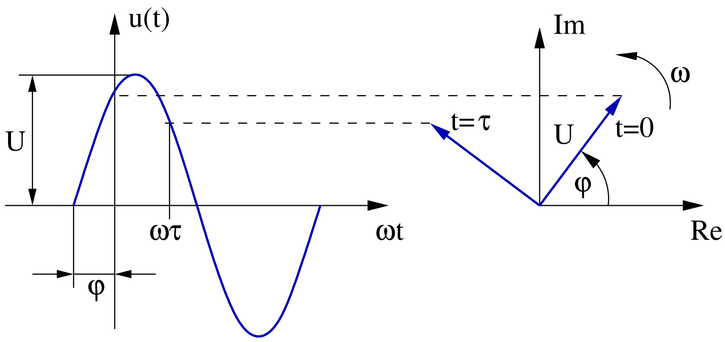


Figure 2.8

side, a vector of the length equal to the value of amplitude rotates counter-clock-wise with angular velocity ω in the complex plane. Sinusoidal function describes the projection of the vector on imaginary axis. The quantity $\underline{U} = U\,e^{j\varphi}$, defined as a *complex amplitude* of a signal, represents the position of rotating vector in the time instant $t = 0$. It is a complex number of modulus and argument corresponding to amplitude and initial phase of the signal. Inserting complex form of signals into general form of constitutive relations and taking into account that only component $e^{j\omega t}$ depends on time, the following relations are obtained:

$$\underline{I}_R\, e^{j\omega t} = -G\underline{U}_R\, e^{j\omega t}$$
$$\underline{I}_C\, e^{j\omega t} = -C\frac{d}{dt}\underline{U}_C\, e^{j\omega t} = -j\omega C\,\underline{U}_C\, e^{j\omega t}$$
$$\underline{I}_L\, e^{j\omega t} = -\frac{1}{L}\int \underline{U}_L\, e^{j\omega t}dt = -\frac{1}{j\omega L}\underline{U}_L\, e^{j\omega t}$$

Disregarding time component, which is the same for all considered signals (or assuming $t = 0$), algebraic relations between currents and voltages with respect to complex amplitudes are obtained. In matrix notation, they can be formulated as follows:

$$\underline{\mathbf{i}}_{\mathrm{R}} = -\mathbf{Y}_{\mathrm{R}}\,\underline{\mathbf{u}}_{\mathrm{R}};\quad \underline{\mathbf{i}}_{\mathrm{C}} = -\mathbf{Y}_{\mathrm{C}}\,\underline{\mathbf{u}}_{\mathrm{C}};\quad \underline{\mathbf{i}}_{\mathrm{L}} = -\mathbf{Y}_{\mathrm{L}}\,\underline{\mathbf{u}}_{\mathrm{L}}; \tag{2.13}$$

where the diagonal matrices of admittances are:

$$\mathbf{Y}_{\mathrm{R}} = \mathbf{G};\quad \mathbf{Y}_{\mathrm{C}} = j\omega\,\mathbf{C};\quad \mathbf{Y}_{\mathrm{L}} = \frac{1}{j\omega}\,\mathbf{L}^{-1}; \tag{2.14}$$

Modulus of admittance is a ratio of amplitudes and argument defines a phase shift between current and voltage. Admittance of capacitor and coil depends on frequency – capacitor attenuates low-frequency signals while coil attenuates high-frequency signals. In resistor, current and voltage are in counterphase ($\varphi_i = -\varphi_u$), in ideal capacitor current is delayed by a quarter of period ($\varphi_i = \varphi_u - \frac{\pi}{2}$) while in ideal coil current is ahead of voltage by a quarter of period ($\varphi_i = \varphi_u + \frac{\pi}{2}$).

Complex amplitudes describe instantenous states of signals, hence they have to comply with topological constraints of the circuit. Kirchhoff's current law expressed with respect to complex amplitudes is as follows:

$$\overset{\mathrm{R}}{\mathbf{M}}\,\underline{\mathbf{i}}_{\mathrm{R}} + \overset{\mathrm{C}}{\mathbf{M}}\,\underline{\mathbf{i}}_{\mathrm{C}} + \overset{\mathrm{L}}{\mathbf{M}}\,\underline{\mathbf{i}}_{\mathrm{L}} + \overset{\mathrm{E}}{\mathbf{M}}\,\underline{\mathbf{i}}_{\mathrm{E}} + \overset{\mathrm{J}}{\mathbf{M}}\,\underline{\mathbf{j}} = [\mathbf{0}]\,. \tag{2.15}$$

Applying voltage–potential and constitutive relations, as well as nodal condition related with voltage sources, the following system of equations can be formulated:

$$\begin{bmatrix} \tilde{\mathbf{Y}} & -\overset{\mathrm{E}}{\mathbf{M}} \\ \overset{\mathrm{E}}{\mathbf{M}}{}^{\mathrm{T}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{v}} \\ \underline{\mathbf{i}}_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} \overset{\mathrm{J}}{\mathbf{M}}\,\underline{\mathbf{j}} \\ \underline{\mathbf{e}} \end{bmatrix} \tag{2.16}$$

where the aggregated admittance matrix is:

$$\tilde{\mathbf{Y}} = \overset{\mathrm{R}}{\mathbf{M}}\,\mathbf{Y}_{\mathrm{R}}\,\overset{\mathrm{R}}{\mathbf{M}}{}^{\mathrm{T}} + \overset{\mathrm{C}}{\mathbf{M}}\,\mathbf{Y}_{\mathrm{C}}\,\overset{\mathrm{C}}{\mathbf{M}}{}^{\mathrm{T}} + \overset{\mathrm{L}}{\mathbf{M}}\,\mathbf{Y}_{\mathrm{L}}\,\overset{\mathrm{L}}{\mathbf{M}}{}^{\mathrm{T}}$$

It is a linear system of algebraic equations in complex number domain which can be expressed in more compact form as:

$$\underline{\mathbf{A}}\,\underline{\mathbf{x}} = \underline{\mathbf{z}} \tag{2.17}$$

The same as in the DC case, the main matrix $\underline{\mathbf{A}}$ is singular because KCL equations (2.15) have been formulated with respect to all nodes. In order to solve the system, ground conditions need to be applied. The procedure is exactly the same as in the DC case.

Example 2

To find base AC solution for the circuit of configuration presented in Figure 2.7. Parameters of sources are: E = 5V, J = $2j$ mA and frequency 200Hz.

Admittances of elements are: $\mathrm{Y_R} = 1$ mS, $\mathrm{Y_C} = 1.26j$ mS, $\mathrm{Y_L} = -796j$ mS. A solution to the system of equations (2.16) is:

$$\underline{\mathbf{v}} = [\,0 \quad 5 \quad 2.57 + 0.25j \quad 3.60 - 1.42j \quad 3.79 - 0.87j \quad 5.00 - 0.004j\,]^{\mathrm{T}}$$

$$\underline{\mathbf{i}}_{\mathrm{E}} = [4.36 + 4.77j] \times 10^{-3}$$

Let also calculate responses in capacitors:

$$\underline{\mathbf{u}}_{\mathrm{C}} = \overset{\mathrm{C}}{\mathbf{M}}{}^{\mathrm{T}}\underline{\mathbf{v}} = \begin{bmatrix} 3.60 - 1.42j \\ 2.44 - 0.26j \end{bmatrix} \qquad \underline{\mathbf{i}}_{\mathrm{C}} = -\mathbf{Y}_{\mathrm{C}}\underline{\mathbf{u}}_{\mathrm{C}} = \begin{bmatrix} -1.79 - 4.52j \\ -0.32 - 3.06j \end{bmatrix} \times 10^{-3}$$

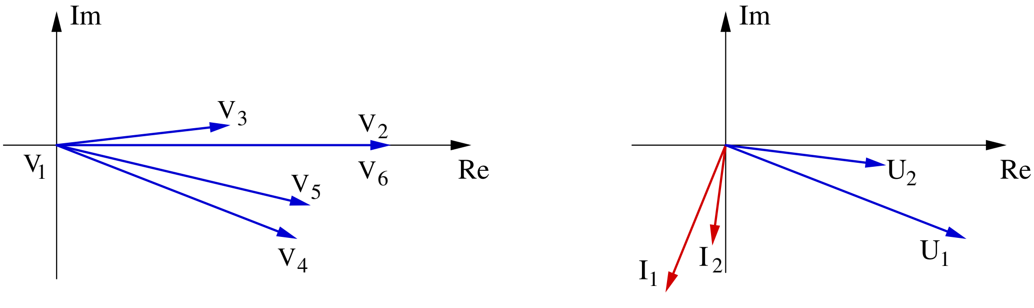


Figure 2.9: AC solution for Example 2

Solutions for nodal potentials and responses in capacitors, in the form of rotating vectors in complex plane (also called *phasors*), are presented in Figure 2.9. Length of the phasor defines amplitude and angle with respect to real axis – initial phase of the signal.

Transient analysis in discrete time domain

In a general case of arbitrary input signals, constitutive relations for circuit elements have the following form:

$$\mathbf{i}_{\mathrm{R}}(t) = -\mathbf{G}\,\mathbf{u}_{\mathrm{R}}(t); \quad \mathbf{i}_{\mathrm{C}}(t) = -\mathbf{C}\,\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{u}_{\mathrm{C}}(t); \quad \mathbf{i}_{\mathrm{L}}(t) = -\mathbf{L}^{-1}\int \mathbf{u}_{\mathrm{L}}(t)\,\mathrm{dt} \tag{2.18}$$

Inserting them into Kirchhoff's current law (Equation 2.6), along with the voltage–potential relation, gives the following relation:

$$\tilde{\mathbf{C}}\,\dot{\mathbf{v}}(t) + \tilde{\mathbf{G}}\,\mathbf{v}(t) + \tilde{\mathbf{K}}\int \mathbf{v}(t)\mathrm{dt} - \overset{\mathrm{E}}{\mathbf{M}}\,\mathbf{i}_{\mathrm{E}}(t) = \overset{\mathrm{J}}{\mathbf{M}}\,\mathbf{j}(t) \tag{2.19}$$

where aggregated matrices of parameters are:

$$\tilde{\mathbf{C}} = \overset{\mathrm{C}}{\mathbf{M}}\,\mathbf{C}\,\overset{\mathrm{C}}{\mathbf{M}}^{\mathrm{T}}; \quad \tilde{\mathbf{G}} = \overset{\mathrm{R}}{\mathbf{M}}\,\mathbf{G}\,\overset{\mathrm{R}}{\mathbf{M}}^{\mathrm{T}}; \quad \tilde{\mathbf{K}} = \overset{\mathrm{L}}{\mathbf{M}}\,\mathbf{L}^{-1}\,\overset{\mathrm{L}}{\mathbf{M}}^{\mathrm{T}}$$

Once again, currents in voltage sources have to be assigned as additional unknown variables and related nodal conditions have to be introduced. Ultimately, the following system of second-order differential equations can be obtained:

$$\begin{bmatrix}\tilde{\mathbf{C}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\ddot{\mathbf{x}}(t) + \begin{bmatrix}\tilde{\mathbf{G}} & -\overset{\mathrm{E}}{\mathbf{M}}\\ \overset{\mathrm{E}}{\mathbf{M}}^{\mathrm{T}} & \mathbf{0}\end{bmatrix}\dot{\mathbf{x}}(t) + \begin{bmatrix}\tilde{\mathbf{K}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{x}(t) = \mathbf{z}(t) \tag{2.20}$$

where the new variables are:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix}\mathbf{v}(t)\\ \mathbf{i}_{\mathrm{E}}(t)\end{bmatrix}; \quad \mathbf{z}(t) = \begin{bmatrix}\overset{\mathrm{J}}{\mathbf{M}}\,\mathbf{j}(t)\\ \mathbf{e}(t)\end{bmatrix}; \tag{2.21}$$

Ground conditions are of the form $\dot{x}_i(t) = 0$, hence i-th row and i-th colmun in all matrices should be set to zero, as well as $z_i(t) = 0$, except an entry $\tilde{G}_{ii} = 1$.

Solution for the arbitrary dynamic input signals, defined over uniformly discretized time domain, will be obtained using the Newmark algorithm (implicit numerical integration). Newmark algorithm enables to directly integrate the obtained system of equations, other integration procedures (e.g. Runge-Kutta) would require to transform the system into the first-order differential equations in the normal form (which may require reformulation with respect to state variables using the SVD).

Example 3

To find nodal potentials for the circuit of configuration presented in Figure 2.7, for input signals presented in Figure 2.10. Voltage source supplies one sine wave of amplitude 5V and current source supplies step function of intensity 4mA. Time instants are: t_1 = 2ms, t_2 = 5ms, t_3 = 8ms. Assume total time of analysis T=10ms and zero initial conditions (non-energetic state).

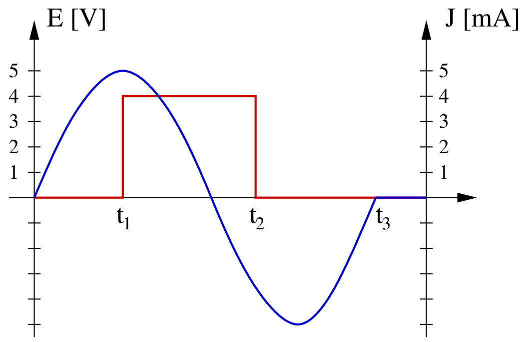


Figure 2.10: Scenario of dynamic excitation

Time of analysis has been discretized into 100 time steps of 0.1ms. The results are presented in Figure 2.11.

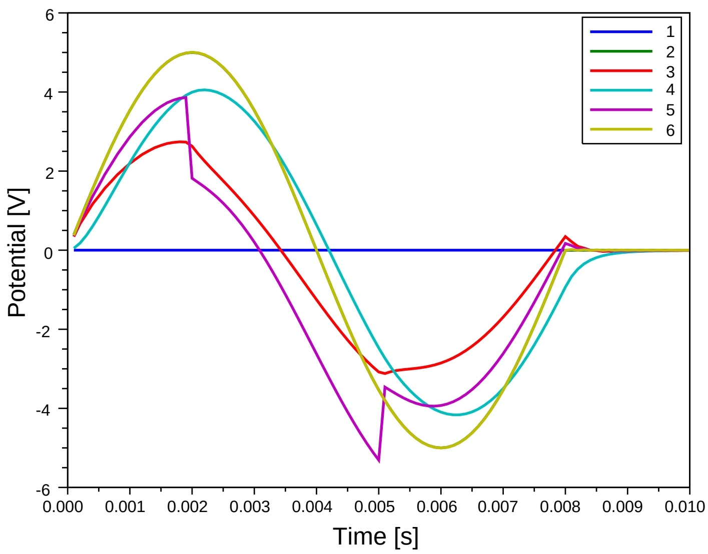


Figure 2.11: Results of transient analysis

2.4 Electro-mechanical analogies

It is a common occurrence that simple models of various physical systems (mechanical, electrical, magnetic, hydraulic, acoustic) are described by relations of the same mathematical form. The similarities can be noticed both in the form of constitutive relations, which describe models of elementary components, as well as in the form of physical constraints, which govern global system behavior. For example, truss structures, resistive circuits or water pipelines may be considered as discrete systems of skeletal topology (represented in a form of a graph), whose elements/branches are described by linear constitutive relations (Hooke's law, Ohm's law, flow equation) and global behavior is governed by topological constraints (equilibrium of forces, currents and flows in nodes and compatibility of deformations, voltages and pressures around loops). As a consequence, they all can be analyzed using general network theory and the obtained global states of responses are mutually equivalent (states of strains/stresses, voltages/currents, pressure heads/flow rates).

Electro-mechanical analogies [30, 31, 32] is a well known and established concept, developed at the beginning of the 20th century. At that time, studies on mechanical and electrical vibratory systems were strongly motivated by the development of electro-mechanical and electro-acoustic transducers and devices such as telephone, telegraph or motion pictures [33]. Since the methods of circuit analysis were much better developed, analysis of mechanical and electromechanical systems was carried out using network theory. Namely, the methodology was to formulate an electrical equivalent circuit, analyze it and translate the solution back into mechanical terms. The basis of the methodology was recognition of similarities between parameters and physical quantities occurring in mathematical description of electrical and mechanical systems. In the first approach, depicted later as a *direct analogy*, a single-degree-of-freedom mass-spring-damper system, excited by an external force, was considered an mechanical equivalent of a serial connection of resistor, capacitor and coil, supplied by voltage source (Figure 2.12).

$$
\begin{aligned}
f(t) &= m\ddot{x}(t) + c\dot{x}(t) + kx(t) \\
e(t) &= L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t)
\end{aligned}
\tag{2.22}
$$

Dynamic behavior of both systems is expressed in a form of ordinary, second-order differential equations (2.22), where independent variables are displacement $x(t)$ of a mass and electric charge $q(t)$ in capacitor. In mechanical system, the relation describes a state of equilibrium between the external

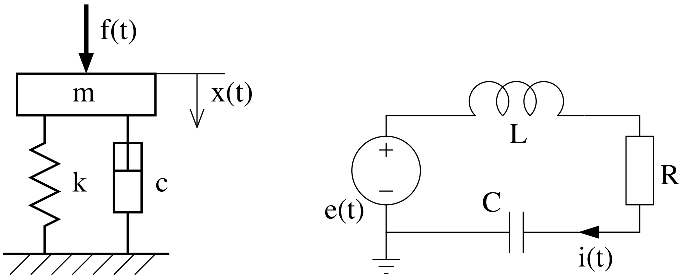


Figure 2.12: Direct electro-mechanical analogy

force and the sum of inertial, elastic and damping forces, while in electrical system, the relation expresses Kirchhoff's voltage law (electromotive force of a source is balanced by the sum of voltages across RLC components). It means that equilibrium condition in node is equivalent to continuity condition along a loop. As a result, in direct analogy, junctions in mechanical system have to be modeled as loops in electrical circuit and vice-versa. To overcome this drawback, the concept of *inverse analogy* was proposed, where the same mechanical system had an equivalent in parallel RLC circuit, supplied by a current source (Figure 2.13).

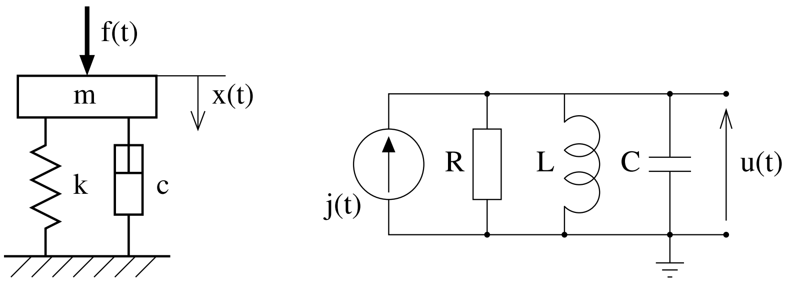


Figure 2.13: Inverse electro-mechanical analogy

$$\begin{aligned} f(t) &= m\ddot{x}(t) + c\dot{x}(t) + kx(t) \\ j(t) &= C\ddot{\varphi}(t) + \frac{1}{R}\dot{\varphi}(t) + \frac{1}{L}\varphi(t) \end{aligned} \tag{2.23}$$

This time, nodal principle of force equilibrium is equivalent with Kirchhoff's Current Law (Equation 2.23), where independent variable for electrical system is magnetic flux $\varphi(t)$ in coil. Electric charge or magnetic flux are not convenient variables in circuit analysis, hence expressing the relations with respect to velocity $v(t)$, current $i(t)$ in serial circuit and voltage $u(t)$ in parallel circuit, the following equivalent equations can be obtained:

$$
\begin{aligned}
f(t) &= m\frac{\mathrm{d}v(t)}{\mathrm{dt}} + cv(t) + k\int v(t)dt \\
e(t) &= L\frac{\mathrm{d}i(t)}{\mathrm{dt}} + Ri(t) + \frac{1}{C}\int i(t)dt \\
j(t) &= C\frac{\mathrm{d}u(t)}{\mathrm{dt}} + \frac{1}{R}u(t) + \frac{1}{L}\int u(t)dt
\end{aligned}
\tag{2.24}
$$

In analysis of oscillating systems, there is also an intuitive analogy from the point of view of energetic tranformations. In mechanical case, kinetic energy of a mass changes into potential energy accumulated in a spring and is partialy dissipated by a damper. In electrical case, energy is transferred between capacitor (electrical field) and coil (magnetic field) and is partialy dissipated by resistor (heat).

Truss-circuit analogy

Electromechanical inverse analogy ensures topological conformity between models of truss structures and electrical circuits. In order to define full set of equivalent quantities, simple static truss structures and resistive DC circuits will be compared. Let consider two examples of one-dimensional structures made of two bar members of the same initial length L and different stiffnesses k_1 and k_2. In the first configuration (Figure 2.14), elements are in parallel,

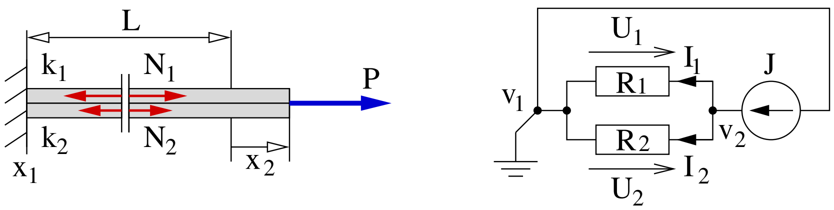


Figure 2.14: Truss - circuit analogy: parallel system

fixed at one end and sharing the same node at free end, and are subjected to external force P. Both elements are equally strained and internal forces are in equilibrium with external force:

$$
\varepsilon_1 = \varepsilon_2\,; \qquad N_i = k_i\varepsilon_i L\,; \qquad N_1 + N_2 = P \tag{2.25}
$$

Geometrical relations and boundary conditions are:

$$
\varepsilon_i L = x_2 - x_1\,; \quad x_1 = 0 \tag{2.26}
$$

An equivalent electrical circuit consists of two resistors in parallel, supplied by ideal current source. Elements share the same voltage and the state of currents follows the KCL:

$$U_1 = U_2; \qquad I_i = \frac{U_i}{R_i}; \qquad I_1 + I_2 = J \tag{2.27}$$

Relation between voltage and nodal potentials, as well as ground conditions are:

$$U_i = v_2 - v_1; \qquad v_1 = 0 \tag{2.28}$$

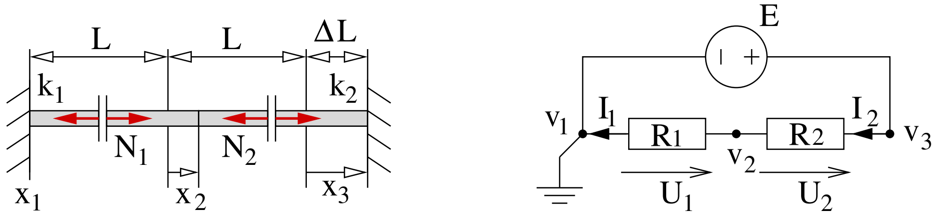


Figure 2.15: Truss - circuit analogy: serial system

In the second configuration (Figure 2.15), bar elements are in series and are subjected to some initial deformation $\varepsilon^0 = \Delta L/2L$. This time, internal forces are equal but strains are different:

$$N_1 = N_2; \qquad \varepsilon_i L = N_i/k_i; \qquad \varepsilon_1 L + \varepsilon_2 L = 2L\varepsilon^0 \tag{2.29}$$

Geometrical relations are:

$$\varepsilon_1 L = x_2 - x_1; \quad \varepsilon_2 L = x_3 - x_2; \quad x_1 = 0 \tag{2.30}$$

In equivalent circuit, resistors are also in series and are supplied by ideal voltage source. Elements share the same current and the state of voltages follows the KVL:

$$I_1 = I_2; \qquad U_i = R_i I_i; \qquad U_1 + U_2 = E; \tag{2.31}$$

Voltage-potential relations and ground conditions are:

$$U_1 = v_2 - v_1; \qquad U_2 = v_3 - v_2; \qquad v_1 = 0 \tag{2.32}$$

Comparing the relations, a full system of equivalent parameters, quantities and governing principles between static truss structures and resistive DC circuits can be established (Table 2.3). Naturally, such ideal system of

	Truss	Circuit
Parameter	Stiffness	Conductance
External load	Force Initial strain	Current source Voltage source
Responses	Strain Axial force	Voltage Current
Nodal quantity	Displacement	Electric potential
Constitutive relation	Hooke's law	Ohm's law
Global principles	Equilibrium of forces Continuity of deformations	Kirchhoff's Current Law Kirchhoff's Voltage Law
Boundary conditions	Blocked displacement	Ground

Table 2.3: Truss-circuit analogy

analogies exists only for simple one dimensional systems. Forces and displacements are vector quantities, hence in the case of 2D plane structures, there are two degrees of freedom in every node and two independent equilibrium conditions. Currents and electric potentials are scalars, hence regardless of topology and spatial arrangement of circuit components, there is always a single "degree of freedom" in a node. Another intriguing aspect is the condition of truss stability, i.e. mutual relation between numbers of members, joints and reactions makes a structure statically determinate, indeterminate or a mechanism. In electrical circuits, numbers of nodes and elements aren't related by any similar condition – circuit can always be solved based on Kirchhoff's laws. Statically determinate structure can be solved based only on force equilibrium equations – in extreme assumption, an equivalent circuit would have to be solved based only on Kirchhoff's current laws, but such a circuit couldn't contain any loops and any topological similarities would be lost. However, there can be defined an analogy concerning local static determinacy. This is a case when deformation or load imposed on truss member influences only geometry of a structure (modifies nodal displacements) without generating strains or stresses in other members. An electrical equivalent is a component with imposed source that doesn't generate responses in other components, only modifies distribution of potentials in nodes. Such scenario may occur if component has one "loose" terminal, not connected to any node of a circuit. There are also some other minor differences which imply that exact physical interpretation of analogies may be sometimes tricky, difficult or even impossible. The examples are:

- An equivalent of ideal source of voltage inserted between a pair of circuit nodes is a rigid truss member which enforces fixed distance between nodes or degrees of freedom and may carry arbitrary axial force.

- Source of current is a two-terminal element, hence current supplied through one node of a circuit is taken back from another node. Depending on boundary conditions, analogous excitation by external force imposed on a junction of a truss structure may be globally compensated by several reactions in different nodes. System of load–reactions may be then considered like a multi-terminal current source of one output and several inputs.

- Masses of truss members are lumped in nodes so that way inertial forces are reduced to nodes. Nodes in circuits cannot be associated with any electrical parameter.

The most significant problem is however the interpretation of dynamical analogies. In the case of static truss/DC circuit, stiffness has been assumed to be an equivalent of conductance. However, from Equation (2.24) results that in dynamic system, conductance should be considered as an equivalent of damping coefficient (both damper and resistors are elements which dissipate energy). To keep the compatibility with statics, let rewrite dynamic equations for inverse analogy with respect to new independent variables: displacement $x(t)$ and electric potential $v(t)$ in free nodes of the systems:

$$\begin{aligned} f(t) &= m\ddot{x}(t) + c\dot{x}(t) + kx(t) + \quad \dots \\ j(t) &= \quad \dots \quad + C\dot{v}(t) + \frac{1}{R}v(t) + \frac{1}{L}\int v(t)dt \end{aligned} \tag{2.33}$$

In such interpretation, resistor corresponds to spring, capacitor to damper but no electrical equivalent of mass and no mechanical equivalent of inductor are defined. Although the discrepancy of parameters, consistent analogies for basic electrical and mechanical quantities are obtained: currents are equivalents of forces, voltages of strains, electric potentials of nodal displacements, supply by current source is an equivalent of external force and supply by voltage source is an equivalent of initial deformation.

FEM vs. MNA

Analogies between trusses and circuits can be also noticed in the form of equations obtained from standard methods of analysis. In the simplest case of plane truss structure subjected to static load and resistive DC circuit supplied by current sources, global systems of equations produced by Direct Stiffness Method and classical Nodal Analysis are:

$$\tilde{\mathbf{K}}\mathbf{x} = \mathbf{f} \qquad \tilde{\mathbf{G}}\mathbf{v} = \mathbf{j}$$

Global stiffness matrix $\tilde{\mathbf{K}}$ corresponds to global conductance matrix $\tilde{\mathbf{G}}$, vector of unknown nodal displacements $\mathbf{x}$ to vector of unknown electric potentials $\mathbf{v}$ and input vector of external nodal forces $\mathbf{f}$ to input vector of source currents $\mathbf{j}$. It is evident that all quantities comply with the assumed system of truss-circuit analogies (Table 2.3). Global matrices of parameters are aggregated in the following way:

$$\tilde{\mathbf{K}} = \mathbf{B}^{\mathrm{T}}\mathbf{K}\,\mathbf{B} \qquad \tilde{\mathbf{G}} = \mathbf{M}\,\mathbf{G}\,\mathbf{M}^{\mathrm{T}}$$

where $\mathbf{B}$ is geometric matrix, $\mathbf{M}$ is incidence matrix and $\mathbf{K}$ and $\mathbf{G}$ are diagonal matrices of stiffnesses and conductances in elements. The natural difference is that for trusses both topological and geometrical transformation from local to global coordination system is required while in circuits only topological association of elements with nodes takes place.

In a more general case, Modified Nodal Analysis may be viewed as an equivalent of the Finite Element Method. The similarities are apparent in mathematical form of global system of equations, general scheme of matrix aggregation or handling boundary conditions (for example, system of equations (2.20) resembles equations of motion for mechanical systems). Virtual Distortion Method operates on FEM-based models of truss or frame structures. It introduces some concepts and quantities which are strictly associated with parameters of mechanical model and its input and output functions. In spite of some discrepancies in details, the overall consistency of assumed analogies between trusses and circuits not only will enable to adapt the concepts of VDM but will also provide their simple and intuitive interpretation.

Chapter 3

Virtual Distortion Method in circuit analysis

Virtual Distortion Method is classified as a technique of fast, exact, static and dynamic re-analysis, but its scope of applications includes many other numerical problems (e.g. sensitivity analysis, topological optimization or damage identification). In essence, the method uses a discrete model of a system in the original configuration and introduces a field of *virtual distortions* to simulate effects caused by modifications of structural parameters. Virtual distortions can be interpreted as some external perturbations imposed locally on system elements which affect the global state of responses. In structural mechanics, distortions are associated with deformation modes possible for a given kind of finite element (e.g. initial strains in bars). Simulated behavior of modified system is obtained as a superposition of primary responses produced by real excitations and residual responses invoked by virtual distortions. From the perspective of numerics, global matrices of parameters occurring in mathematical description of a system remain unchanged since the modifications are introduced through equivalent input vectors. That way, not only time-consuming matrix operations can be avoided, most of applied algorithms can be formulated with regard to pre-selected sets of elements and responses, hence dimensions of considered problems can be substantially reduced. It was demonstrated in [22] that VDM-based approach complies with a general Sherman-Morrison and Woodbury formulas, which enble to calculate an inversion of a matrix subjected to variations. In this sense, VDM belongs to the family of methods of exact re-analysis which are especially effective if the number of modifications is significantly lower then the dimension of original problem. Similar methods from the same group are *second theorem of structural variation* [23] and *pseudoforce method* [24] which, in general scheme, are almost identical but have been developed independently and hence use different notions and nomenclature. A distinctive feature of the VDM is the concept of *influence matrix* which reflects local-global interrelations between normalized states of virtual distortions and selected system responses. Influence matrices comprise a computational core for all numerical algorithms formulated within the method – they are used to evaluate distortions, update responses and calculate sensitivity. Also, their properties provide crucial information about system characteristics or conditioning of considered problem.

Foundations of the Virtual Distortion Method were initiated in early works by Holnicki-Szulc et al. [34, 35, 36]. First engineering applications included remodeling, simulation of progressive collapse, topological optimization and sensitivity analysis. VDM operates mainly on models of skeletal structures (trusses, frames) where the number of independent distortions to be imposed on finite element is small, i.e one for bars (initial strains), three for beams (axial, bending and bending/shear modes). At a current stage of development, VDM enables to simulate modifications of stiffness and mass, as well as the occurrence of plastic thresholds (i.e. elements of characteristic approximated by piecewise linear sections may be included in analysis). In recent years, VDM has been successfully applied to solve some inverse problems including model updating, damage identification and load reconstruction. Procedures of damage identification have been formulated for statics [37], dynamics in time domain [38] and quasi-statics in frequency domain [39]. Problem of dynamic load reconstruction was discussed in [40] while the combined load and damage identification procedure was demonstrated in [41]. Other recent developments include design of adaptive structures for impact loads [42, 43], modeling and detection of delamination [44] and model-less identification of mass modifications [45]. Thanks to structural analogies, the method has been also adapted to analysis of other engineering systems. In paper [46], an application to leakage detection in water networks was presented. First attempts to apply VDM concepts in the area of electrical circuit analysis were demonstrated in [47, 48]. The most comprehensive and up-to-date overview of the method and its application in various engineering systems can be found in [49] or [50].

3.1 Overview for truss structures

Main notions and concepts of the VDM are easiest to demonstrate on the example of linear plane truss structure, of some redundancy, subjected to static load. In this case, virtual distortions are interpreted as a field of *initial* *strains* imposed on the model of original structure in order to simulate modifications of stiffness. A conceptual scheme of the method is demonstrated in Figure 3.1. If the original unloaded structure (Fig 3.1a) is subjected to

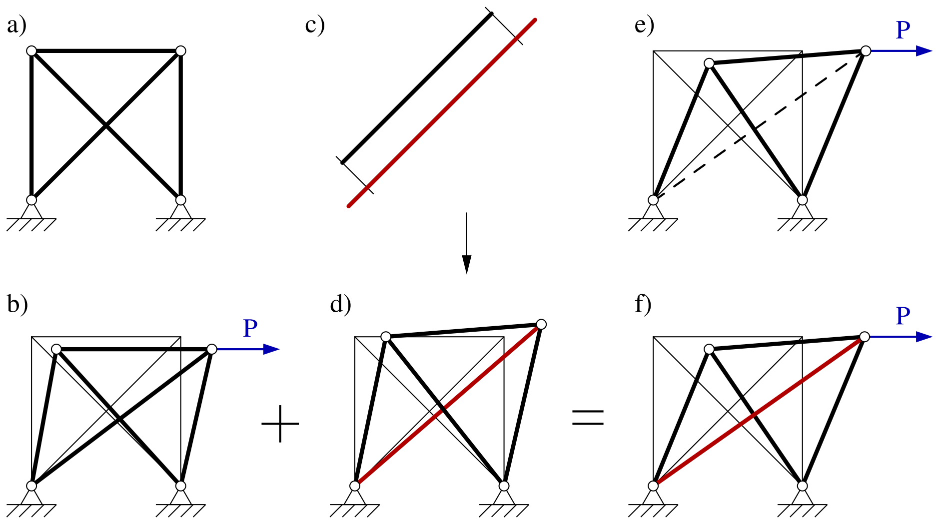


Figure 3.1: Conceptual scheme of the VDM: a) original structure, b) loaded structure, c) virtual distortion, d) pre-stressed structure, e) modified structure, f) simulated structure

some external load, for example static force P depicted in Fig 3.1b, then it will undergo deformation and the accompanying state of strains/forces in elements, called a *linear response*, will occur. Changes of shape depicted in the figure are naturally exaggerated just to better visualize the concept, in any case small deformations, i.e. geometrical constancy and elastic range of strains/stresses, are assumed. If properties of some members were different, for example there was a loss of stiffness in bar element depicted by a slashed line in Fig 3.1e, then under the same load and boundary conditions, a *modified response* would be produced. Let suppose that the same element was set free from the original structure and initially strained (Fig 3.1c). It is said that virtual distortion, which can be interpreted as a thermal expansion caused by homogenous heating, have been imposed on the element. If such initially strained element is assembled again in the original, unloaded structure, then as a result of geometrical incompatibility, some *residual response* will occur – a structure is said to be in a *pre-stressed* state (Fig 3.1d). *Simulated response* obtained in the case of simultaneously pre-stressed and loaded structure (superposition of linear and residual responses – Fig 3.1f) is assumed to be the same as the response in modified structure.

Basically, VDM postulates that there exists a field of virtual distortions, which for a fixed load scenario, introduces the same global changes in structural responses as modifications of parameters. The postulated equality between models of modified and simulated structure is valid for multiple, simultaneous modifications and arbitrary kind of external load. In dynamic analysis, virtual distortions become time-dependent functions (generally, form of distortions imitates the form of response functions). Computational efficiency of the VDM results from the fact that responses generated by external loads and virtual distortions are considered independently – linear responses are evaluated from the finite element model at the initial stage of analysis while for every set of modifications (e.g. damage scenarios), VDM-based algorithms are used to calculate equivalent field of virtual distortions and the resulting state of residual responses. Another upside of the approach is the capability to analytically calculate gradients of circuit responses with respect to virtual distortions, hence they may serve as effective steering variables in various optimization procedures.

In static or quasi-static analysis (constant, harmonic loads), residual response is simply a linear combination of virtual distortions, where coefficients of the combination, comprising an influence matrix, are stationary and evaluated from the model of original structure. State of strains in a truss structure under static load with imposed field of virtual distortions can be expressed as follows:

$$\varepsilon_i = \varepsilon_i^L + \sum_j D_{ij}\varepsilon_j^0 \tag{3.1}$$

where ε_i^L is a vector of linear responses, ε_j^0 is a vector of virtual distortions and D_{ij} is the influence matrix. Columns of the matrix, called *influence vectors*, store strain responses produced by *unit virtual distortions* ($\varepsilon^0 = 1$) imposed selectively on truss members in the case of original, unloaded structure. Comparing strain-force relations in modified and simulated configurations, the following relation for virtual distortions can be derived:

$$\varepsilon_i^0 = (1 - \mu_i)\,\varepsilon_i \tag{3.2}$$

where parameter μ_i is a measure of stiffness modification (a ratio of modified versus original value), which may as well describe modifications of Young's modulus E or cross-sectional area A of bar element:

$$\mu_i = \frac{\tilde{k}_i}{k_i} = \frac{\tilde{E}_i\tilde{A}_i}{E_iA_i} \tag{3.3}$$

In Equation (3.2), virtual distortion is related with simulated strain response which, vide Equation (3.1), depends on all active distortions. Combining both equations, a system of linear equations which directly relates distortions with modifications parameters can be derived:

$$\sum_j \Big[\mathbb{I}_{ij} - (1 - \mu_i) D_{ij}\Big] \varepsilon_j^0 = (1 - \mu_i)\, \varepsilon_i^L \tag{3.4}$$

The above system may be formulated locally, with respect to modified elements only. This forms a basis for algorithm of quick re-analysis: instead of full system recalculation with new modified global stiffness matrix, two smaller system need to be solved – one to evaluate distortions and another to update responses.

In dynamic analysis, virtual distortions, being in general continuous, time-dependent functions, are discretized and treated as sequences of impulses. Residual responses are calculated as a superposition of impulse responses, making use of the concept of dynamic influence matrix. Strain responses in simulated structure can be expressed as follows:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \sum_{\tau=0}^{t} \sum_j D_{ij}(t-\tau) \varepsilon_j^0(\tau) \tag{3.5}$$

In this case, influence vectors store dynamic strain responses produced by unit impulse virtual distortions imposed selectively on truss members in the first time instant. Relation for distortion resulting from comparison of strain-force relations in modified and simulated structures is as follows:

$$\varepsilon_i^0(t) = (1 - \mu_i)\, \varepsilon_i(t) \tag{3.6}$$

Again, in order to obtain direct relation between distortions and modification parameters, Equations (3.5) and (3.6) can be combined and a set of linear systems, specified for every time instant, can be derived.

$$\sum_j \Big[\mathbb{I}_{ij} - (1-\mu_i) D_{ij}(0)\Big] \varepsilon_j^0(t) = (1-\mu_i) \Big[\varepsilon_i^L(t) + \sum_{\tau=0}^{t-1} \sum_j D_{ij}(t-\tau) \varepsilon_j^0(\tau)\Big] \tag{3.7}$$

The above set of equations has been formulated is such a way as to obtain the same main matrix for each time step. As a result, it needs to be solved sequentially but only right-hand-side vector needs to updated in successive time steps.

3.2 Electrical virtual distortions

In a most general sense, regardless of physical nature of considered system, virtual distortions may be defined as additional input functions which imitate the influence of local structural modifications on global system response. In structural mechanics, virtual distortions are associated with deformation modes possible for a given kind of finite element. Particularly, in a case of truss structures, they take a form of initial strains which simulate the influence of stiffness modifications on strain/stress distribution. In water networks (see [46] for reference), virtual distortions are interpreted as additional pressure heads enforced in network branches, which simulate effects exerted by modifications of hydraulic compliance or local leakages on pressure/flow distribution. Generally, introduction of virtual distortions results in generation of some additional states of responses, which can be next superposed with primary responses produced by real excitations (acting loads, active pumps) in order to obtain simulated behavior of modified system.

Following the idea, electrical virtual distortions may be considered as some perturbations which affect the state of currents/voltages in the same way as modifications of electrical parameters in passive elements of a circuit. Implementation of the concept will be done using the system of electromechanical analogies (vide Section 2.4). A base for considerations is a model of linear truss structure under static load and a comparative model of linear DC circuit. In trusses, virtual distortion simulates modification of stiffness k and is considered as an initial strain ε^0 imposed on the bar element taken out of a structure (Figure 3.2). State of the element with imposed virtual

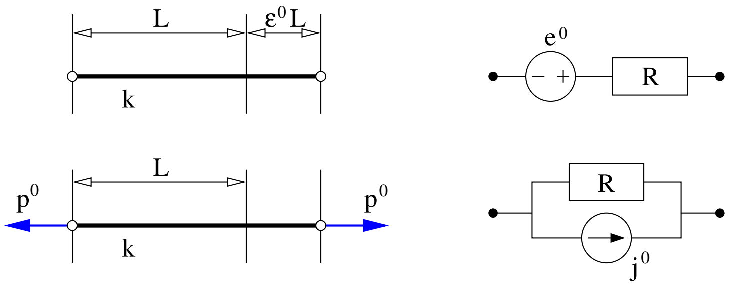


Figure 3.2: Virtual distortions in trusses and circuits

distortion can be interpreted as an elongation/shrinkage caused by homogeneous heating/cooling – it means that element is deformed ($\varepsilon = \varepsilon^0$) without the occurence of internal force ($N = 0$). The same state of initial deformation may be also realized by a pair of self-equilibrated virtual forces $p^0 = k\,\varepsilon^0 L$

applied in the nodes associated with the element. Practically, this is the way of introducing initial strain into the finite element model of a structure. Even though virtual forces introduce both strain and internal force in a freed element ($\varepsilon = \varepsilon^0, N = p^0$), overall effect exerted on a structure, after reassembly of the distorted element, will be exactly the same for both kinds of perturbations. Naturally, strains and stresses will be invoked in other truss members only in the case of statically undetermined structure. In the case of local static determinacy, imposed distortion will only affect geometry of a structure (displacements of nodes).

According to the defined truss–circuit analogy (Table 2.3), electrical equivalent of initial strain in bar element is ideal source of voltage e^0 inserted in series with resistor, while equivalent of a pair of axial forces is ideal source of current j^0 inserted in parallel. In this arrangement, a direct equivalent of stiffness is conductance. Let notice that resistor with imposed virtual distortion can be considered as a model of real source, where resistance of component acts as an inner resistance of source. Current and voltage distortions can be then easily interchanged, i.e. the condition $e^0 = R\,j^0$ need to be satisfied. Although responses in a component taken out of a circuit are different for different kind of imposed distortion ($U = e^0, I = 0$ for voltage distortion, $U = e^0, I = j^0$ for current distortion), overall impact on circuit reponses by both perturbations will be the same.

VDM-based algorithms used in truss structure analysis are usually formulated with regard to virtual distortions in a form of initial strains. Hence, following strictly the analogy, further consideration should be proceed using voltage distortions. However, in the MNA-based formulation of circuit equations (vide Section 2.3), current sources are much easier to handle numerically, since only the right-hand side vector of excitations need to be updated, while aggregation of voltage sources would require to extend system dimension by introducing additional nodal constraints. For this reason, in further considerations, electrical virtual distortions will be implemented only in the form of additional current sources.

Let now extend the concept onto more general case of linear RLC circuits, supplied by arbitrary kind of excitation signals, where modifications of both conductance and capacitance are assumed. On one hand, let consider a model of modified circuit, where some components, due to damage or a purposeful change, are of modified values of conductance $\widehat{G}$ or capacitance $\widehat{C}$ (Figure 3.3). On the other hand, let introduce an equivalent VDM-based model, where modified components are simulated by original components

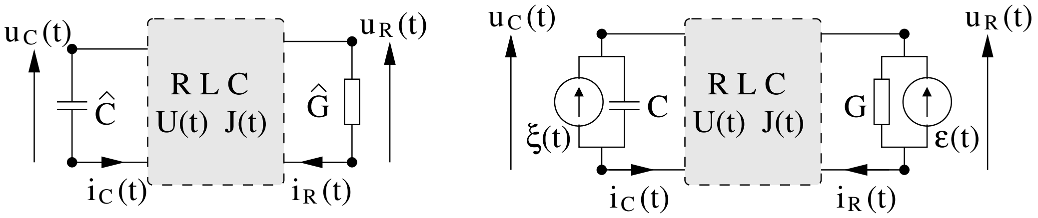


Figure 3.3: Modified vs. simulated model of a circuit

(G or C) coupled with virtual distortions $\varepsilon(t)$ (simulating modification of conductance) or $\xi(t)$ (simulating modification of capacitance). Both $\varepsilon(t)$ and $\xi(t)$ are considered as ideal current sources, generally time-dependent, inserted in parallel. A sufficient condition for the equivalence of both models is that currents and voltages registered on terminals of modified/simulated components are identical. If the condition is fulfilled then all global relations (resulting from Kirchhoff's laws) are intact and consequently, all responses in other parts of a circuit remain unchanged. The following relations can be written:

	Resistor	Capacitor
Modified component	$i_R(t) = -\widehat{G}\, u_R(t)$	$i_C(t) = -\widehat{C}\, \dot{u}_C(t)$
Distorted component	$i_R(t) - \varepsilon(t) = -G\, u_R(t)$	$i_C(t) - \xi(t) = -C\, \dot{u}_C(t)$

Assuming equality of corresponding currents and voltages, the following conditions can be derived:

$$\varepsilon(t) = (1-\mu)\, G\, u_R(t) \tag{3.8a}$$

$$\xi(t) = (1-\nu)\, C\, \dot{u}_C(t) \tag{3.8b}$$

where parameters μ and ν define magnitudes of conductance and capacitance modifications:

$$\mu = \frac{\widehat{G}}{G}; \qquad \nu = \frac{\widehat{C}}{C} \tag{3.9}$$

Shape of a function describing virtual distortion is the same as the shape of voltage response in resistor $u_R(t)$ or derivative of voltage in capacitor $\dot{u}_C(t)$. Since these responses refer to modified/simulated circuit configuration, they depend on all introduced modifications/distortions. Direct relations between virtual distortions and modification parameters will be derived in Section 3.4, after the adaptation of the concept of influence matrix. Temporarily, it can be concluded that virtual distortions appear in simulated circuit in exactly the same elements where modifications of parameters take place in modified circuit. However, their values depend not only on magnitudes of simulated modifications but also on the presence of other distortions. Naturally, the form of distortions is strictly related with the type of input signals produced by real sources. In the steady-state DC analysis, relations (3.8) will have the following form:

$$\varepsilon = (1 - \mu)\, G\, U_R \tag{3.10a}$$

$$\xi = 0 \tag{3.10b}$$

Since the value of capacitance doesn't influence DC circuit responses (it only affects the amount of charge stored in capacitor), the concept of virtual distortions is of no use – only modifications of conductance can be simulated. Virtual distortions naturally take the form of ideal current sources of constant intensity. In the steady-state AC analysis, the relations are:

$$\underline{\varepsilon} = (1 - \mu)\, Y_R\, \underline{U}_R \tag{3.11a}$$

$$\underline{\xi} = (1 - \nu)\, Y_C\, \underline{U}_C \tag{3.11b}$$

In this case, virtual distortions, as all other signals, are expressed in a form of complex amplitudes – it means they are harmonic sources of some amplitude and phase and of frequency conforming with the frequency of acting real sources.

In any case, introduction of virtual distortions into the model of a circuit will result in generation of some additional states of currents/voltages. Adapting the VDM-based nomenclature, the following notions concerning circuit responses will be used:

Linear response is a state of response (currents, voltages, nodal potentials) generated by *real sources* in a circuit of *original* configuration.

Modified response is a state of response generated by *real sources* in a circuit of *modified* configuration.

Residual response is a state of response generated by *virtual distortions* in a circuit of *original* configuration with real sources discarded (i.e. sources of voltage are replaced by short-circuits, sources of current are replaced by breaks).

Simulated response is a state of response generated by *real sources* and *virtual distortions* in a circuit of *original* configuration.

Reiterating the main VDM postulate, responses in modified circuit are assumed to be globally equal with simulated responses, which are superpositions of linear and residual responses.

3.3 Electrical influence matrices

In general idea, influence matrices define local-global interrelations between selected system responses and normalized states of virtual distortions. For example, a single entry D_{ij} of the static strain influence matrix (vide Equation 3.1) defines strain response in the i-th element of truss structure, invoked by unit virtual distortion (unit initial strain) imposed on the j-th element. Values of responses storied in influence matrices are pure virtual since the unit strain implies that the distorted element is twice its initial length. Not only it doesn't reflect any physically possible modification of stiffness, it violates the assumption of small deformations. Influence matrix is just a collection of lineary scaled responses on globally normalized perturbation, identical for all considered locations. Depending on the content and application, different types of influence matrices can be defined. In truss structure analysis, the notions of *strain influence matrix*, which is used to evaluate distortions, and *general influence matrix*, which is used to update responses or calculate gradients, are distinguished.

Taking into account the interpretation of electrical virtual distortion, a single influence vector, comprising the column of *electrical influence matrix*, may be considered as a set of circuit responses caused by unit ideal current source (constant, harmonic or impulse, depending on type of analysis) inserted in parallel with selected component. Again, entries of the influence matrix should be treated as lineary scaled responses on globally normalized input function, which are pure virtual and hence don't violate any physical limitations (e.g. power-rating of circuit components or ampacity of connecting wires). In DC/AC case, the relation for influence matrix can be derived directly from circuit equations describing its original, modified and simulated configuration. In dynamic case, interpretation and procedure of computing the dynamic influence matrix will be based on the idea of Duhamel's integral and analogies with truss structures.

DC case

Let consider a steady-state DC circuit with assumed modifications of conductances. In the original configuration, circuit is described by the MNA-based system of linear algebraic equations (vide Section 2.3, Equation 2.11). In the case of modified circuit, a new main matrix, reflecting the introduced changes of conductances, has to be assembled. In the case of simulated circuit, the main matrix is the same as for the initial configuration but additional input vector, reflecting the influence of virtual distortions, has to be set up. Equations describing initial, modified and simulated circuit configurations are as follows:

Initial circuit	Modified circuit	Simulated circuit
$\mathbf{A}\mathbf{x}^{\mathrm{L}} = \mathbf{z}$	$\tilde{\mathbf{A}}\mathbf{x} = \mathbf{z}$	$\mathbf{A}\mathbf{x} = \mathbf{z} + \mathbf{z}^{\varepsilon}$

$\mathbf{A}$ and $\tilde{\mathbf{A}}$ are main matrices assembled for the original and modified sets of parameters, $\mathbf{x}^{\mathrm{L}}$ is a base solution for original circuit configuration (linear response), $\mathbf{x}$ is a modified/simulated response, $\mathbf{z}$ is an input vector comprising active sources and $\mathbf{z}^{\varepsilon}$ is an additional input vector comprising virtual distortions. Since virtual distortions are implemented as ideal current sources inserted in parallel with resistors, the additional input vector can be formulated as:

$$\mathbf{z}^{\varepsilon} = \begin{bmatrix} \overset{\mathrm{R}}{\mathbf{M}}\boldsymbol{\varepsilon} \\ \mathbf{0} \end{bmatrix} \tag{3.12}$$

where the number of additional zero entries is equal to the total number of voltage sources and coils (i.e. it is the number of additional unknown variables beside nodal potentials). Vector of distortions $\boldsymbol{\varepsilon}$ is defined globally with respect to all resistors. It is also assumed that matrices $\mathbf{A}$ and $\tilde{\mathbf{A}}$, as well as vectors $\mathbf{z}$ and $\mathbf{z}^{\varepsilon}$, include ground conditions (i.e. entries in the vector $\mathbf{z}^{\varepsilon}$ associated with grounded nodes should be set to zero).

A base solution for the simulated circuit configuration can be calculated as:

$$\mathbf{x} = \mathbf{A}^{-1}(\mathbf{z} + \mathbf{z}^{\varepsilon}) = \mathbf{x}^{\mathrm{L}} + \mathbf{D}^{x}\boldsymbol{\varepsilon} \tag{3.13}$$

In accordance with the VDM concept, response in simulated circuit is a superposition of linear response (generated by real sources) and residual response (generated by virtual distortions). Residual response can be expressed as a linear combination of distortions, where matrix of coefficients $\mathbf{D}^{x}$, called from now on a *base influence matrix*, is described by the following relation:

$$\mathbf{D}^{x} = \mathbf{A}^{-1} \begin{bmatrix} \overset{\mathrm{R}}{\mathbf{M}} \\ \mathbf{0} \end{bmatrix} \tag{3.14}$$

Column of the matrix $\mathbf{D}^{x}$ can be interpreted as a base solution obtained for the original main matrix and input vector equal to the column of incidence matrix $\overset{\mathrm{R}}{\mathbf{M}}$ (extended by a number of zero entries to equalize dimensions). Such an input vector can be in turn interpreted as an excitation produced by ideal current source of unit intensity inserted in parallel with selected resistor. An entry D^{x}_{ij} of the matrix can be then considered as a scaling factor which defines the inflence of virtual distortion imposed in j-th component on i-th circuit response. Let notice that influence matrix depends only on original circuit configuration (more precisely, only on the features reflected in the main matrix $\mathbf{A}$ – that is topology, parameters of passive components, ground conditions but not the values of active sources).

The concept of influence matrix and residual responses can be easily extended on any kind of circuit responses. Vector $\mathbf{f}$ containing arbitrary signals (currents, voltages or potentials) can be expressed as:

$$\mathbf{f} = \mathbf{f}^{\mathrm{L}} + \mathbf{D}^{f}\,\boldsymbol{\varepsilon}; \quad \mathbf{f}^{\mathrm{L}} = \mathbf{T}\,\mathbf{x}^{\mathrm{L}}; \quad \mathbf{D}^{f} = \mathbf{T}\,\mathbf{D}^{x} \tag{3.15}$$

where $\mathbf{T}$ is a certain matrix of linear transformation which reflects relations between arbitrary circuit responses and a base solution. In order to distinguish between various kind of influence matrices, the additional sub- and superscripts will used in further considerations. Superscript will denote the type of quantities storied in the matrix, first subscript – a set of locations these quantities refer to (aprioprate component, nodes or mixed set) and second subscript – a set of components where virtual distortions have been imposed (resistors or capacitors). According to the notation, matrix $\mathbf{D}^{v}_{\mathrm{nR}}$ means DC influence matrix of nodal potentials for virtual distortions imposed on resistors, while $\underline{\mathbf{D}}^{i}_{\mathrm{RC}}$ will mean AC influence matrix of currents in resistors for virtual distortions imposed on capacitors. The following relations between various kinds of DC influence matrices can be written:

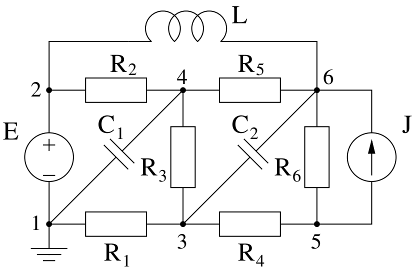
$$\mathbf{D}^{x}_{\mathrm{iR}} = \begin{bmatrix} \mathbf{D}^{v}_{\mathrm{nR}} \\ \mathbf{D}^{i}_{\mathrm{LR}} \\ \mathbf{D}^{i}_{\mathrm{ER}} \end{bmatrix}; \quad \mathbf{D}^{u}_{\mathrm{RR}} = \overset{\mathrm{R}}{\mathbf{M}}{}^{\mathrm{T}}\,\mathbf{D}^{v}_{\mathrm{nR}}; \quad \mathbf{D}^{i}_{\mathrm{RR}} = \mathbf{I} - \mathbf{G}\mathbf{D}^{u}_{\mathrm{RR}} \tag{3.16}$$

Subscripts n and i refer accordingly to nodes and arbitrary, mixed set of locations. Identity matrix $\mathbf{I}$ occuring in relation for the influence matrix of currents in resistors $\mathbf{D}^{i}_{\mathrm{RR}}$ results from the fact that distortion imposed on the element is also a constituent of the simulated current response in this element.

Example 4

To find DC base influence matrix $\mathbf{D}^{x}_{\mathrm{iR}}$ and DC voltage influence matrix $\mathbf{D}^{u}_{\mathrm{RR}}$ for a circuit presented in Figure 3.4.

Main matrix $\mathbf{A}$ of the circuit has been previously presented in Example 1,



$$
\begin{aligned}
R_i &= 1\,\mathrm{k\Omega} \qquad i = 1 \dots 6\\
C_1 &= C_2 = 1\,\mu\mathrm{F}\\
L &= 1\,\mathrm{mH}
\end{aligned}
$$

Figure 3.4: Exemplary circuit

page 24. Base influence matrix, calculated from Equation (3.14), is as follows:

$$
\mathbf{D}^x_{\mathrm{iR}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
461.54 & 153.85 & -307.69 & -230.77 & -153.85 & -230.77\\
153.85 & 384.62 & 230.77 & -76.92 & -384.62 & -76.92\\
230.77 & 76.92 & -153.85 & 384.62 & -76.92 & -615.38\\
0 & 0 & 0 & 0 & 0 & 0\\
-0.385 & -0.462 & -0.077 & -0.308 & -0.538 & -0.308\\
-0.538 & 0.154 & -0.308 & -0.231 & -0.154 & -0.231
\end{bmatrix}
$$

First six rows of the matrix $\mathbf{D}^x_{\mathrm{iR}}$ refer to potentials in nodes, seventh row to current in coil and eighth row to current in voltage source. First column refers to unit virtual distortion imposed on first resistor, second column to unit virtual distortion imposed on second resistor etc.. First node is grounded, second node is connected with the first one through short-circuited voltage source and sixth node is short-circuited with the second one through coil, hence electric potentials in all these nodes are equal zero.

Influence matrix of voltages in resistors is:

$$
\mathbf{D}^u_{\mathrm{RR}} = \begin{bmatrix}
461.54 & 153.85 & -307.69 & -230.77 & -153.85 & -230.77\\
153.85 & 384.62 & 230.77 & -76.92 & -384.62 & -76.92\\
-307.69 & 230.77 & 538.46 & 153.85 & -230.77 & 153.85\\
-230.77 & -76.92 & 153.85 & 615.38 & 76.92 & -384.62\\
-153.85 & -384.62 & -230.77 & 76.92 & 384.62 & 76.92\\
-230.77 & -76.92 & 153.85 & -384.62 & 76.92 & 615.38
\end{bmatrix}
$$

Not coincidentally, the matrix is symmetric, which is a result of reciprocity principle. Matrix $\mathbf{D}^u_{\mathrm{RR}}$ is an equivalent of strain influence matrix – it will be used to calculate distortions.

AC case

Procedure to derive the relation for influence matrix is quite similar as in the DC case, but this time two vectors of distortions will be taken into consideration – one simulating modifications of conductance $\underline{\boldsymbol{\varepsilon}}$ and one simulating modifications of capacitance $\underline{\boldsymbol{\xi}}$. Systems of equations for different circuit configurations (vide Section 2.3, Equation 2.17) are as follows:

Initial circuit	Modified circuit	Simulated circuit
$\underline{\mathbf{A}}\,\underline{\mathbf{x}}^{\mathrm{L}} = \underline{\mathbf{z}}$	$\tilde{\underline{\mathbf{A}}}\,\underline{\mathbf{x}} = \underline{\mathbf{z}}$	$\underline{\mathbf{A}}\,\underline{\mathbf{x}} = \underline{\mathbf{z}} + \underline{\mathbf{z}}^{\varepsilon} + \underline{\mathbf{z}}^{\xi}$

In a case of modified circuit, changes of both conductance and capacitance are reflected in the modified main matrix $\tilde{\underline{\mathbf{A}}}$, while in the case of simulated circuit, two additional input vectors are introduced:

$$\underline{\mathbf{z}}^{\varepsilon} = \begin{bmatrix} \overset{\mathrm{R}}{\mathbf{M}}\,\underline{\boldsymbol{\varepsilon}} \\ \mathbf{0} \end{bmatrix} \qquad \underline{\mathbf{z}}^{\xi} = \begin{bmatrix} \overset{\mathrm{C}}{\mathbf{M}}\,\underline{\boldsymbol{\xi}} \\ \mathbf{0} \end{bmatrix} \tag{3.17}$$

Simulated response can be calculated as a superposition of linear and residual responses:

$$\underline{\mathbf{x}} = \underline{\mathbf{x}}^{\mathrm{L}} + \underline{\mathbf{D}}^{x}_{\mathrm{iR}}\,\underline{\boldsymbol{\varepsilon}} + \underline{\mathbf{D}}^{x}_{\mathrm{iC}}\,\underline{\boldsymbol{\xi}} \tag{3.18}$$

where the base influence matrices are:

$$\underline{\mathbf{D}}^{x}_{\mathrm{iR}} = \underline{\mathbf{A}}^{-1} \begin{bmatrix} \overset{\mathrm{R}}{\mathbf{M}} \\ \mathbf{0} \end{bmatrix}; \quad \underline{\mathbf{D}}^{x}_{\mathrm{iC}} = \underline{\mathbf{A}}^{-1} \begin{bmatrix} \overset{\mathrm{C}}{\mathbf{M}} \\ \mathbf{0} \end{bmatrix} \tag{3.19}$$

Again, column of the influence matrix may be interpreted as a response of original circuit on unit ideal current source (unit distortion) inserted in parallel with a given component. Unit distortion is here interpreted as a source of *unit complex amplitude*, i.e. it has an unit amplitde, zero phase and the same frequency as active sources. Relations between various kinds of AC influence matrices are as follows:

$$\underline{\mathbf{D}}^{x}_{\mathrm{iR}} = \begin{bmatrix} \underline{\mathbf{D}}^{v}_{\mathrm{nR}} \\ \underline{\mathbf{D}}^{i}_{\mathrm{ER}} \end{bmatrix}; \quad \underline{\mathbf{D}}^{u}_{\mathrm{RR}} = \overset{\mathrm{R}}{\mathbf{M}}^{\mathrm{T}}\,\underline{\mathbf{D}}^{v}_{\mathrm{nR}}; \quad \underline{\mathbf{D}}^{i}_{\mathrm{RR}} = \mathbf{I} - \mathbf{Y}_{\mathrm{R}}\underline{\mathbf{D}}^{u}_{\mathrm{RR}} \tag{3.20}$$

$$\underline{\mathbf{D}}^{x}_{\mathrm{iC}} = \begin{bmatrix} \underline{\mathbf{D}}^{v}_{\mathrm{nC}} \\ \underline{\mathbf{D}}^{i}_{\mathrm{EC}} \end{bmatrix}; \quad \underline{\mathbf{D}}^{u}_{\mathrm{CC}} = \overset{\mathrm{C}}{\mathbf{M}}^{\mathrm{T}}\,\underline{\mathbf{D}}^{v}_{\mathrm{nC}}; \quad \underline{\mathbf{D}}^{i}_{\mathrm{CC}} = \mathbf{I} - \mathbf{Y}_{\mathrm{C}}\underline{\mathbf{D}}^{u}_{\mathrm{CC}} \tag{3.21}$$

The same as in the DC case, any simulated circuit response can be expressed as a sum of linear response and combination of virtual distortions. For example, voltages across capacitors may be calculated as:

$$\underline{\mathbf{u}}_{\mathrm{C}} = \underline{\mathbf{u}}^{\mathrm{L}}_{\mathrm{C}} + \underline{\mathbf{D}}^{u}_{\mathrm{CR}}\,\underline{\boldsymbol{\varepsilon}} + \underline{\mathbf{D}}^{u}_{\mathrm{CC}}\,\underline{\boldsymbol{\xi}} \tag{3.22}$$

Example 5

To find influence matrices $\underline{\mathbf{D}}^{x}_{\text{iC}}$ and $\underline{\mathbf{D}}^{i}_{\text{CC}}$ for a circuit from the Example 4 (Figure 3.4, page 49). Frequency of real sources f = 200Hz.

Base influence matrix for virtual distortions simulating modifications in capacitors is calculated from Equation (3.19):

$$\underline{\mathbf{D}}^{x}_{\text{iC}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 68.3-95.7j & -325.7+202.5j \\ 291.7-153.9j & -68.2+96.3j \\ 34.2-47.6j & -162.7+101.6j \\ 0.15+0.56j & 0.27+0.65j \\ -0.74+0.27j & -0.45+0.12j \end{bmatrix}$$

First six rows refer to potentials in nodes and last row to current in voltage source. Influence matrix of currents in capacitors is obtained from Equation (3.21):

$$\underline{\mathbf{D}}^{i}_{\text{CC}} = \begin{bmatrix} 0.807-0.367j & 0.121+0.086j \\ 0.121+0.086j & 0.746-0.410j \end{bmatrix}$$

Dynamic case

In any linear, time-invariant system, an output $y(t)$ can be calculated as a convolution of a given input $x(t)$ and unit-impulse response function $h(t)$:

$$y(t) = \int_0^{\infty} h(t-\tau)\, x(\tau) d\tau \tag{3.23}$$

Function $h(t)$ is a response of non-energized system on impulse input signal modeled by Dirac delta function $\delta(t)$. In mechanical and electrical engineering, relation (3.23) is called a Duhamel's integral. In discrete-time domain, the integral can be approximated by a series:

$$y[t] = \sum_{\tau=t_0}^{t} \Delta\tau\, h[t-\tau]\, x[\tau] \tag{3.24}$$

where $\Delta\tau$ is the length of the time step. Square brackets denote that the relation is formulated for discrete time instants $t = \{t_0, t_1 = t_0 + \Delta\tau, \ldots, t_n\}$.

Equation (3.24) constitutes a basis for the concept of *dynamic influence matrix*. In transient analysis, virtual distortions are time-dependent functions. In discrete time domain, they may be approximated by sequences of successive impulses of values being average values of distortions at a given time step (Figure 3.5). Residual response invoked by virtual distortion can be

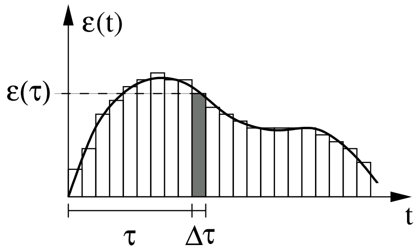


Figure 3.5: Virtual distortion as a sequence of impulses

then considered as a superposition of responses caused by successive impulses. Using the analogous relation for truss structures (Equation 3.7), simulated circuit responses can be formulated as follows:

$$\mathbf{f}[t] = \mathbf{f}^{\mathrm{L}}[t] + \sum_{\tau=t_0}^{t} \Big(\mathbf{D}^{f}_{\mathrm{iR}}[t-\tau]\, \boldsymbol{\varepsilon}[\tau] + \mathbf{D}^{f}_{\mathrm{iC}}[t-\tau]\, \boldsymbol{\xi}[\tau] \Big) \tag{3.25}$$

Dynamic influence matrices $\mathbf{D}^{f}_{\mathrm{iR}}[t]$ and $\mathbf{D}^{f}_{\mathrm{iC}}[t]$ are collections of discretized time responses invoked by unit impulse virtual distortions imposed on circuit elements in the first time step of analysis. The value of residual response at the given time step t_n depends on all impulses of distortion from previous time steps $t_{i<n}$. Dynamic influence matrices are approximated numericaly using the Newmark algorithm – column of the matrix is a solution of circuit equations for discretized input vector $\mathbf{z}[t]$, consisting of unit impulse distortion defined in the first time step (zeros in all other steps), with real sources discarded and zero initial conditions assumed.

As an example, in Figure 3.6, nodal potentials for unit impulse virtual distortions imposed on capacitors C_1 (left) and C_2 (right) of the exemplary circuit, which form the columns of dynamic influence matrix $\mathbf{D}^{v}_{\mathrm{nC}}[t]$, have been presented. Time step $\Delta\tau = 0.1$ms was assumed.

3.4 Evaluation of distortions, re-analysis

Thus far, the concept of virtual distortions has been introduced but without providing a direct procedure for their evaluation. Relations between virtual distortions and modification parameters derived in Section 3.2 include simulated circuit responses which also depend on distortions. Now, knowing that

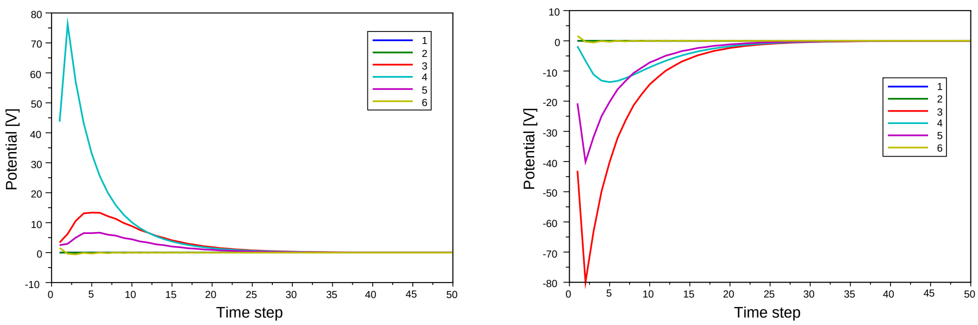


Figure 3.6: Potential responses for unit impulse distortions imposed on capacitors C_1 (left) and C_2 (right) from the exemplary circuit.

simulated responses can be expressed as a superposition of linear and residual responses, it is possible to formulate a system of equations which directly relates virtual distortions with modification parameters, where all other required quantities are obtained from the model of original circuit.

DC circuits

Equations (3.10), limited to modifications of conductance, can be rewritten in matrix notation in the following form:

$$\boldsymbol{\varepsilon} = \big(\mathbf{I} - [\boldsymbol{\mu}]\big)\,\mathbf{G}\,\mathbf{u}_{\mathrm{R}} \tag{3.26}$$

where $\mathbf{I}$ denotes the identity matrix, $[\boldsymbol{\mu}]$ is a diagonal matrix created from the vector of parameters of conductance modifications $\boldsymbol{\mu}$ and $\mathbf{u}_{\mathrm{R}}$ is a vector of voltages across resistors. Vector of distortions $\boldsymbol{\varepsilon}$ is defined with respect to all resistors. Voltages $\mathbf{u}_{\mathrm{R}}$ refer to simulated circuit configuration, hence they can be expressed in terms of virtual distortions:

$$\mathbf{u}_{\mathrm{R}} = \mathbf{u}_{\mathrm{R}}^{\mathrm{L}} + \mathbf{D}_{\mathrm{RR}}^{\mathrm{u}}\,\boldsymbol{\varepsilon} \tag{3.27}$$

Inserting (3.27) into (3.26) and organizing variables with respect to virtual distortions, the following system of linear equations can be obtained:

$$\mathbf{H}\,\boldsymbol{\varepsilon} = \mathbf{b} \tag{3.28}$$

where:

$$\mathbf{H} = \mathbf{I} - \Delta_{\mathrm{G}}\,\mathbf{D}_{\mathrm{RR}}^{u} \tag{3.29a}$$

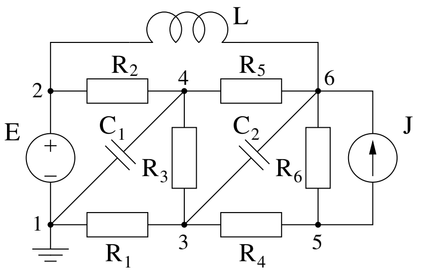
$$\mathbf{b} = \Delta_{\mathrm{G}}\,\mathbf{u}_{\mathrm{R}}^{\mathrm{L}} \tag{3.29b}$$

$$\Delta_{\mathrm{G}} = \big(\mathbf{I} - [\boldsymbol{\mu}]\big)\,\mathbf{G} \tag{3.29c}$$

System of equations (3.28) enables to evaluate virtual distortions $\boldsymbol{\varepsilon}$ which correspond to a given set of modification parameters $\boldsymbol{\mu}$. Other required quantities, which need to be pre-computed from the model of original circuit, are linear voltage responses in resistors $\mathbf{u}^{\mathrm{L}}_{\mathrm{R}}$ and voltage influence matrix $\mathbf{D}^{u}_{\mathrm{RR}}$. Although the system has been formulated globally, it may include only modified components. Let notice that if there is no modification of conductance in the k-th element of a circuit ($\mu_k = 1$) then $H_{kj} = I_{kj}$, $b_k = 0$ and immediately $\varepsilon_k = 0$. This feature is a basis of quick re-analysis algorithm, which enables to find local responses of the circuit on local modifications of parameters. In the first step of the algorithm, virtual distortions are calculated by solving locally formulated system of equations (3.28) and next, selected circuit responses are updated as a superposition of linear and residual responses.

Example 6

To evaluate distortions and recalculate voltages across capacitors in a circuit from Figure 3.7 for the given set of modifications: $\mu_1 = 0.2$ and $\mu_4 = 3$ (conductance in R_1 decreased five times, conductance in R_4 increased three times).



$\mathrm{R}_i = 1\,\mathrm{k}\Omega \qquad i = 1\ldots 6$
$\mathrm{C}_1 = \mathrm{C}_2 = 1\,\mu\mathrm{F}$
$\mathrm{L} = 1\,\mathrm{mH}$
$\mathrm{E} = 5\,\mathrm{V}$
$\mathrm{J} = 2\,\mathrm{mA}$

Figure 3.7: Exemplary circuit

At the beginning, let introduce the index α which defines the set of modified elements (locations of distortions): $\alpha = \{R_1, R_4\}$. In the first step, distortions will be calculated. To this end, linear voltage responses and voltage influence matrix for distortion locations need to pre-computed:

$$\mathbf{u}^{\mathrm{L}}_{\alpha} = \begin{bmatrix} 2.23 \\ 0.38 \end{bmatrix}; \quad \mathbf{D}^{u}_{\alpha\alpha} = \begin{bmatrix} 461.5 & -230.8 \\ -230.8 & 615.4 \end{bmatrix}$$

Formulating and solving the system of equations (3.28), the following values of virtual distortions are obtained:

$$\boldsymbol{\varepsilon}_{\alpha} = \begin{bmatrix} 2.763 \\ 0.227 \end{bmatrix} \times 10^{-3}$$

In the second step, voltage responses across capacitors will be updated. To this end, corresponding linear responses and influence matrix are required:

$$\mathbf{u}_{\mathrm{C}}^{\mathrm{L}} = \begin{bmatrix} 4.08 \\ 2.77 \end{bmatrix}; \quad \mathbf{D}_{\mathrm{C}\alpha}^{u} = \begin{bmatrix} 153.8 & -76.9 \\ -461.5 & 230.8 \end{bmatrix}$$

Simulated responses are superposition of linear and residual responses:

$$\mathbf{u}_{\mathrm{C}} = \mathbf{u}_{\mathrm{C}}^{\mathrm{L}} + \mathbf{D}_{\mathrm{C}\alpha}^{u} \boldsymbol{\varepsilon}_{\alpha} = \begin{bmatrix} 4.48 \\ 1.55 \end{bmatrix}$$

Example 7

For the same exemplary circuit (Figure 3.7), evaluate distortions which correspond to single breaks in resistors.

For every k-th resistor ($k = 1 \ldots 6$), the following procedure is applied:

- a break in a single resistor is assumed: $\mu_k = 0$
- Equation (3.28) is formulated: $\Delta_k = G_k$, $b_k = G_k U_k^L$, $H_{kk} = 1 - G_k D_{kk}^u$
- distortion is calculated: $\varepsilon_k = b_k / H_{kk}$

Virtual distortions which simulate single breaks in a circuit are:

$\varepsilon_1(\mu_1 = 0)$	$\varepsilon_2(\mu_2 = 0)$	$\varepsilon_3(\mu_3 = 0)$	$\varepsilon_4(\mu_4 = 0)$	$\varepsilon_5(\mu_5 = 0)$	$\varepsilon_6(\mu_6 = 0)$
0.0041	-0.0015	0.004	0.001	0.0015	0.0062

Virtual distortion depends on several factors (e.g. conductance and linear voltage response in the related component), hence its plain value doesn't tell much about the magnitude or direction of the modeled modification, or even if it lies wihin the feasible domain. Figure 3.8 presents relations between virtual distortion and parameter of conductance modification for every resistor from the exemplary circuit, for a case of a single modification (i.e distortions are considered independently and are not mutually interrelated).

AC circuits

The procedure leading to formulation of relation between virtual distortions and parameters of modifications is similar as in the DC case, but it is now assumed that both modifications of conductance and capacitance may occur

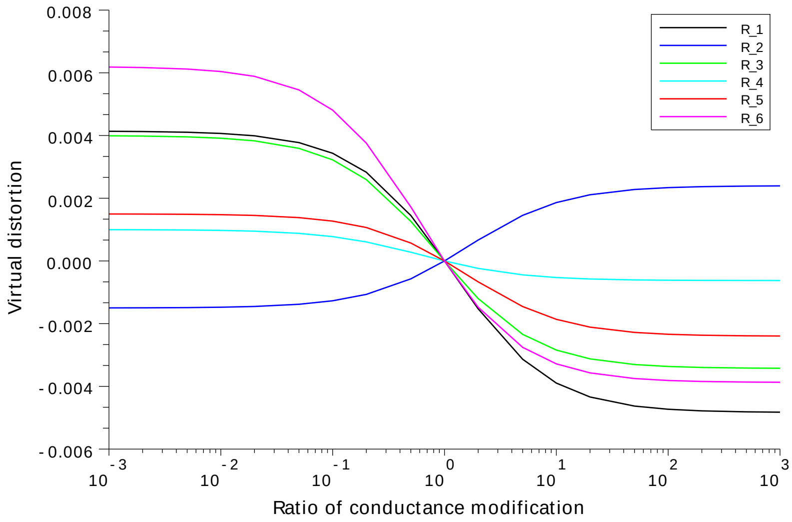


Figure 3.8: Virtual distortions for single modification scenarios

simultaneously. Equations (3.11), combined in a one matrix equation, are as follows:

$$\begin{bmatrix}\underline{\boldsymbol{\varepsilon}}\\ \underline{\boldsymbol{\xi}}\end{bmatrix} = \left(\mathbf{I} - \begin{bmatrix}[\boldsymbol{\mu}] & \mathbf{0}\\ \mathbf{0} & [\boldsymbol{\nu}]\end{bmatrix}\right)\begin{bmatrix}\mathbf{Y}_\mathrm{R} & \mathbf{0}\\ \mathbf{0} & \mathbf{Y}_\mathrm{C}\end{bmatrix}\begin{bmatrix}\underline{\mathbf{u}}_\mathrm{R}\\ \underline{\mathbf{u}}_\mathrm{C}\end{bmatrix} \tag{3.30}$$

Simulated voltage responses in resistors and capacitors are:

$$\begin{bmatrix}\underline{\mathbf{u}}_\mathrm{R}\\ \underline{\mathbf{u}}_\mathrm{C}\end{bmatrix} = \begin{bmatrix}\underline{\mathbf{u}}^\mathrm{L}_\mathrm{R}\\ \underline{\mathbf{u}}^\mathrm{L}_\mathrm{C}\end{bmatrix} + \begin{bmatrix}\underline{\mathbf{D}}^u_\mathrm{RR} & \underline{\mathbf{D}}^u_\mathrm{RC}\\ \underline{\mathbf{D}}^u_\mathrm{CR} & \underline{\mathbf{D}}^u_\mathrm{CC}\end{bmatrix}\begin{bmatrix}\underline{\boldsymbol{\varepsilon}}\\ \underline{\boldsymbol{\xi}}\end{bmatrix} \tag{3.31}$$

Combining and organizing the equations with respect to distortions, the following system is obtained:

$$\underline{\mathbf{H}}\begin{bmatrix}\underline{\boldsymbol{\varepsilon}}\\ \underline{\boldsymbol{\xi}}\end{bmatrix} = \underline{\mathbf{b}} \tag{3.32}$$

where:

$$\underline{\mathbf{H}} = \mathbf{I} - \Delta_\mathrm{Y}\begin{bmatrix}\underline{\mathbf{D}}^u_\mathrm{RR} & \underline{\mathbf{D}}^u_\mathrm{RC}\\ \underline{\mathbf{D}}^u_\mathrm{CR} & \underline{\mathbf{D}}^u_\mathrm{CC}\end{bmatrix} \tag{3.33a}$$

$$\underline{\mathbf{b}} = \Delta_\mathrm{Y}\begin{bmatrix}\underline{\mathbf{u}}^\mathrm{L}_\mathrm{R}\\ \underline{\mathbf{u}}^\mathrm{L}_\mathrm{C}\end{bmatrix} \tag{3.33b}$$

$$\Delta_\mathrm{Y} = \left(\mathbf{I} - \begin{bmatrix}[\boldsymbol{\mu}] & \mathbf{0}\\ \mathbf{0} & [\boldsymbol{\nu}]\end{bmatrix}\right)\begin{bmatrix}\mathbf{Y}_\mathrm{R} & \mathbf{0}\\ \mathbf{0} & \mathbf{Y}_\mathrm{C}\end{bmatrix} \tag{3.33c}$$

The same as in the DC case, the system may be formulated locally, with respect to modified elements only.

Example 8

Evaluate distortions and recalculate responses in a coil in a circuit from Figure 3.4 for the given modifications: $\mu_1 = 3$ and $\nu_1 = 0.25$ (conductance in R_1 increased three times, capacitance in C_1 decreased four times). Parameters of sources are: E = 5V, J = $2j$ mA and frequency 200Hz.

Let the index α define the set of modified elements: $\alpha = \{R_1, C_1\}$. Linear voltage responses and voltage influence matrix required to evaluate distortions are as follows:

$$\underline{\mathbf{u}}^{\mathrm{L}}_{\alpha} = \begin{bmatrix} 2.57 + 0.25j \\ 3.60 - 1.42j \end{bmatrix}; \quad \underline{\mathbf{D}}^{u}_{\alpha\alpha} = \begin{bmatrix} 325.5 - 210.9j & 68.3 - 95.7j \\ 68.3 - 95.7j & 291.7 - 153.9j \end{bmatrix}$$

Solution to the system of equations (3.32) produces the following values of virtual distortions:

$$\begin{bmatrix} \underline{\varepsilon}_1 \\ \underline{\xi}_1 \end{bmatrix} = \begin{bmatrix} -3.21 & - & 1.37j \\ 0.17 & + & 3.63j \end{bmatrix} \times 10^{-3}$$

To update the responses in a coil, the following quantities are required:

$$\underline{\mathbf{u}}^{\mathrm{L}}_{\mathrm{L}} = [4.21 - 3.71j] \times 10^{-3}; \quad \underline{\mathbf{D}}^{u}_{\mathrm{L}\alpha} = [-0.27 + 0.61j \quad 0.15 + 0.56j]$$

$$\underline{\mathbf{i}}^{\mathrm{L}}_{\mathrm{L}} = [2.95 + 3.35j] \times 10^{-3}; \quad \underline{\mathbf{D}}^{i}_{\mathrm{L}\alpha} = [-0.49 - 0.21j \quad 0.45 + 0.12j]$$

Superposition of linear and residual responses produces:

$$\underline{\mathbf{u}}_{\mathrm{L}} = [3.89 - 4.68j] \times 10^{-3}; \quad \underline{\mathbf{i}}_{\mathrm{L}} = [3.72 + 3.10j] \times 10^{-3}$$

Dynamic circuits

In transient case, modifications of conductance and capacitance are modeled by time-dependent virtual distortions (Equations 3.8). Combined in a one matrix equation and defined in discrete time domain, they are as follows:

$$\begin{bmatrix} \boldsymbol{\varepsilon}[t] \\ \boldsymbol{\xi}[t] \end{bmatrix} = \left(\mathbf{I} - \begin{bmatrix} [\boldsymbol{\mu}] & \mathbf{0} \\ \mathbf{0} & [\boldsymbol{\nu}] \end{bmatrix} \right) \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\mathrm{R}}[t] \\ \dot{\mathbf{u}}_{\mathrm{C}}[t] \end{bmatrix} \tag{3.34}$$

Simulated responses in resistors and capacitors are approximated using dynamic influence matrices:

$$\begin{bmatrix} \mathbf{u}_{\mathrm{R}}[t] \\ \dot{\mathbf{u}}_{\mathrm{C}}[t] \end{bmatrix} = \begin{bmatrix} \mathbf{u}^{\mathrm{L}}_{\mathrm{R}}[t] \\ \dot{\mathbf{u}}^{\mathrm{L}}_{\mathrm{C}}[t] \end{bmatrix} + \sum_{\tau=0}^{t} \begin{bmatrix} \mathbf{D}^{u}_{\mathrm{RR}}[t-\tau] & \mathbf{D}^{u}_{\mathrm{RC}}[t-\tau] \\ \dot{\mathbf{D}}^{u}_{\mathrm{CR}}[t-\tau] & \dot{\mathbf{D}}^{u}_{\mathrm{CC}}[t-\tau] \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}[\tau] \\ \boldsymbol{\xi}[\tau] \end{bmatrix} \tag{3.35}$$

Inserting (3.35) into (3.34) and organizing the equations with respect to a given time step, the following sequence of linear systems can be obtained:

$$\mathbf{H}\begin{bmatrix}\boldsymbol{\varepsilon}[t]\\ \boldsymbol{\xi}[t]\end{bmatrix} = \mathbf{b}[t] \tag{3.36}$$

where:

$$\mathbf{H} = \mathbf{I} - \Delta\begin{bmatrix}\mathbf{D}^u_{\mathrm{RR}}[0] & \mathbf{D}^u_{\mathrm{RC}}[0]\\ \dot{\mathbf{D}}^u_{\mathrm{CR}}[0] & \dot{\mathbf{D}}^u_{\mathrm{CC}}[0]\end{bmatrix} \tag{3.37a}$$

$$\mathbf{b}[t] = \Delta\left(\begin{bmatrix}\mathbf{u}^{\mathrm{L}}_{\mathrm{R}}[t]\\ \dot{\mathbf{u}}^{\mathrm{L}}_{\mathrm{C}}[t]\end{bmatrix} + \sum_{\tau=0}^{t-1}\begin{bmatrix}\mathbf{D}^u_{\mathrm{RR}}[t-\tau] & \mathbf{D}^u_{\mathrm{RC}}[t-\tau]\\ \dot{\mathbf{D}}^u_{\mathrm{CR}}[t-\tau] & \dot{\mathbf{D}}^u_{\mathrm{CC}}[t-\tau]\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon}[\tau]\\ \boldsymbol{\nu}[\tau]\end{bmatrix}\right) \tag{3.37b}$$

$$\Delta = \left(\mathbf{I} - \begin{bmatrix}[\boldsymbol{\mu}] & \mathbf{0}\\ \mathbf{0} & [\boldsymbol{\nu}]\end{bmatrix}\right)\begin{bmatrix}\mathbf{G} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}\end{bmatrix} \tag{3.37c}$$

Dynamic virtual distortions are calculated by solving sequentially the system of equations (3.36) for every time step starting from $t=t_0$. Main matrix $\mathbf{H}$ is the same for every step, only left-hand side vector $\mathbf{b}[t]$ need to updated using the values of distortions calculated in previous steps. The same as in steady-state, the systems may be formulated with respect to modified elements only.

Example 9

To calculate dynamic virtual distortion which model single breaks in resistors for a circuit and excitation scenario from the Example 3 (page 29).

Assuming a break in a single resistor ($\mu_k=0$), the system (3.36) reduces to the sequence of equations:

$$\varepsilon_k[t] = \frac{b_k[t]}{H_{kk}};\quad H_{kk} = 1-G_k D^u_{kk}[0];\quad b[t] = G_k\left(u^L_k[t] + \sum_{\tau=0}^{t-1} D^u_{kk}[t-\tau]\varepsilon_k[\tau]\right)$$

Linear responses, influence matrices and the resulting distortions are defined over the previously assumed discrete time domain (100 time steps, 0.1ms each, total time of analysis 10ms). The results are presented in Figure 3.9.

3.5 Sensitivity analysis

The aim of sensitivity analysis is to find derivatives of system responses with respect to selected parameters. It is a crucial issue in many optimization problems, in which the objective function, and hence its gradient, depends

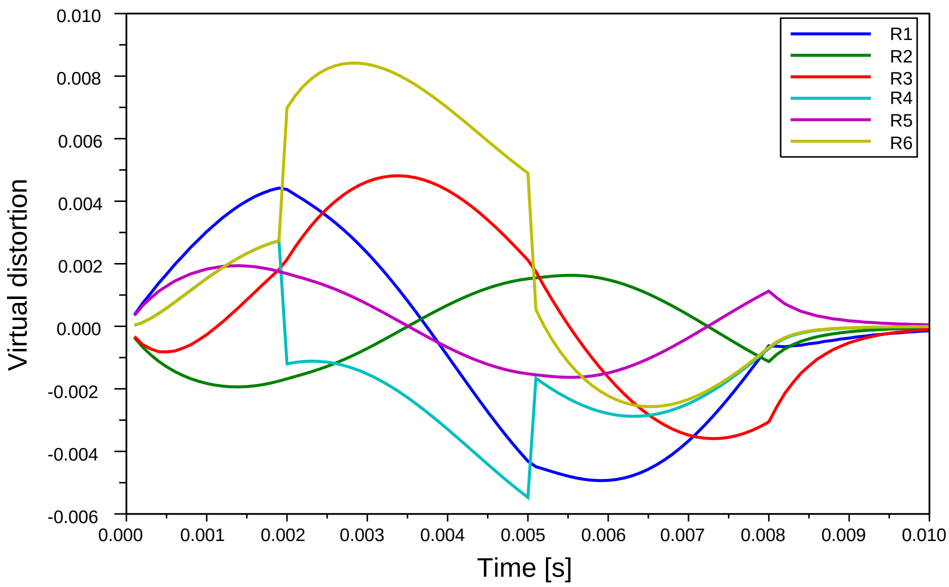


Figure 3.9: Dynamic virtual distortions simulating single breaks in resistors

on responses of the system (e.g. typical problem of fitting the numerical and experimental data in system or input identification). In damage identification problem, sensitivity analysis enables also to select optimal points of measurement (sensor placement). In this section, the VDM-based approach will be applied to find the derivatives of arbitrary circuit responses with respect to parameters of conductance and capacitance modifications:

$$\frac{\partial f_i}{\partial \mu_j} = \dots \qquad \frac{\partial f_i}{\partial \nu_j} = \dots$$

A classical formulation of the problem, Finite Difference Approximation, has the following form:

$$\frac{\partial f_i(\boldsymbol{\mu})}{\partial \mu_j} = \lim_{\Delta\mu \to 0} \frac{f_i(\boldsymbol{\mu};\, \mu_j + \Delta\mu) - f_i(\boldsymbol{\mu})}{\Delta\mu} \tag{3.38}$$

The above formula provides a numerical approximation of the derivative, of accuracy strongly dependent on proper selection of the value of finite increment $\Delta\mu$. Also, the method requires recalculation of the response for every perturbed variable.

DC case

It has been demonstrated in the previous sections that simulated circuit responses can be expressed as linear functions of virtual distortions:

$$\mathbf{f} = \mathbf{f}^{\mathrm{L}} + \mathbf{D}^{f}_{\mathrm{iR}}\, \boldsymbol{\varepsilon} \tag{3.39}$$

Linear responses $\mathbf{f}^{\mathrm{L}}$ and influence matrix $\mathbf{D}^{f}_{\mathrm{iR}}$ are calculated from the model of original circuit, hence they are independent of the introduced modifications. The Jacobi matrix of derivatives of responses with respect to virtual distortions equals then to the influence matrix:

$$\frac{\partial\, \mathbf{f}}{\partial\, \boldsymbol{\varepsilon}} = \mathbf{D}^{f}_{\mathrm{iR}} \tag{3.40}$$

Derivatives of responses with respect to modification parameters $\boldsymbol{\mu}$ can be calculated using the chain rule:

$$\frac{\partial\, \mathbf{f}}{\partial \boldsymbol{\mu}} = \frac{\partial\, \mathbf{f}}{\partial \boldsymbol{\varepsilon}}\, \frac{\partial\, \boldsymbol{\varepsilon}}{\partial \boldsymbol{\mu}} = \mathbf{D}^{f}_{\mathrm{iR}}\, \frac{\partial\, \boldsymbol{\varepsilon}}{\partial \boldsymbol{\mu}} \tag{3.41}$$

Let denote the unknown quantity $\partial\, \boldsymbol{\varepsilon}/\partial \boldsymbol{\mu}$ as a *gradient of distortions*. Component of the gradient defines the influence of conductance modification in one resistor on the value of virtual distortion in other resistor. Since values of distortions are mutually interrelated, so do components of the gradient and global approach is required. To derive the formula for the gradient of distortions, let differentiate both sides of the Equation (3.26):

$$\frac{\partial\, \boldsymbol{\varepsilon}}{\partial \boldsymbol{\mu}} = \frac{\partial}{\partial \boldsymbol{\mu}} \Big[\Delta_{\mathrm{G}}\, \mathbf{u}_{\mathrm{R}} \Big] \tag{3.42}$$

where $\Delta_{\mathrm{G}} = \big(\mathbf{I} - [\boldsymbol{\mu}]\big)\, \mathbf{G}$ and $\mathbf{u}_{\mathrm{R}} = \mathbf{u}^{\mathrm{L}}_{\mathrm{R}} + \mathbf{D}^{u}_{\mathrm{RR}}\, \boldsymbol{\varepsilon}$. Calculating the derivative of the right-hand side of equation we obtain:

$$\frac{\partial\, \boldsymbol{\varepsilon}}{\partial \boldsymbol{\mu}} = -\mathrm{diag}\big\{ \mathbf{G}\mathbf{u}_{\mathrm{R}} \big\} + \Delta_{\mathrm{G}}\, \mathbf{D}^{u}_{\mathrm{RR}}\, \frac{\partial\, \boldsymbol{\varepsilon}}{\partial \boldsymbol{\mu}} \tag{3.43}$$

The term "diag$\{\mathbf{x}\}$" denotes a diagonal matrix created from the vector $\mathbf{x}$. Organizing the variables with respect to components of the gradient, the following system of linear equations can be formulated:

$$\mathbf{H}\, \frac{\partial\, \boldsymbol{\varepsilon}}{\partial \boldsymbol{\mu}} = \mathbf{B} \tag{3.44}$$

where:

$$\mathbf{H} = \mathbf{I} - \Delta_{\mathrm{G}}\, \mathbf{D}^{u}_{\mathrm{RR}} \tag{3.45a}$$

$$\mathbf{B} = \mathrm{diag}\Big\{-\mathbf{G}\left(\mathbf{u}^{\mathrm{L}}_{\mathrm{R}} + \mathbf{D}^{u}_{\mathrm{RR}}\,\boldsymbol{\varepsilon}\right)\Big\} \tag{3.45b}$$

$$\Delta_{\mathrm{G}} = \big(\mathbf{I} - [\boldsymbol{\mu}]\big)\,\mathbf{G} \tag{3.45c}$$

Matrix $\mathbf{H}$ is the same matrix as in the system of equations used to calculate distortions (Equation 3.29a) and $\mathbf{B}$ is a diagonal matrix which depends on the simulated voltage responses in resistors. Components of the gradient are strictly related with the actual state of distortions. In the case of original circuit configuration where $[\boldsymbol{\mu}] = \mathbf{I}$, $\mathbf{H} = \mathbf{I}$ and $\boldsymbol{\varepsilon} = [0]$, components of the gradient of distortions can be found from the simplified equation:

$$\frac{\partial\,\boldsymbol{\varepsilon}}{\partial\boldsymbol{\mu}} = \mathrm{diag}\left\{-\mathbf{G}\,\mathbf{u}^{\mathrm{L}}_{\mathrm{R}}\right\} \tag{3.46}$$

Example 10

To calculate sensitivity of voltages across capacitors for the exemplary circuit in the original configuration (Figure 3.4, page 49).

According to Equation (3.41):

$$\frac{\partial\,\mathbf{u}_{\mathrm{C}}}{\partial\boldsymbol{\mu}} = \mathbf{D}^{u}_{\mathrm{CR}}\,\frac{\partial\,\boldsymbol{\varepsilon}}{\partial\boldsymbol{\mu}}$$

and gradient of distortion is can be calculated from the simplified relation (3.46). Ultimately, the following results are obtained:

$$\frac{\partial\,\mathbf{u}_{\mathrm{C}}}{\partial\boldsymbol{\mu}} = \begin{bmatrix} -0.34 & 0.36 & -0.43 & 0.03 & 0.36 & 0.18 \\ 1.02 & -0.14 & -0.57 & -0.09 & -0.14 & -0.55 \end{bmatrix}$$

AC case

Simulated responses are functions of distortions:

$$\underline{\mathbf{f}} = \underline{\mathbf{f}}^{\mathrm{L}} + \underline{\mathbf{D}}^{f}_{\mathrm{iR}}\,\underline{\boldsymbol{\varepsilon}} + \underline{\mathbf{D}}^{f}_{\mathrm{iC}}\,\underline{\boldsymbol{\xi}} \tag{3.47}$$

From the point of view of mathematics, partial derivative of AC response (complex number) with respect to parameter of conductance or capacitance modification (real numbers) is a directional derivative (Gâteaux). Since linear responses and influence matrices are independent of modifications, the derivatives are as follows:

$$\frac{\partial\,\underline{\mathbf{f}}}{\partial\boldsymbol{\mu}} = \underline{\mathbf{D}}^{f}_{\mathrm{iR}}\,\frac{\partial\,\underline{\boldsymbol{\varepsilon}}}{\partial\boldsymbol{\mu}} + \underline{\mathbf{D}}^{f}_{\mathrm{iC}}\,\frac{\partial\,\underline{\boldsymbol{\xi}}}{\partial\boldsymbol{\mu}}; \qquad \frac{\partial\,\underline{\mathbf{f}}}{\partial\boldsymbol{\nu}} = \underline{\mathbf{D}}^{f}_{\mathrm{iR}}\,\frac{\partial\,\underline{\boldsymbol{\varepsilon}}}{\partial\boldsymbol{\nu}} + \underline{\mathbf{D}}^{f}_{\mathrm{iC}}\,\frac{\partial\,\underline{\boldsymbol{\xi}}}{\partial\boldsymbol{\nu}}$$

Again, the problem is to formulate the relation for gradients of distortions. To this end, let consider the Equation (3.30), which relates distortions with modification parameters, and Equation (3.31), which describes voltage responses in resistors and capacitors. Particular components of the gradient of distortions may be obtained by differentiation of appropriate constituents of the Equation (3.30):

$$\begin{aligned}
\frac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\mu}} &= \frac{\partial}{\partial \boldsymbol{\mu}}\{\Delta_{\mathrm{R}}\underline{\mathbf{u}}_{\mathrm{R}}\} = -\mathrm{diag}\{\mathbf{Y}_{\mathrm{R}}\underline{\mathbf{u}}_{\mathrm{R}}\} + \Delta_{\mathrm{R}}\frac{\partial \underline{\mathbf{u}}_{\mathrm{R}}}{\partial \boldsymbol{\mu}} \\
\frac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\mu}} &= \frac{\partial}{\partial \boldsymbol{\mu}}\{\Delta_{\mathrm{C}}\underline{\mathbf{u}}_{\mathrm{C}}\} = \Delta_{\mathrm{C}}\frac{\partial \underline{\mathbf{u}}_{\mathrm{C}}}{\partial \boldsymbol{\mu}} \\
\frac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\nu}} &= \frac{\partial}{\partial \boldsymbol{\nu}}\{\Delta_{\mathrm{R}}\underline{\mathbf{u}}_{\mathrm{R}}\} = \Delta_{\mathrm{R}}\frac{\partial \underline{\mathbf{u}}_{\mathrm{R}}}{\partial \boldsymbol{\nu}} \\
\frac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\nu}} &= \frac{\partial}{\partial \boldsymbol{\nu}}\{\Delta_{\mathrm{C}}\underline{\mathbf{u}}_{\mathrm{C}}\} = -\mathrm{diag}\{\mathbf{Y}_{\mathrm{C}}\underline{\mathbf{u}}_{\mathrm{C}}\} + \Delta_{\mathrm{C}}\frac{\partial \underline{\mathbf{u}}_{\mathrm{C}}}{\partial \boldsymbol{\nu}}
\end{aligned}$$

where $\Delta_{\mathrm{R}} = (\mathbf{I} - [\boldsymbol{\mu}])\mathbf{Y}_{\mathrm{R}}$ and $\Delta_{\mathrm{C}} = (\mathbf{I} - [\boldsymbol{\nu}])\mathbf{Y}_{\mathrm{C}}$. Derivatives of voltage responses in elements are:

$$\frac{\partial \underline{\mathbf{u}}_{\mathrm{R}}}{\partial \boldsymbol{\mu}} = \underline{\mathbf{D}}^{u}_{\mathrm{RR}}\frac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\mu}} + \underline{\mathbf{D}}^{u}_{\mathrm{RC}}\frac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\mu}}; \qquad \frac{\partial \underline{\mathbf{u}}_{\mathrm{R}}}{\partial \boldsymbol{\nu}} = \underline{\mathbf{D}}^{u}_{\mathrm{RR}}\frac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\nu}} + \underline{\mathbf{D}}^{u}_{\mathrm{RC}}\frac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\nu}}$$

$$\frac{\partial \underline{\mathbf{u}}_{\mathrm{C}}}{\partial \boldsymbol{\mu}} = \underline{\mathbf{D}}^{u}_{\mathrm{CR}}\frac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\mu}} + \underline{\mathbf{D}}^{u}_{\mathrm{CC}}\frac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\mu}}; \qquad \frac{\partial \underline{\mathbf{u}}_{\mathrm{C}}}{\partial \boldsymbol{\nu}} = \underline{\mathbf{D}}^{u}_{\mathrm{CR}}\frac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\nu}} + \underline{\mathbf{D}}^{u}_{\mathrm{CC}}\frac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\nu}}$$

Combining the above relations and organizing the variables with respect to components of the gradient of distortions, the following system of equations can be formulated:

$$\underline{\mathbf{H}}\begin{bmatrix} \dfrac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\mu}} & \dfrac{\partial \underline{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\nu}} \\ \dfrac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\mu}} & \dfrac{\partial \underline{\boldsymbol{\xi}}}{\partial \boldsymbol{\nu}} \end{bmatrix} = -\mathrm{diag}\left\{\begin{bmatrix} \mathbf{Y}_{\mathrm{R}}\underline{\mathbf{u}}_{\mathrm{R}} \\ \mathbf{Y}_{\mathrm{C}}\underline{\mathbf{u}}_{\mathrm{C}} \end{bmatrix}\right\} \tag{3.49}$$

The main matrix of the system $\underline{\mathbf{H}}$ is the same matrix as in the system of equations for distortion calculation (Equation 3.32).

Dynamic case

Simulated transient responses of the circuit are:

$$\mathbf{f}[t] = \mathbf{f}^{\mathrm{L}}[t] + \sum_{\tau=t_0}^{t}\Big(\mathbf{D}^{f}_{\mathrm{iR}}[t-\tau]\,\boldsymbol{\varepsilon}[\tau] + \mathbf{D}^{f}_{\mathrm{iC}}[t-\tau]\,\boldsymbol{\xi}[\tau]\Big) \tag{3.50}$$

Partial derivatives of responses with respect to impulses of distortions are equal to appropriate components of the dynamic influence matrices:

$$\frac{\partial\, \mathbf{f}[t]}{\partial\, \boldsymbol{\varepsilon}[\tau]} = \mathbf{D}^{f}_{\mathrm{iR}}[t-\tau]; \qquad \frac{\partial\, \mathbf{f}[t]}{\partial\, \boldsymbol{\xi}[\tau]} = \mathbf{D}^{f}_{\mathrm{iC}}[t-\tau] \tag{3.51}$$

Applying the chain rule of differentiation, derivatives of responses with respect to modification parameters can be calculated as follows:

$$\frac{\partial\, \mathbf{f}[t]}{\partial\, \boldsymbol{\mu}} = \sum_{\tau=0}^{t} \left[\mathbf{D}^{f}_{\mathrm{iR}}[t-\tau]\, \frac{\partial\, \boldsymbol{\varepsilon}[\tau]}{\partial\, \boldsymbol{\mu}} + \mathbf{D}^{f}_{\mathrm{iC}}[t-\tau]\, \frac{\partial\, \boldsymbol{\xi}[\tau]}{\partial\, \boldsymbol{\mu}} \right] \tag{3.52}$$

$$\frac{\partial\, \mathbf{f}[t]}{\partial\, \boldsymbol{\nu}} = \sum_{\tau=0}^{t} \left[\mathbf{D}^{f}_{\mathrm{iR}}[t-\tau]\, \frac{\partial\, \boldsymbol{\varepsilon}[\tau]}{\partial\, \boldsymbol{\nu}} + \mathbf{D}^{f}_{\mathrm{iC}}[t-\tau]\, \frac{\partial\, \boldsymbol{\xi}[\tau]}{\partial\, \boldsymbol{\nu}} \right] \tag{3.53}$$

Once again, the problem reduces to formulation of relations for components of the gradient of distortions The appropriate formulas can be obtained by differentiation of the Equation (3.34):

$$\frac{\partial\, \boldsymbol{\varepsilon}[t]}{\partial\, \boldsymbol{\mu}} = -\mathrm{diag}\big\{\mathbf{G}\, \mathbf{u}_{\mathrm{R}}[t]\big\} + \Delta_{\mathrm{G}}\, \frac{\partial\, \mathbf{u}_{\mathrm{R}}[t]}{\partial\, \boldsymbol{\mu}} \tag{3.54a}$$

$$\frac{\partial\, \boldsymbol{\xi}[t]}{\partial\, \boldsymbol{\mu}} = \Delta_{\mathrm{C}}\, \frac{\partial\, \dot{\mathbf{u}}_{\mathrm{C}}[t]}{\partial\, \boldsymbol{\mu}} \tag{3.54b}$$

$$\frac{\partial\, \boldsymbol{\varepsilon}[t]}{\partial\, \boldsymbol{\nu}} = \Delta_{\mathrm{G}}\, \frac{\partial\, \mathbf{u}_{\mathrm{R}}[t]}{\partial\, \boldsymbol{\nu}} \tag{3.54c}$$

$$\frac{\partial\, \boldsymbol{\xi}(t)}{\partial\, \boldsymbol{\nu}} = -\mathrm{diag}\big\{\mathbf{C}\, \dot{\mathbf{u}}_{\mathrm{C}}[t]\big\} + \Delta_{\mathrm{C}}\, \frac{\partial\, \dot{\mathbf{u}}_{\mathrm{C}}(t)}{\partial\, \boldsymbol{\nu}} \tag{3.54d}$$

where $\Delta_{\mathrm{G}} = (\mathbf{I} - [\boldsymbol{\mu}])\mathbf{G}$ and $\Delta_{\mathrm{C}} = (\mathbf{I} - [\boldsymbol{\nu}])\mathbf{C}$ and derivatives of responses $\mathbf{u}_{\mathrm{R}}[t]$ and $\dot{\mathbf{u}}_{\mathrm{C}}[t]$ can be calculated based on Equations (3.52) and (3.53). The general procedure is to combine all the relations and organize the variables with respect to components of the gradient at the given time step t, similarly as it has been done in the system of equations for distortion evaluation (Equation 3.36). Ultimately, the following sequence of linear systems can be obtained:

$$\mathbf{H} \begin{bmatrix} \dfrac{\partial\, \boldsymbol{\varepsilon}[t]}{\partial \boldsymbol{\mu}} & \dfrac{\partial\, \boldsymbol{\varepsilon}[t]}{\partial \boldsymbol{\nu}} \\ \dfrac{\partial\, \boldsymbol{\xi}[t]}{\partial \boldsymbol{\mu}} & \dfrac{\partial\, \boldsymbol{\xi}[t]}{\partial \boldsymbol{\nu}} \end{bmatrix} = \mathbf{B}^{d}[t] + \mathbf{B}^{g}[t-1] \tag{3.55}$$

where:

$$\mathbf{H} = \mathbf{I} - \Delta \begin{bmatrix} \mathbf{D}^u_{\mathrm{RR}}[0] & \mathbf{D}^u_{\mathrm{RC}}[0] \\ \dot{\mathbf{D}}^u_{\mathrm{CR}}[0] & \dot{\mathbf{D}}^u_{\mathrm{CC}}[0] \end{bmatrix} \tag{3.56a}$$

$$\mathbf{B}^d[t] = -\mathrm{diag}\left\{ \begin{bmatrix} \mathbf{G}\mathbf{u}_{\mathrm{R}}[t] \\ \mathbf{C}\dot{\mathbf{u}}_{\mathrm{C}}[t] \end{bmatrix} \right\} \tag{3.56b}$$

$$\mathbf{B}^g[t-1] = \Delta \sum_{\tau-0}^{t-1} \begin{bmatrix} \mathbf{D}^u_{\mathrm{RR}}[t-\tau] & \mathbf{D}^u_{\mathrm{RC}}[t-\tau] \\ \dot{\mathbf{D}}^u_{\mathrm{CR}}[t-\tau] & \dot{\mathbf{D}}^u_{\mathrm{CC}}[t-\tau] \end{bmatrix} \begin{bmatrix} \dfrac{\partial \boldsymbol{\varepsilon}[\tau]}{\partial \boldsymbol{\mu}} & \dfrac{\partial \boldsymbol{\varepsilon}[\tau]}{\partial \boldsymbol{\nu}} \\ \dfrac{\partial \boldsymbol{\xi}[\tau]}{\partial \boldsymbol{\mu}} & \dfrac{\partial \boldsymbol{\xi}[\tau]}{\partial \boldsymbol{\nu}} \end{bmatrix} \tag{3.56c}$$

$$\Delta = \left(\mathbf{I} - \begin{bmatrix} [\boldsymbol{\mu}] & \mathbf{0} \\ \mathbf{0} & [\boldsymbol{\nu}] \end{bmatrix} \right) \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \tag{3.56d}$$

The system needs to be solved sequentially for every time step. Main matrix $\mathbf{H}$ is time-invariant and the same the for every step. Diagonal matrix $\mathbf{B}^d[t]$ depends on simulated circuit responses. Matrix $\mathbf{B}^g[t]$ depends on values of gradients from previous time steps.

Chapter 4

Inverse analysis and damage identification

4.1 The ELGRID system – general concept

The acronym ELGRID refers to a concept of structural health monitoring system which postulates to utilize an electrical network as an embedded distributed sensor of mechanical defects. The idea is that the branches of the network sustain electrical faults along with the occurrence or growth of structural damages like cracks, delamination or debonding. The problem of detection and localization of structural damages is simply formulated as an identification of coupled electrical faults in the sensing network. Schematically, the concept of the ELGRID system is presented in Figure 4.1

The electrical sensing network (1), deposited on the surface or embedded within the matrix of composite material, is a compact, passive, analog circuit which contains a plurality of resistive or capacitive sensors, arranged in some regular pattern to uniformly cover the monitored area of a structure. Topology and electrical characteristics of the network in the original configuration are known in advance and it is expected that occurring damages will result in local modifications of resistance and/or capacitance. The network is periodically excited by the predefined test stimuli from the signal generation unit (2) and responses of the network are collected by the data acquisition unit (3). Diagnosis of the network is performed in the signal processing unit (4), which compares the registered responses with the reference signature and runs a dedicated algorithm to localize the damaged branches.

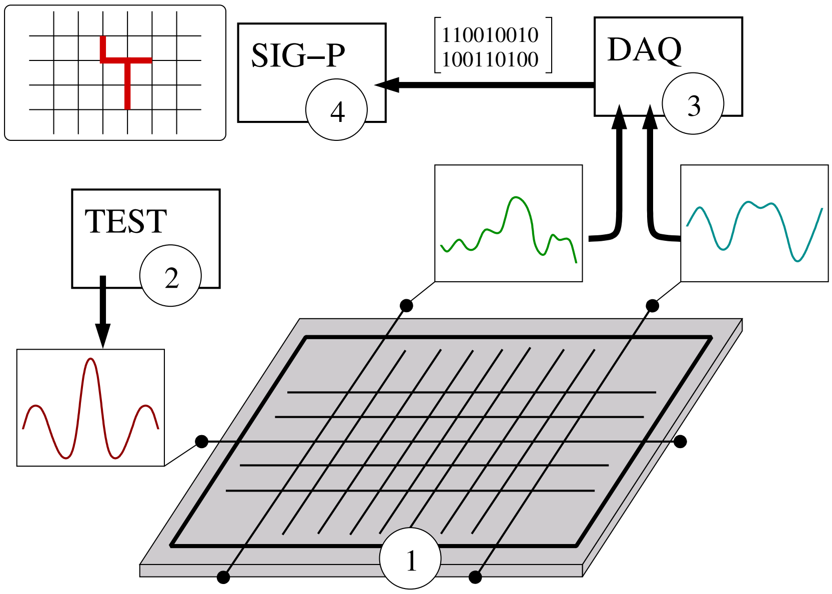


Figure 4.1: The ELGRID system - concept

Hardware considerations

The concept of the ELGRID system doesn't refer to any particular type of sensors or technological solutions. The sensing layer is generally considered as an electrical network in which parameters of branches are modified by the occurring structural damages. In the simplest but also the most applicable implementation, the sensing branch includes a crack wire. Break of the wire introduces a discontinuity in the electrical path and, if connected with other electrical component, results in local loss of conductance or capacitance. There are several technologies of crack wires which may be applied – plain resistive wires, conductive paths printed or sprayed on elastic substrate, conductive adhesives, polymers with metallic or carbon particles or nanomaterials. According to the system concept, the electrical model of the sensing network is viewed as an analog RC circuit which, in the ultimate form, will be diagnosed using dynamic inverse analysis in time domain. Hence, it need to include components of relatively high values of resistances and capacitances in order to obtain time constants enabling non-stationary responses to be invoked and registered. On the other hand, since the network is assumed to be embedded within the composite material, all components should be of the smallest possible dimensions. Resistors and capacitors made in the surface mount technology are planned to be utilized. From practical reasons, inductors, although included in the theoretical formulation of the problem, are not going to be used at all.

Sensing network is considered to be a passive circuit, excited periodically from the external test generation unit. Low power signals are going to be applied, with maximum amplitudes of voltages around 20 volts and maximum amplitudes of currents around 5 miliamperes. Responses of the network will be measured by the multi-channel data acquisition unit, of required sampling rate up to 100kHz. From the practical perspective, the easiest measurable quantities are electric potentials – the least number of connections is required (one for every potential plus one to a common ground). Measurements of currents will not be included since they require an integration within the circuit (inserting shunt resistor) which may not be possible because of unbreakable connections.

Model assumptions

It is assumed that the electrical model of the sensing network is linear, time-invariant circuit, consisting of ideal lumped resistors, capacitors and coils, supplied by ideal sources of voltage or current.

Linearity implies that parameters of passive components (resistance, capacitance, inductance) are independent of the level of current or voltage or ambient conditions (temperature, humidity). Flow of electric current is always accompanied by energy dissipation on the resistance and heating of components. For most metals, resistance increases with temperature. Since the network is projected to work with low power signals, it may be assumed that the system operates within the linear part of global characteristic.

Ideal components are characterized by a single parameter which describe its dominant feature. In practice, parasitic properties may reveal. Models of real-life components consists of ideal elements which represent these additional features. In Figure 4.2, the exemplary models of real capacitor and coil with ferromagnetic core are presented:

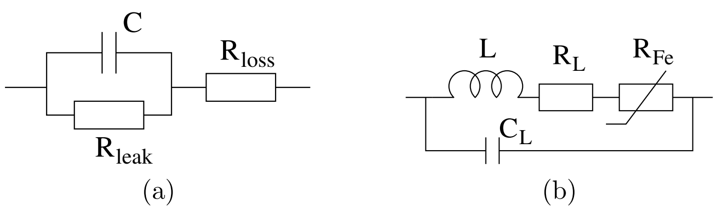


Figure 4.2: Real models of capacitor (a) and coil (b)

In the case of real capacitor, R_{leak} simulates finite resistance of the insulator an the related leakage current, while R_{loss} simulate resistances of contacts and electrodes. In the case of real coil with ferromagnetic core, R_L represents resistance of coil wire, C_L represents the in-between-turns capacitance, while R_{Fe} (non-linear!) simulates losses in the core related with hysteresis and eddy currents. Parasite parameters, as well as non-linearities, usually occur in high frequency range (MHz), which is not the considered case. However, even if parasitic parameters cannot be neglected, they may be fairly easy incorporated into the MNA-based system of equations. In the steady-state analysis, an equivalent conductance or admittance of the model of real component need only to be evaluated. In dynamic analysis, additional internal nodes in the models of components may be required to be introduced into global system of circuit equations.

4.2 Problem formulation

The main goal of the work is to solve the problem of damage identification in a circuit being the model of electrical sensing network. The term *damage* is understood as a modification of conductance or capacitance within the network, invoked by physical failure of constituent component (e.g. caused by its gradual degradation, fracture or break of connections). It is assumed that numerical model of the network is known in advance (it takes a form of linear RLC circuit supplied by independent sources) and it is possible to simulate any response on any scenario of excitation (using the MNA-based procedures of DC, AC or transient, time domain analysis). The input data includes also a set of responses measured for the damaged configuration of the network, invoked by known pre-defined test signals.

From the numerical perspective, the task can be formulated as an inverse problem of model updating: the goal is to find modifications of parameters which ensure a good fit between responses simulated from the numerical model and responses measured in real, damaged system. Preferably, the solution should involve both localization and quantification of simultaneously occurring damages, without restrictions on their number and intensity. Moreover, it is desired that the least possible number of testing signals and measurements should be involved, both from the practical perspective and because of limited accessibility of internal nodes of the network. Under such conditions, even for typical and relatively simple topologies of networks, the problem is ill-posed (cannot be uniquely solved). In order to ensure well-posedness, some additional constraints and limitations need to be introduced.

Some of them are natural consequences resulting from conceptual assumptions and practical functioning of the monitoring system, other will have to be enforced through adequate selection of network topology, parameters of elements, test signals and measurement strategy. These rules/guidelines of network design will be formulated based on theoretical conditions of circuit testability (topological analysis) and conclusions drawn from the VDM-based inverse analysis.

The concept of virtual distortions is well suited to formulate and solve the problem of damage identification – virtual distortions appear in exactly the same elements where modifications of parameters take place, simulated responses are linear functions of distortions and gradients of responses may be calculated. As a result, the problem can be considered as a search for the equivalent distribution of virtual distortions and solved from the transformed system of linear equations or through the gradient-based optimization. Let introduce the following notions which enable to formulate the problem in VDM nomenclature:

Distortion locations are a set of circuit components where the possibility of damage occurrence is assumed and hence, distortions are allowed to appear during the identification process. Components not included in a set of distortion locations are considered unalterable.

Reference responses are a set of selected circuit responses which are the input data for the identification procedure. It is assumed that reference response can be measured in a real circuit.

Linear responses are a set of reference responses obtained for the original circuit configuration. They are calculated from the numerical model or are measured in undamaged circuit.

Modified responses are a set of reference responses measured for the damaged circuit configuration.

Test stimuli is a set of input test signals which invoke the reference responses.

The assumption of limited number of possible distortion locations is natural since the considered circuits are models of sensing networks where some elements are expected to be damaged while other just play an auxiliary role. Number of distortion locations determines the dimension of the problem hence its reduction facilitates the practical implementation (e.g. lowers the number of required measurements and numerical cost of algorithm). It may be however assumed that all components of the circuit belong to the set of distortion locations. In theoretical considerations, reference responses may include arbitrary electrical signals: potentials in nodes, voltages or currents in any kind of component. Practically, as the number, kind and locations of possible response measurements and input signal supply are determined by the accessibility of network nodes, nodal potentials will be preferred as reference responses. The kind of test stimuli is strictly related with the identifications procedure. Steady-state DC/AC and dynamic test signals will be considered. As a measure of damage intensity, parameters of conductance and capacitance modifications μ and ν, introduced in Section 3.2 (Equation 3.9) will be used. Natural constraints are $\mu, \nu \geqslant 0$, where zero means electrical break. Practically, physical deterioration of real components usually leads to losses of conductance and capacitance, hence the scope of parameter modifications can be constrained to $\mu, \nu \in (0; 1)$.

Let consider a circuit shown in Figure (4.3) as an example of a model of very simple sensing network made of resistive components. It is assumed that 7 inner resistors (depicted in black) are embedded within the structural material and operate as sensors of cracks – they form a set of distortion locations. The remaining components are just additional connections and are assumed to be unalterable. It is further assumed that only 6 nodes on the outer edges of the network are accessible. One of them will be used to set a common ground and one to supply a test stimuli. Electric potentials in the remaining nodes ($v_1 \div v_4$) are assigned as a set of reference responses.

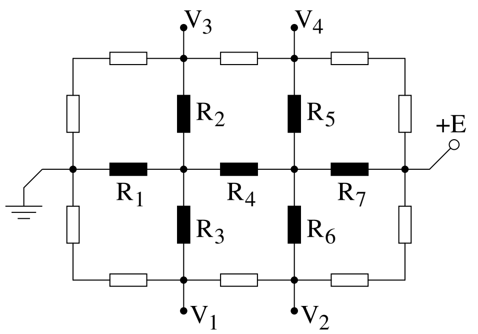


Figure 4.3: Simple model of sensing network

The question to be answered in the following sections is: is it possible to uniquely solve the problem for the arbitrary scenario of occurred damages? And if not – what conditions (concerning topology, parameters, test signal) have to be fulfilled in order for the network to be diagnosable?

4.3 Steady-state inverse analysis

Let start the analysis of the identification problem for a case where both test stimuli and reference responses are steady-state, constant or single-frequency harmonic signals. The discussion will be carried out with respect to DC circuits, but most of the derived relations and resulting conclusions also apply to AC circuits. Obviously, in the AC case, the equivalent relations would be defined with respect to complex amplitudes of signals, as well as both modifications of conductance and capacitance could be considered. The additional subscripts appearing in the following relations refer to the assumed sets of reference responses (α) and distortion locations (β).

Let suppose that a set of modified responses $\mathbf{f}_\alpha^{\mathrm{M}}$, invoked by a fixed DC test stimuli, has been registered in a circuit. These responses refer to a certain unknown state of parameters $\boldsymbol{\mu}_\beta$ which describe the occurred damages in terms of conductance modifications. Using the VDM approach, the influence of damages on circuit responses can be simulated by the equivalent vector of virtual distortions $\boldsymbol{\varepsilon}_\beta$ imposed on the model of original circuit:

$$\mathbf{f}_\alpha^{\mathrm{M}} = \mathbf{f}_\alpha(\boldsymbol{\mu}_\beta) = \mathbf{f}_\alpha(\boldsymbol{\varepsilon}_\beta) \tag{4.1}$$

Simulated responses of a circuit consist of linear and residual part:

$$\mathbf{f}_\alpha(\boldsymbol{\varepsilon}_\beta) = \mathbf{f}_\alpha^{\mathrm{L}} + \mathbf{D}_{\alpha\beta}^{f}\,\boldsymbol{\varepsilon}_\beta \tag{4.2}$$

Linear responses $\mathbf{f}_\alpha^{\mathrm{L}}$ and influence matrix $\mathbf{D}_{\alpha\beta}^{f}$ are calculated from the model of original circuit. By simple substitution and transformation, vector of distortions can be calculated as:

$$\boldsymbol{\varepsilon}_\beta = \left(\mathbf{D}_{\alpha\beta}^{f}\right)^{-1}\left(\mathbf{f}_\alpha^{\mathrm{M}} - \mathbf{f}_\alpha^{\mathrm{L}}\right) \tag{4.3}$$

Naturally, this operation is allowed and produces a unique solution only if the influence matrix $\mathbf{D}_{\alpha\beta}^{f}$ is square and non-singular. Dimensions of the matrix depend on the number of reference responses (rows) and distortion locations (columns) while rank of the matrix depends on mutual independence of reference responses, their sensitivity on occurring modifications and distinguishability of distortion locations. It may be concluded from the general analysis of topological constraints that the full-rank influence matrix can be obtained if all the following conditions are fulfilled:

1. Reference responses are not determined exclusively by supply or ground conditions.

2. Reference responses are sensitive to possible damage scenarios (i.e. modification of parameter in any distortion locations is observable in a set of reference responses).

3. Reference responses are linearly independent (i.e. any reference response cannot be determined from the others using Kirchhoff's laws or constitutive equations)

4. Distortion locations are distinct (i.e. distortions simulating modifications in different elements are not related with the same pair of nodes).

Let discuss the above conditions taking into account the exemplary circuit (Figure 4.4) and its base influence matrix $\mathbf{D}^{x}_{\mathrm{iR}}$ (taken from the Example 4, page 49).

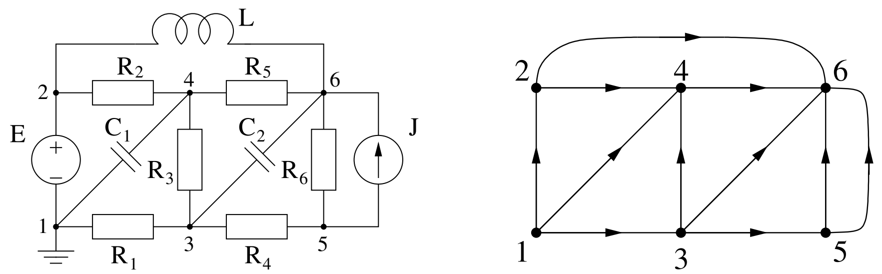


Figure 4.4: Exemplary circuit and its oriented graph

$$
\mathbf{D}^{x}_{\mathrm{iR}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
461.54 & 153.85 & -307.69 & -230.77 & -153.85 & -230.77 \\
153.85 & 384.62 & 230.77 & -76.92 & -384.62 & -76.92 \\
230.77 & 76.92 & -153.85 & 384.62 & -76.92 & -615.38 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-0.385 & -0.462 & -0.077 & -0.308 & -0.538 & -0.308 \\
-0.538 & 0.154 & -0.308 & -0.231 & -0.154 & -0.231
\end{bmatrix}
$$

First of the conditions simply excludes all the responses which are not affected by the occurring modifications: these are potentials in grounded nodes (e.g. 1-st row in the exemplary matrix), voltages in components in parallel with ideal voltage source ($U = E$) or currents in components in series with ideal current source ($I = J$). Second condition refer to a general rule which states that influence of parameter modification may be of local range and changes in responses registered in distant points may be of similar magnitude as the errors resulting from measurements or inaccuracy of numerical model. Hence, in a case of large network with a small number of remotely placed points of response measurements, the influence matrix may be extremely ill-conditioned. Third condition refers to topological constraints which govern global states of currents and voltages and determine their mutual dependencies. Simply put – a set of reference responses cannot contain voltages across components which form a loop and cannot contain currents in components which form a node cutset. If the rule is not fulfilled then the associated rows of the influence matrix will be linearly dependent (in accordance with Kirchhoff's laws, their sum will be zero). Also, current and voltage in the same component can be included in the set of reference responses only if the component belongs to the set of distortion locations (in other case, value of parameter is invariable and quantities are related by constitutive equation). Potentials in nodes are mutually dependent if they are connected by ideal voltage source ($V_i = V_j + E$) or are shorted ($V_i = V_j$) (e.g. the 2-nd and 6-th row in the exemplary matrix). The last condition brings up the fact that responses generated by unit distortions imposed on components which are in parallel or in series are almost identical. In the exemplary circuit, resistors R_2 and R_5 are in fact in parallel (they share the 4-th node while 2-nd and 6-th node are shorted), whereas resistors R_4 and R_6 are in series. As a result, the corresponding columns of the influence matrix differ only by the values of current in coil (7-th row) or potential in 5-th node. If these responses were omitted from the influence matrix, the corresponding columns would be linearly dependent.

The procedure of solving the inverse problem based on the Equation (4.3) requires that for the assumed set of distortion locations, an equal number of independent reference responses need to be selected. Selection of reference responses can be based on graph analysis and the following rules, which result from topological constraints:

1. The number of independent voltage responses equals to the number of independent nodal potentials, which is equal to the number of independent Kirchhoff's current laws.

2. The number of independent current responses equals to the number of independent mesh currents, which is equal to the number of independent Kirchhoff's voltage laws.

Let consider the process of selection of reference responses for the exemplary circuit in the DC and AC case, assuming that the set of distortion locations includes all resistors. Full graph of the circuit, shown in Figure 4.4, consists of 11 edges, 6 nodes and 6 loops. The goal is to distinguish dependent nodes, which are determined by ground conditions and voltage constraints, and eliminate redundant loops, which are determined by current constraints. To this end, let eliminate from the graph all *break-equivalent edges* for which currents are of fixed values – that means edges representing ideal current sources (I=J) and capacitors in DC case (I=0). Secondly, let distinguish all *short-equivalent edges* for which voltages are of fixed values – that means edges representing ideal voltage sources (U=E) and coils in DC case (U=0). The applied names of edges refer to the fact that during calculation of the influence matrix, the associated components act like breaks or short-circuits. At last, let distinguish all dependent nodes – that means grounded ones and one out of each pair connected by the short-equivalent edge. The reduced graphs for DC and AC case are presented in Figure 4.5 (short-equivalent edges and dependent nodes are depicted in red). Numbers of independent nodes and elementary loops in reduced graph define numbers of independent voltage and current responses. In the DC case, 3 independent nodes and 3 elementary loops are obtained, while in the AC case, there are 4 independent nodes and 5 elementary loops. To identify changes of resistances in all resistors, six independent reference responses need to be selected. In DC case, the only choice is to measure 3 voltages and 3 currents, while in AC case, 6 out of 9 available independent responses may be used. To determine which branch

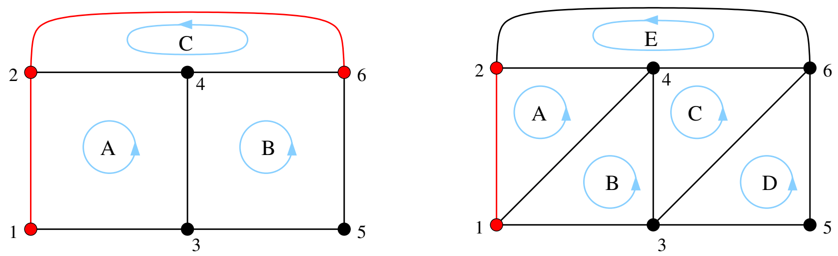


Figure 4.5: Reduced graphs for DC (left) and AC case (right)

voltages and branch currents are independent of each other, the concept of a *tree* of the graph may be applied. Tree is a set of edges which connect all independent nodes of a graph but don't comprise a loop. The remaining edges belong to the set of closing branches (short-equivalent edges are not taken into account). Any closing branch comprises an independent loop with tree edges. Tree of a graph may be distinguished in multiple variants – the examples of trees in the reduced graph for the exemplary circuit in AC case are presented in Figure 4.6 (edges of a tree are depicted by thick solid lines and closing branches by thin dashed lines). Voltages in tree edges and currents in closing branches comprise a complete set of mutually independent responses.

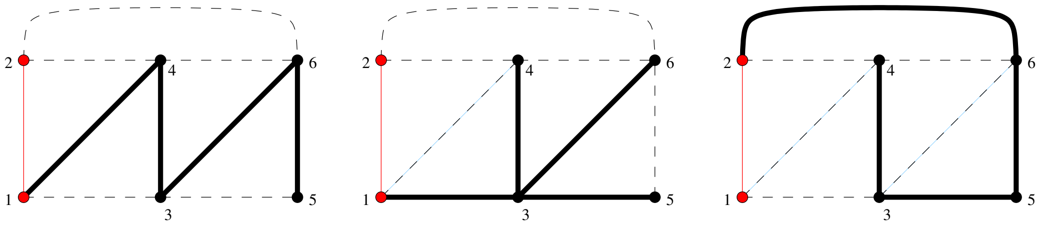


Figure 4.6: Examples of trees in reduced graph (AC case)

Despite the fact that Equation (4.3) enables to uniquely solve the inverse problem, its practical application is limited to the case of small number of distortion locations. For numerous locations, the same amount of independent reference responses need to acquired, which may be difficult or even impossible (measurements of voltage and current may be necessary, both amplitude and phase are required in AC case, internal nodes of the circuit may be inaccessible). Moreover, modified responses will be perturbed by measurement errors while linear responses and influence matrix by numerical errors and model inaccuracy (e.g. tolerances of circuit elements). If additionally the influence matrix is ill-conditioned, the obtained solution may be highly inaccurate and even lie beyond the natural constraints. As an alternative, an iterative, sensitivity-based optimization procedure may be proposed.

Gradient based approach

Let the vector of distance functions $\mathbf{d}_\alpha$ defines differences between responses simulated by a temporary state of virtual distortions and responses measured in modified circuit:

$$\mathbf{d}_\alpha = \mathbf{f}_\alpha(\boldsymbol{\varepsilon}_\beta) - \mathbf{f}^{\mathrm{M}}_\alpha \tag{4.4}$$

Objective function g is defined as the least square problem:

$$g = (\mathbf{d}_\alpha)^{\mathrm{T}}\mathbf{d}_\alpha = \sum_{i\,\in\,\alpha}(d_i)^2 \tag{4.5}$$

As optimization variables, virtual distortions $\boldsymbol{\varepsilon}_\beta$ or modification parameters $\boldsymbol{\mu}_\beta$ may be assigned. Gradients of the objective function with respect to distortions or modification parameters can be calculated from the following relations:

$$\frac{\partial g}{\partial \boldsymbol{\varepsilon}_\beta} = 2\left(\frac{\partial \mathbf{d}_\alpha}{\partial \boldsymbol{\varepsilon}_\beta}\right)^{\mathrm{T}}\mathbf{d}_\alpha; \qquad \frac{\partial g}{\partial \boldsymbol{\mu}_\beta} = 2\left(\frac{\partial \mathbf{d}_\alpha}{\partial \boldsymbol{\mu}_\beta}\right)^{\mathrm{T}}\mathbf{d}_\alpha \tag{4.6}$$

where partial derivatives of the distance functions are:

$$\frac{\partial \mathbf{d}_\alpha}{\partial \boldsymbol{\varepsilon}_\beta} = \frac{\partial \mathbf{f}_\alpha}{\partial \boldsymbol{\varepsilon}_\beta} = \mathbf{D}^{\mathrm{f}}_{\alpha\beta}; \qquad \frac{\partial \mathbf{d}_\alpha}{\partial \boldsymbol{\mu}_\beta} = \mathbf{D}^{f}_{\alpha\beta} \frac{\partial \boldsymbol{\varepsilon}_\beta}{\partial \boldsymbol{\mu}_\beta} \tag{4.7}$$

Gradient of objective function with respect to modification parameter is more demanding numerically as it requires that gradient of distortions to be calculated at every iteration. However, with modification parameters as optimization variables, constraints can be easily defined.

Let formulate the algorithm of damage identification making use of the steepest-descent approach. In every iteration, optimization variables (e.g. modification parameters) will be updated according to the formula:

$$\boldsymbol{\mu}^{(p)}_\beta = \boldsymbol{\mu}^{(p-1)}_\beta - \lambda^{(p)} \frac{\partial g}{\partial \boldsymbol{\mu}_\beta} \tag{4.8}$$

where p denotes the iteration step and $\lambda^{(p)}$ is a non-negative step length. Full procedure is as follows:

Initial settings and data:

- reference responses α and distortion locations β
- measured reference responses $\mathbf{f}^{\mathrm{M}}_\alpha$

Initial computations:

- Linear responses $\mathbf{u}^{\mathrm{L}}_\beta$ and $\mathbf{f}^{\mathrm{L}}_\alpha$
- Influence matrices $\mathbf{D}^{u}_{\beta\beta}$ and $\mathbf{D}^{f}_{\alpha\beta}$

Initialization: $\boldsymbol{\varepsilon}_\beta = [0]$; $\boldsymbol{\mu}_\beta = [1]$; $\mathbf{f}_\alpha = \mathbf{f}^{\mathrm{L}}_\alpha$

Main procedure : in every iteration step p:

- Calculate distance functions: $\mathbf{d}_\alpha = \mathbf{f}_\alpha - \mathbf{f}^{\mathrm{M}}_\alpha$
- Calculate gradient: $\dfrac{\partial g}{\partial \boldsymbol{\mu}_\beta} = 2 \left(\dfrac{\partial \mathbf{d}_\alpha}{\partial \boldsymbol{\mu}_\beta} \right)^{\mathrm{T}} \mathbf{d}_\alpha$
- Update optimization variables: $\boldsymbol{\mu}^{(p)}_\beta = \boldsymbol{\mu}^{(p-1)}_\beta - \lambda^{(p)} \dfrac{\partial g}{\partial \boldsymbol{\mu}_\beta}$
- Calculate distortions (Eq. 3.28): $\left[\mathbf{I} - \Delta_{\mathrm{G}}\, \mathbf{D}^{u}_{\beta\beta}\right] \boldsymbol{\varepsilon}_\beta = \Delta_{\mathrm{G}} \mathbf{u}^{\mathrm{L}}_\beta$
- Update simulated responses: $\mathbf{f}_\alpha = \mathbf{f}^{\mathrm{L}}_\alpha + \mathbf{D}^{f}_{\alpha\beta} \boldsymbol{\varepsilon}_\beta$

The procedure of gradient-based optimization usually also demand that the number of reference responses to be equal to the number of distortion locations (if the system is under-determined, then optimization may result in local minimum of objective function). Nevertheless, let run the procedure for the example of a network defined during the formulation of identification problem (Figure 4.7). Distortion locations were assumed in seven inner

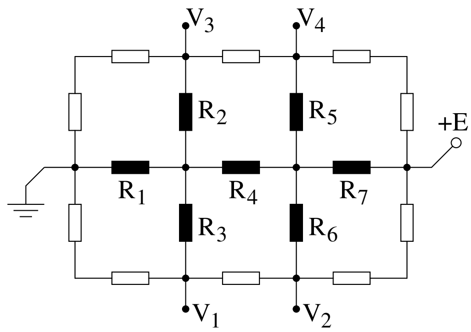


Figure 4.7: Simple model of sensing network

resistors and potentials in four external nodes were assigned as reference responses. Let all resistances are 10kΩ and DC test stimuli is E = +20V. Let the simulated damage scenario is defined by the following modifications: $\mu_2 = 0.1$ and $\mu_6 = 0.2$. Linear and simulated reference responses are:

$$\mathbf{f}^{\mathrm{L}}_{\beta} = \begin{bmatrix} 7.742 \\ 12.258 \\ 7.742 \\ 12.258 \end{bmatrix}; \qquad \mathbf{f}^{\mathrm{M}}_{\beta} = \begin{bmatrix} 7.486 \\ 11.822 \\ 8.260 \\ 12.527 \end{bmatrix}$$

while the response influence matrix is:

$$\mathbf{D}^{f}_{\alpha\beta} = \begin{bmatrix} 2323 & -897 & -3865 & -645 & -436 & -1469 & -1677 \\ 1677 & -436 & -1469 & 645 & -897 & -3865 & -2323 \\ 2323 & 3865 & 897 & -645 & 1469 & 436 & -1677 \\ 1677 & 1469 & 436 & 645 & 3864 & 897 & -2323 \end{bmatrix}$$

The procedure of optimization was run as an unconstrained problem, with distortions as optimized variables. The results, after 20 iterations, are presented in Figures 4.8 and 4.9 and in Table 4.1. Even though the objective function was completely minimized ($g(0) = 0.597, g(20) = 1.27 \times 10^{-11}$), the identified state of damages is far from the actual. However, an interesting features may be noticed in the vector of virtual distortions which is equivalent to the identified state of damages.

Distortion location	$\boldsymbol{\mu}_\beta$ simulated	$\boldsymbol{\mu}_\beta$ identified	$\boldsymbol{\varepsilon}_\beta$ simulated	$\boldsymbol{\varepsilon}_\beta$ identified
1	1	0.963	0	0.0000262
2	0.1	0.124	0.0001230	0.0000968
3	1	1.793	0	0.0000262
4	1	1.078	0	−0.0000444
5	1	1.520	0	0.0000182
6	0.2	0.234	0.0000988	0.0000807
7	1	0.975	0	0.0000182

Table 4.1: Simulated and identified parameters

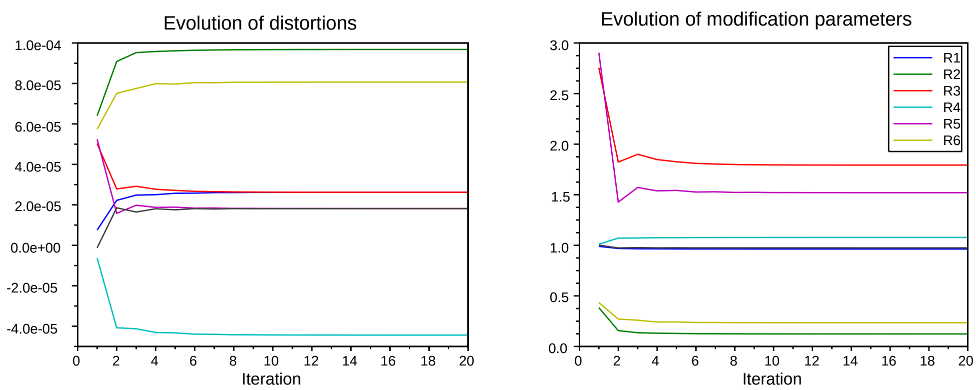


Figure 4.8: Evolution of parameters during optimization

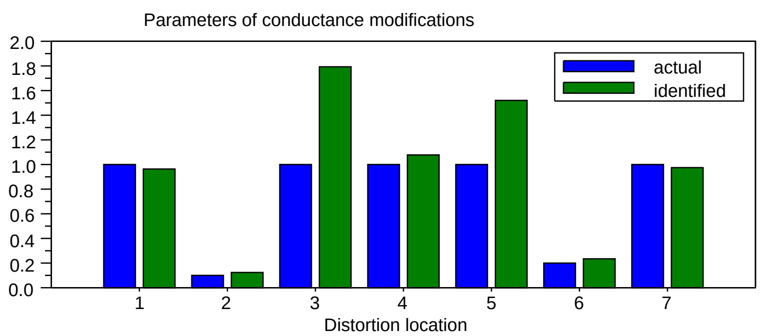


Figure 4.9: Identified state of damages

Impotent states of distortions

Let the vector of distortions $\hat{\varepsilon}_\beta$ refers to the actual, simulated damage scenario, while ε_β refers to damages identified during optimization. From the numerical experiment (Table 4.1) results that both vectors may be related as follows:

$$\varepsilon_\beta = \hat{\varepsilon}_\beta + 0.0000262\,\varepsilon_1^0 - 0.0000182\,\varepsilon_2^0$$

where the vectors of *impotent virtual distortions* are:

$$\varepsilon_1^0 = \begin{bmatrix}1 & -1 & 1 & -1 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$
$$\varepsilon_2^0 = \begin{bmatrix}0 & 0 & 0 & 1 & -1 & 1 & -1\end{bmatrix}^{\mathrm{T}}$$

Vectors ε_1^0 and ε_2^0 belong to the null-space of the response influence matrix:

$$\mathbf{D}^f_{\alpha\beta}\,\varepsilon_1^0 = [0]; \qquad \mathbf{D}^f_{\alpha\beta}\,\varepsilon_2^0 = [0]$$

It means that they don't generate responses which can be observed in the selected set of reference responses. Another such vector is:

$$\varepsilon_3^0 = \begin{bmatrix}1 & 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}^{\mathrm{T}}$$

If the impotent states of distortions are referred to the subgraph of distortion locations (Figure 4.10), it will occur that they comprise node cutsets or loops (vector ε_3^0 comprises a loop through a shorted voltage source). Hence, they

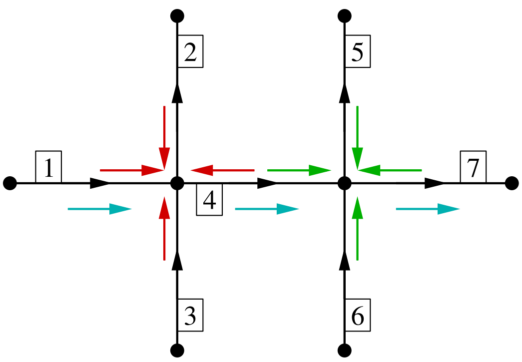


Figure 4.10: Impotent states of distortions

may be interpreted in the same way as impotent sources (see Section 2.2). Namely, impotent distortions comprising node cutset don't affect state of currents (only potential in the node and voltages in incident branches), while impotent distortions comprising loop don't affect state of voltages (only currents in the loop). This fundamental impotent states occur in strictly defined configurations: in loop-type impotent states, distortions are of equal values and the same direction with respect to loop circulation, while in cutset-type impotent states, their values divided by branch conductances are equal and are of the same orientation with respect to the common node (i.e. equivalent voltage distortions would be of the same values and polarity).

The solution obtained from optimization may be perturbed by the arbitrary linear combination of impotent distortions:

$$\boldsymbol{\varepsilon}_\beta = \hat{\boldsymbol{\varepsilon}}_\beta + \sum_i \lambda_i \boldsymbol{\varepsilon}_i^0$$

where $\boldsymbol{\varepsilon}_i^0$ is fundamental vector of impotent distortions (e.g. it defines a loop or node cutset within the set of distortion locations) and λ_i is a scaling factor defining the intensity of distortions in a given impotent state. Generally, λ_i can be of arbitrary value and not all fundamental vectors can be easily distinguished, hence extraction of actual solution from the optimized one is generally impossible. However, the concept behind the proposed methodology of damage identification is to design the sensing network in such a way as to eliminate the possibility of generation of impotent states during optimization (e.g. by proper selection of distortion locations and reference responses).

4.4 Transient inverse analysis (time domain)

The relation for simulated responses with dynamic influence matrix (Equation 3.25), which is a discrete version of convolution operation, could have been transformed into the linear system with lower-triangular block matrix of Toeplitz blocks, but it would be of huge dimension (original dimension times number of time steps), probably extremely ill-conditioned and iterative methods would have to be applied to calculate distortions. From this reason, the problem of damage identification in dynamic case will only be approached using gradient-based optimization.

Let the vectors of distance functions $\mathbf{d}_\alpha[t]$ define differences between instantaneous values of responses simulated by the actual state of virtual distortions and responses measured in the modified circuit:

$$\mathbf{d}_\alpha[t] = \mathbf{f}_\alpha[t] - \mathbf{f}_\alpha^{\mathrm{M}}[t] \tag{4.9}$$

The objective function g is defined as the least square problem:

$$g = \sum_t \mathbf{d}_\alpha^{\mathrm{T}}[t]\, \mathbf{d}_\alpha[t] \tag{4.10}$$

Gradients of the objective function with respect to parameters of conductance and capacitance modifications can be calculated from the following relations:

$$\frac{\partial g}{\partial \boldsymbol{\mu}_\beta} = 2\sum_t \left[\frac{\partial \mathbf{f}_\alpha[t]}{\partial \boldsymbol{\mu}_\beta}\right]^T \mathbf{d}_\alpha[t]; \qquad \frac{\partial g}{\partial \boldsymbol{\nu}_\beta} = 2\sum_t \left[\frac{\partial \mathbf{f}_\alpha[t]}{\partial \boldsymbol{\nu}_\beta}\right]^T \mathbf{d}_\alpha[t] \tag{4.11}$$

The procedure of calculation of derivatives of responses with respect to modification parameters was presented in the Section 3.5 (Equations 3.52, 3.53 and 3.55). A general scheme of optimization procedure, based on steepest descent approach, may be formulated similarly as in the steady-state.

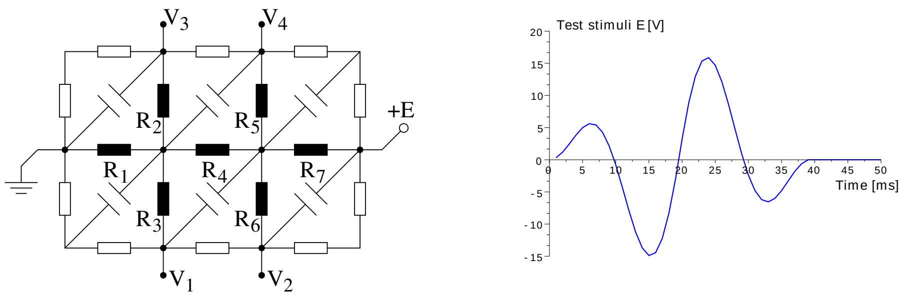


Figure 4.11: Simple model of dynamic sensing network

Let apply the procedure to solve the identification problem for the example of a sensing network excited by dynamic test stimuli shown in Figure 4.11. Naturally, dynamic analysis only makes sense if the circuit is also dynamic, hence the model of a network has been equipped with additional capacitors. Distortion locations are in 7 internal resistors, reference responses are potentials in 4 external nodes and the assumed damage scenario is $\mu_2 = 0.1$ and $\mu_6 = 0.2$. The results of identification are presented in Figures 4.12 and

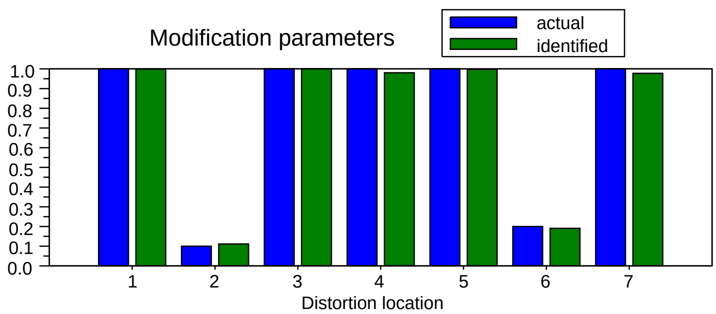


Figure 4.12: Identified state of damages

4.13. An almost exact solution have been reached – the applied procedure hasn't used step length control hence the solution oscillated around global minimum. Let notice that impotent states of distortions (node cutset type) couldn't be generated in this case because internal nodes were incident with inalterable capacitors.

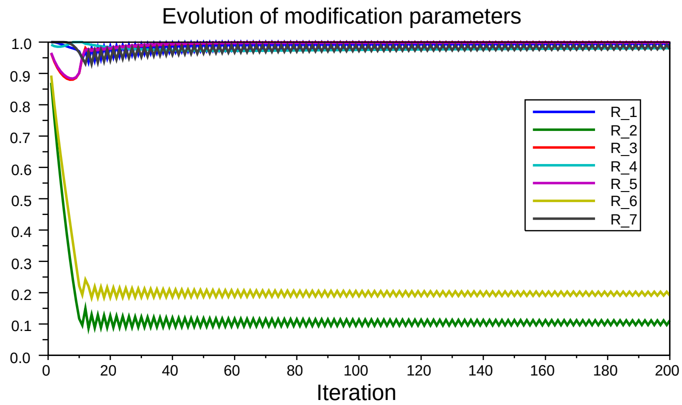


Figure 4.13: Evolution of parameters during optimization

Experimental verification

In order to check the algorithms of simulation and verify the developed methodology of damage identification, an experimental setup has been assembled. The setup included a programmable function generator to supply test stimuli and multi-channel data acquisition board connected to PC computer with NI Signal Express software for data handling. Numerical algorithms have been implemented and executed under Scilab environment. The circuit under test (Figure 4.14), simulating physical model of a sensing network, consisted of 40 discrete resistors ($10\text{k}\Omega \pm 10\%$), arranged in rectangular grid of 4-on-4 blocks, and 16 capacitors ($1\mu\text{F}\pm10\%$) across every block. Every component was equipped with hardware jumper (placed on the other side of a board), which enabled to simulate a break in the component or to rearrange the topology of the circuit. All internal nodes were accessible.

To ensure a good agreement between numerical and experimental results, parameters of all components have been precisely measured and fed into the numerical model. In Figure 4.15, an example of measured reference responses for dynamic test stimuli and a comparison with responses simulated numerically, are presented. The obtained differences were below 1%.

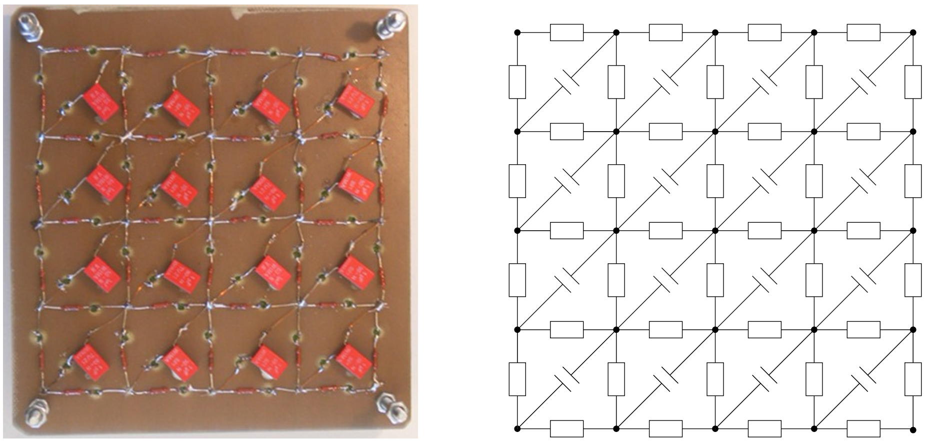


Figure 4.14: Experimental grid – a view and circuit diagram

Examples of damage identification

Experimental setup enabled to investigate the damage identification problem in typical grid network for various configurations of test stimuli and response measurements. The problem was formulated as a search for damages in 24 internal resistors (depicted in black – Figure 4.16) while locations of test stimuli and reference responses were assumed only in external nodes of a network. The exemplary results of identification for two different scenarios of damages (breaks in resistors depicted in red) and locations of test stimuli are presented in Figures 4.17 and 4.18 – red bars correspond to assumed modifications of conductances and green bars are the results of identification. In all cases, the set of reference responses included electric potentials in four external nodes (A,B,C,D) on every side of the network and a windowed sine voltage signal of base frequency 50Hz and two-period length was used as a test stimuli (Fig. 4.16). An equal number of iterations was performed, with the value of objective function reduced approximately by the factor of 10^4.

The general conclusion derived from the conducted experiments is that grid configuration is not diagnosable. Regardless of the shape and location of applied test stimuli (square, triangle and ramp waves, as well as combinations of harmonic functions were applied) it was impossible to identify single fault scenarios as well as multiple, separated damages. In a case of several clustered damages (Fig. 4.17), the procedure of identification sometimes produced good results, but it strictly depended on location of applied stimuli.

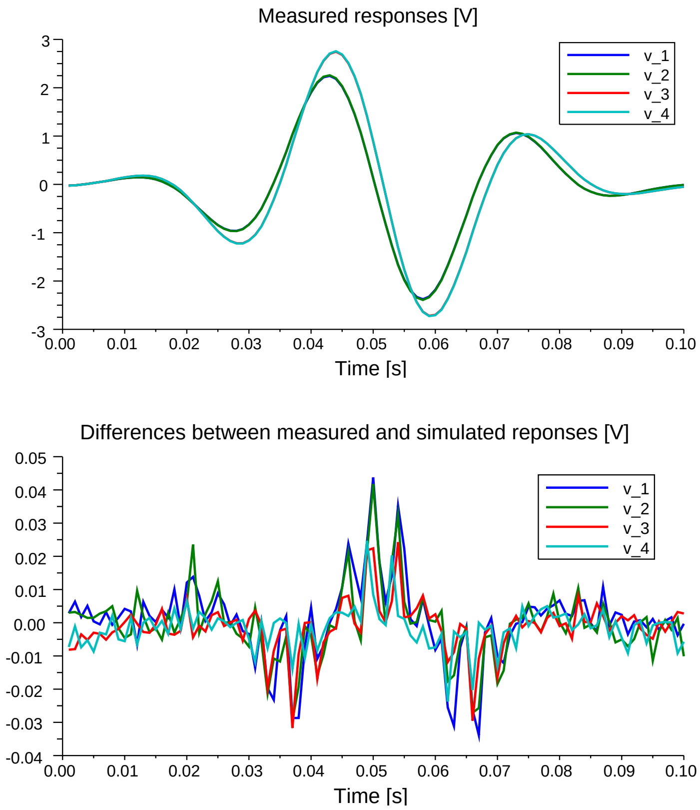


Figure 4.15: Experimental results vs. numerical simulation

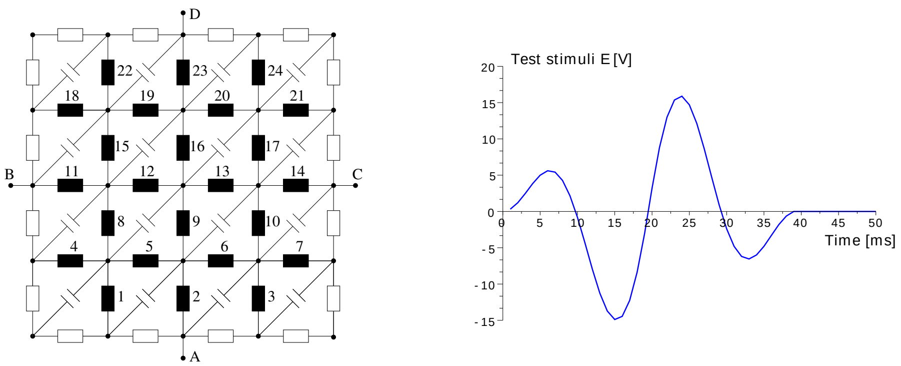


Figure 4.16: Exemplary network of grid topology

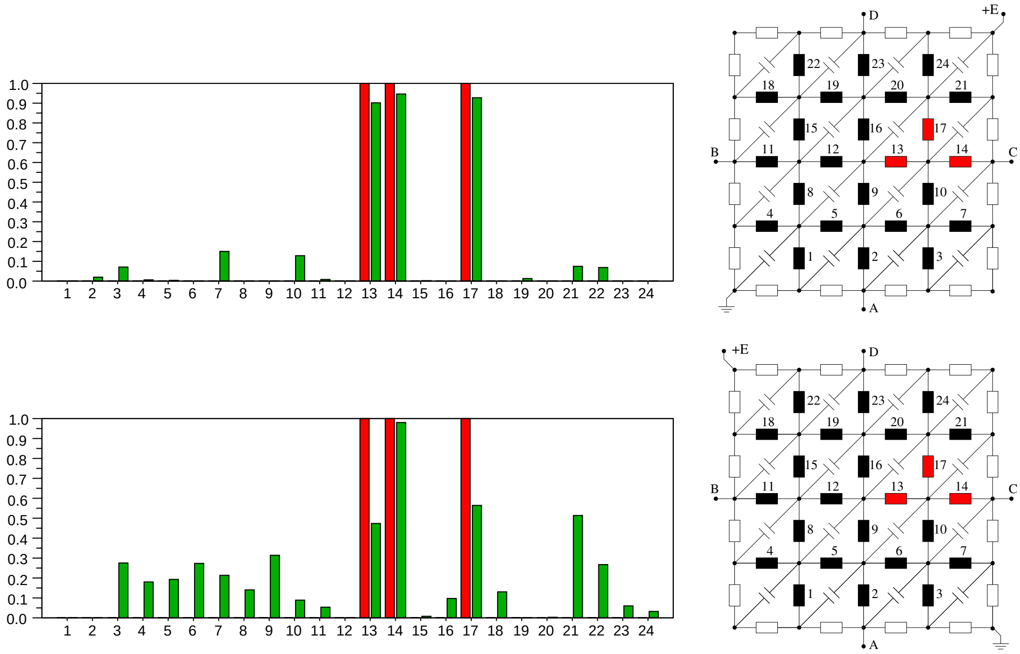


Figure 4.17: Identification in grid network, damage scenario 1

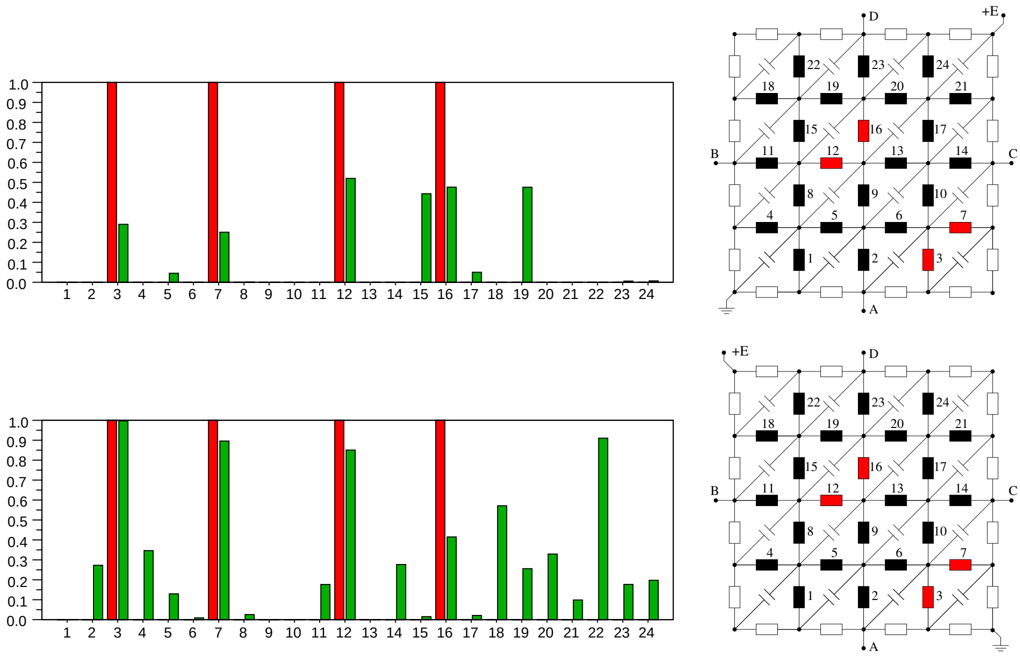


Figure 4.18: Identification in grid network, damage scenario 2

4.5 Optimal network topology

Numerical simulations and experimental tests confirmed the expected fact that for typical regular configurations of networks, the problem of damage identification is ill-posed – the solution obtained from the gradient-based optimization, although minimizes the objective function, may not be equal to the actual damage scenario. VDM-based inverse analysis gives a simple and intuitive interpretation for one of the reasons of ambiguity of the problem of damage identification – vector of virtual distortions, which corresponds to the vector of optimized modification parameters, is a sum of exact and spurious solutions. Exact solution is associated with virtual distortions which simulate the occurred damages while spurious solution is associated with impotent states of virtual distortions. Impotent states of distortions belong to the null-space of the response influence matrix. As a result, they don't invoke any changes in the selected set of reference responses. Consequently, they don't affect the value of objective function and may freely evolve during optimization (although will be suppressed if violate the imposed constraints). Impotent states of distortions form two different configurations: they appear in loops or in node cutsets. Values and directions of distortions comprising an impotent state are mutually coupled by topological constrains (i.e. they need to cancel each other in Kirchhoff's laws).

Based on this knowledge, it may be deduced that impotent states may be eliminated by imposing topological restrictions on the set of distortion locations or by applying an adequate measuring strategy. Let denote elements which belong to the set of distortion locations as *sensors* (such is their assumed function in the sensing network) and other elements as *links*. Topological restrictions dictate that a subgraph defined by sensors cannot include loops and node cutsets. In other words, in full graph of the sensing network, distortion locations should comprise a section of a tree and every node should be incident with at least one link. These are however not sufficient conditions because impotent states may be also formed through links – let consider such instance illustrated in Figure (4.19). If a link completes node cutsets in adjacent nodes, then despite the fact that distortions cannot be generated in links, impotent states may appear in such configurations that distortions associated with links cancel each other.

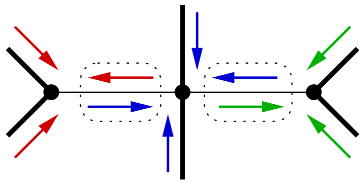


Figure 4.19: Impotent states through links

Although impotent states barely influence global circuit responses, they do introduce some local perturbations. Namely, loop-type impotent states changes currents in associated elements and node cutset-type states changes voltages in associated elements, as well as potential in the node. If the set of reference responses includes current measurement in sensor loop and voltage/potential measurement in sensor node cutset, generation of impotent states during optimization will be suppressed.

Rules of design for diagnosability

The conclusion of the analysis is that the topology of the sensing network should be designed in such a way as to eliminate all configurations which introduce ambiguity of solution. First of all, the generation of impotent states of distortions during optimization should be suppressed. Secondly, indistinct distortion locations should be eliminated. To accomplish these objectives, layout of the sensing network should comply with the following set of rules:

- No two sensors are in series or in parallel.
- Linear responses in every sensor are non-zero (no bridge configurations).
- Sensors don't comprise a loop or current in the loop is included in the set of reference responses.
- Sensors don't comprise a node cutset or potential in the node is included in the set of reference responses.
- Links cannot complete node cutsets.

The problem of finding optimal network layout may be considered as a task of topological optimization based on graph or system theory [51, 52]. Namely, starting from the base topology of sensing network, where all elements are included in the set of distortion locations (e.g. sensors arranged in dense grid pattern), the aim of optimization is to exclude a minimal number of elements to obtain topological configuration which satisfies the postulated conditions.

The above rules cover only topological conditions of diagnosability. Uniqueness and accuracy of identification are also related with sensitivity of reference responses on occurring damages. This problem can be in turn considered as task of selection of optimal shape and location of test stimuli and optimal number and locations of reference responses. In general, design of optimal network layout can be considered as a combination of input identification, system optimization and sensor placement problems, which include continuous and discrete variables and opposing objective functions, expressed in terms of both system characteristics and responses. Because of its complexity, formulation of such a problem has not be attempted. The issue of finding optimal network configurations has been approached through case studies.

Case studies

Grids

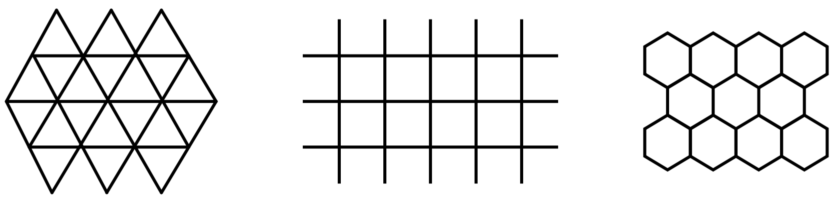


Figure 4.20: Grid patterns

From the point of view of health monitoring, branches of the sensing network which are expected to operate as sensors of damages, should be preferably arranged in some regular fashion in order to uniformly cover the monitored area. Typical grid patterns of triangular, rectangular or hexagonal type ideally meet this request. However, from the point of view of damage identification, grids are the worst possible topologies to diagnose: they contain loops, node cutsets, symmetry and with additional assumption of inaccessible internal nodes for response measurements, can be considered as non-diagnosable. The results obtained for a simple grid network (Figures 4.17 and 4.18) confirm the fact of high ambiguity of damage identification. Grid patterns provide however good base structures which can be redesigned in order to obtain optimal topological configuration (from the point of view of diagnosability).

Trees

Tree structures are natural configurations satisfying the defined rules of design. Starting from the base grid pattern, let select the same amount of 24 distortion locations, re-arranged in a form a tree (there are no internal loops and most of possible node cutsets have been concentrated around points where reference response are measured). Exemplary results of identification for various damage scenarios are presented in Figures 4.21 and 4.22.

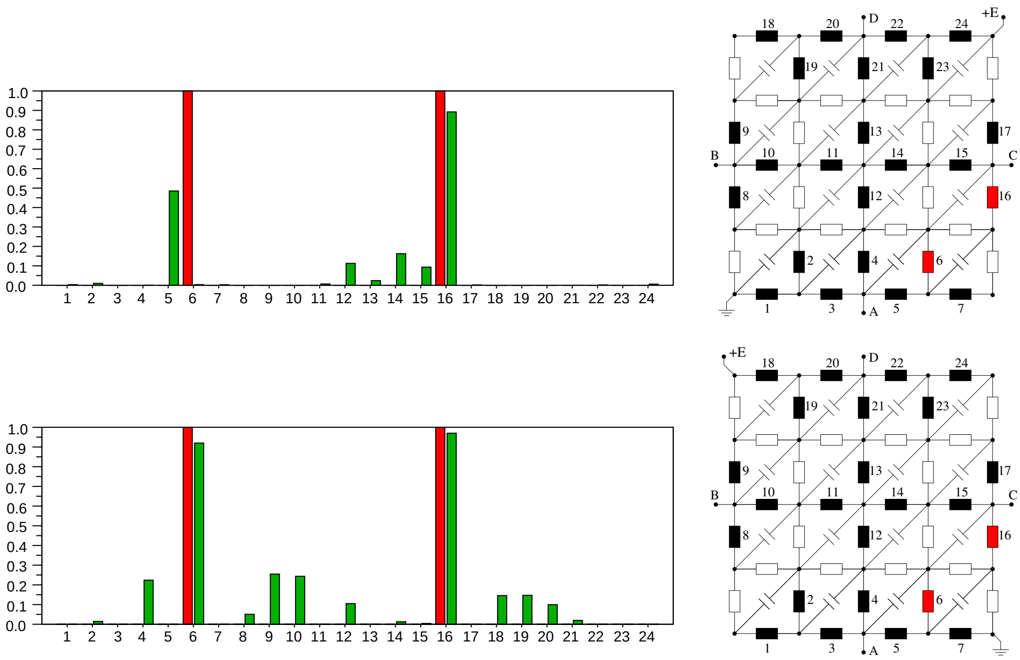


Figure 4.21: Identification in tree network, damage scenario 1

Again, the results of identification strongly depend on the location of test stimuli. In most cases, a solution close to the actual have been obtained for at least one test signal configuration. This suggests that accuracy of solution is more affected by sensitivity of responses than topological conditions and the tree networks could have been possibly uniquely diagnosed if the locations of test stimuli and response measurements had been optimized.

Ladders and ribbons

In the case of two-dimensional networks, without an access to the internal nodes, the main problem is low sensitivity of measured responses on damages occurring within the network. Alternative idea is to design the network

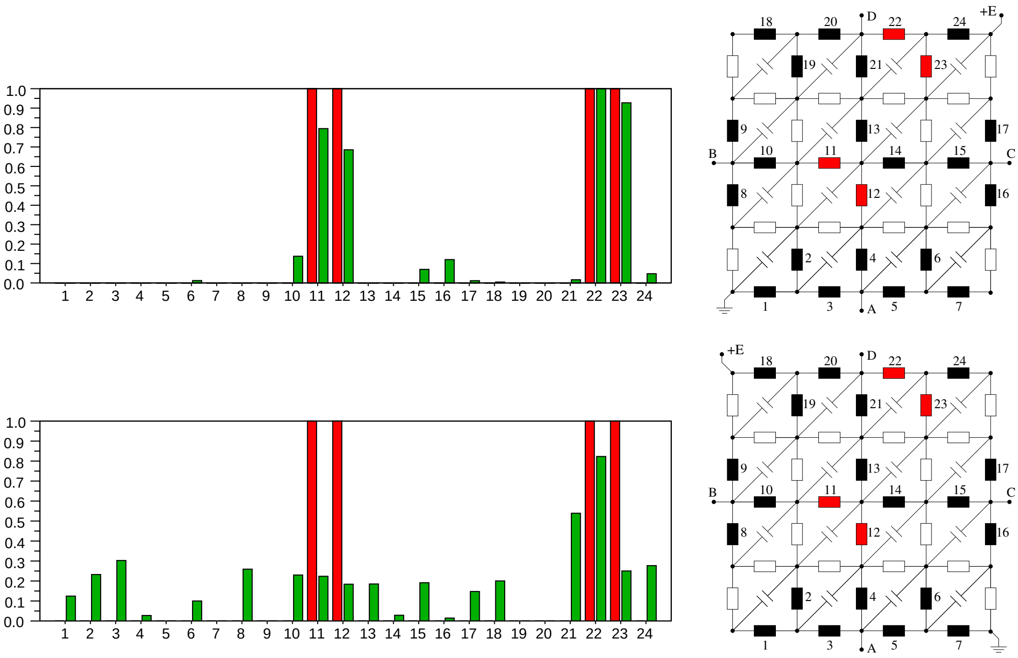


Figure 4.22: Identification in tree network, damage scenario 2

as an elongated structure, where possible damage lie on the path of test signal propagation and hence introduces significant perturbations in circuit responses. The possible configuration of ladder- and ribbon-like topologies are presented in Figure 4.23 (resistors in black define a set of distortion locations). Such configurations fulfill the proposed rules of design – there are no loops and node-cutsets comprised by distortion locations.

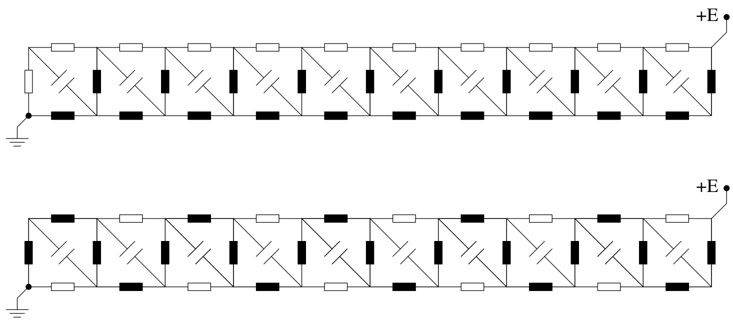


Figure 4.23: Ladder and ribbon patterns

In Figure 4.24, the exemplary results of damage identification for simple 6-block ladder network with 11 distortion locations are presented. Based only on two reference responses (nodes A, B), it was possible to identify arbitrary single fault scenario, but in case of multiple damages the procedure produced false results. In a case o three reference responses located regularly along the network (nodes A,B,C), an arbitrary damage scenario was exactly identified.

Concept of switchable networks

The general conclusion of the study is that the problem of damage identification in electrical networks can be uniquely solved only for a case of simple, one-dimensional topological configurations like ladders or ribbons. Networks of typical grid topologies are not diagnosable (problem is ill-posed) and re-design of grid (e.g. into tree), without simultaneous selection of optimal test stimuli and measuring strategy, may also not ensure the correctness of solution because of low sensitivity of responses on occurring damages. An answer to this problem is the concept of *switchable sensing networks.* According to the concept, the base sensing network of arbitrary topology is controlled by another circuitry which separates (isolates electrically) easily diagnosable parts or blocks for local search of damages. For example, blocks of ladder-like topologies are isolated from the base grid network (Figure 4.25).

The problem of damage identification in global base network is formulated as local searches of damages in isolated subnetworks. Decomposition of the base network need to ensure that any global distortion location is included in one of the locally defined sets of distortion locations. It is also assumed that all isolated subnetworks use the same terminals for test stimuli supply and response measurements. In the example, any subnetwork is supplied through the same pair of nodes and is monitored using two out of four available reference responses. Practically, subnetworks localized close to external edges of base network can be diagnosed more precisely (e.g. because greater number of reference responses may be available), hence the identified elements which are common for different subnetworks, can be excluded from distortion locations in successive isolations.

At the time of the writing, technical solutions for the controlling circuitry are still under development (patent procedure) hence will not be discussed in detail. It can be only mentioned that isolation of subnetworks will be done through a system of electrical switches localized in the nodes of the base network. The concept of switchable network, of configuration as in the Figure 4.25, was positively verified on experimental setup (a system of jumpers was utilized to isolate successive ladder subnetworks from the base grid).

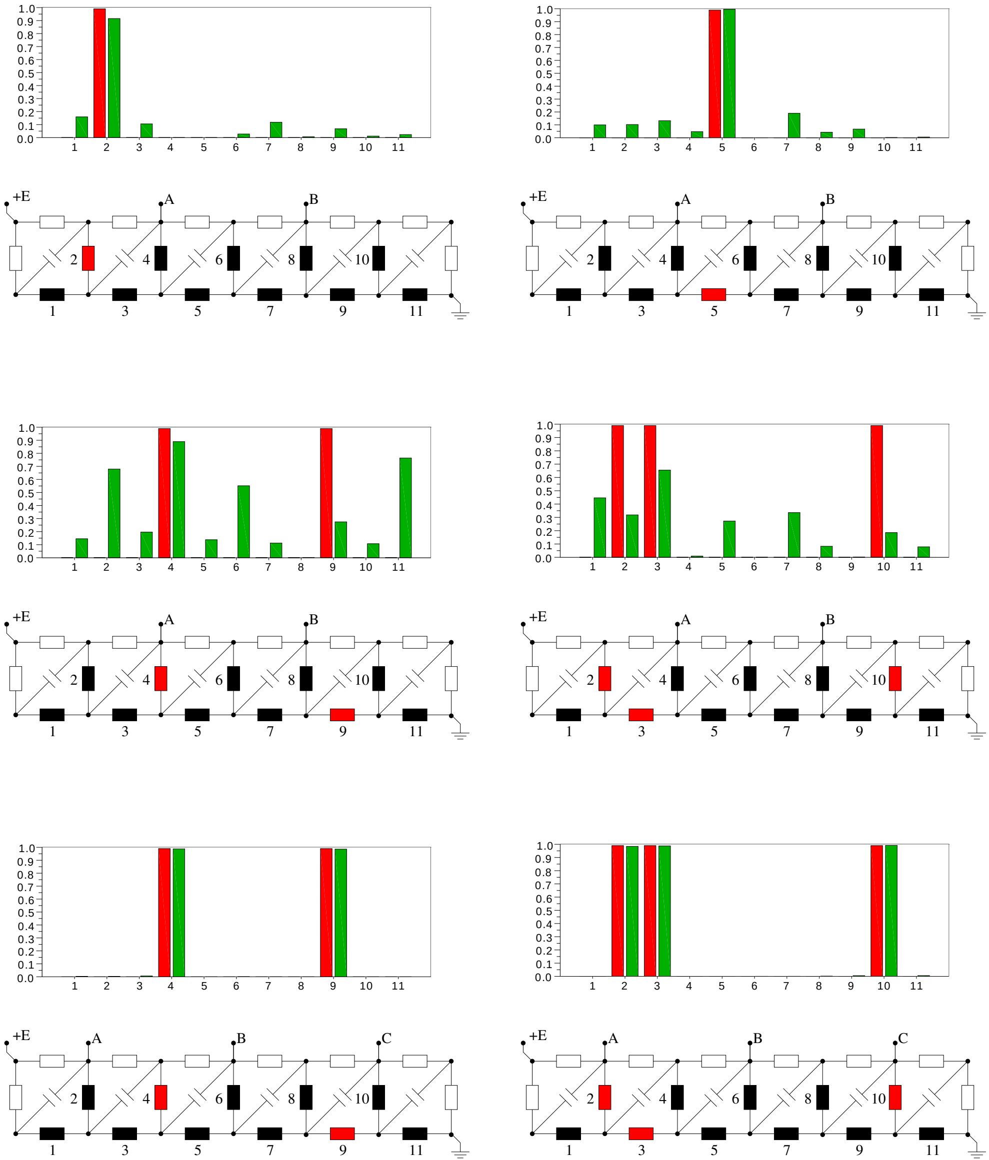


Figure 4.24: Identification in ladder network for various damage scenarios

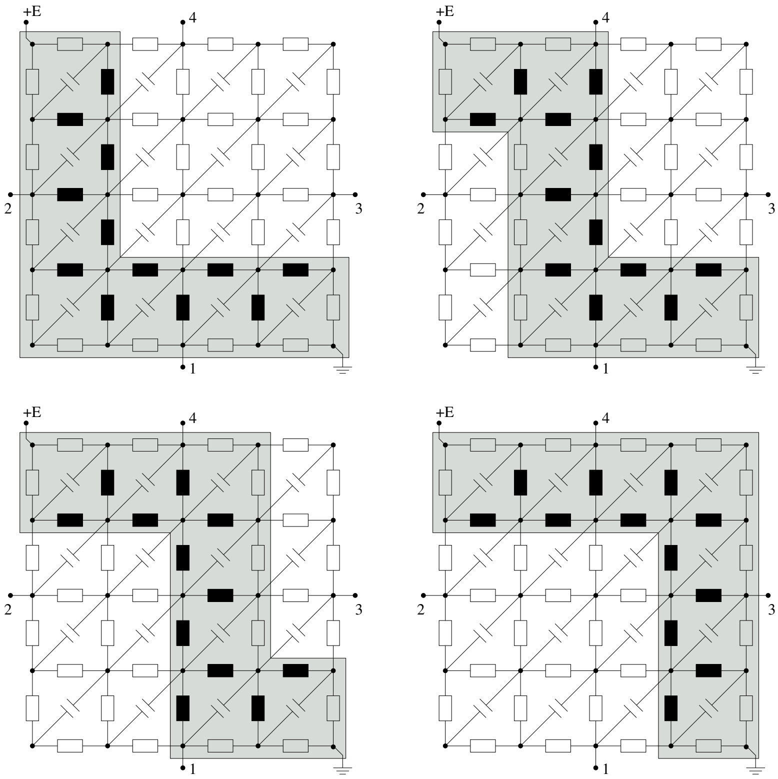


Figure 4.25: Concept of switchable networks

Summary

Original achievements

The main thesis of the work postulated that the problem of damage identification in electrical networks can be solved through dynamic inverse analysis, providing that configuration of the network complies with some pre-defined conditions. In the course of dissertation, an original methodology of diagnosis, based on the adapted algorithms of the Virtual Distortion Method, has been developed, verified and successfully applied to identify damages in electrical networks of tree, ladder and ribbon topology, which all satisfy the introduced topological conditions of diagnosability. Also, indirectly, based on the concept of switchable networks, the developed methodology can be applied to diagnose networks of grid topologies. In author opinion, the thesis has been validated and all assumed objectives have been accomplished.

Particularly, the following original achievements can be distinguished:

- The concept of electrical virtual distortions, which enable to simulate modifications of conductance and/or capacitance, has been adapted to DC, AC and dynamic circuit analysis.
- The concept of electrical influence matrices has been adapted.
- VDM-based procedures of reanalysis, which enables to quickly recalculate selected circuit responses on local modifications of parameters, have been implemented.
- VDM-based procedures of sensitivity analysis with respect to parameters of conductance or capacitance modifications, have been implemented.
- A gradient-based optimization procedures for solving the problem of damage identification in steady-state and dynamic circuits have been implemented.

- The concept of impotent states of distortions, which enables to explain the source of non-uniqueness of solution to damage identification problem, has been introduced.

- A set of rules for design of diagnosable sensing networks, has been formulated.

- Topological configurations of trees, ladders and ribbons have been identified as possible applicable solutions of electrical sensing networks. Electrical sensors in the form of tree and ladder networks, as well as the method of monitoring of structural elements with the said sensors, are patent pending solutions[1]

- The concept of structural health monitoring (ELGRID) system based on electrical sensing networks (plain and switchable) has been introduced.

- New problems of topological and multi-criteria optimization for diagnosability and design of switchable networks have been identified.

In practical aspect, a package of numerical tools for analysis and diagnosis of electrical sensing networks have been developed and implemented in Scilab [56] environment.

Further development

The assumed model of electrical sensing network was intentionally limited to simple models of linear RLC circuits. However, capabilities of both MNA and VDM are much wider and the developed methodology of network diagnosis may be extended to include more complicated circuit configurations. On the one hand, all linear components or devices which can be handled by the Modified Nodal Analysis (e.g. controlled sources, mutual inductors or operational amplifiers) may be incorporated. On the other hand, Virtual Distortion Method utilizes the concept of plastic-like virtual distortions which enable to simulate finite elements with piece-wise linear characteristics. As a result, simple models of diodes, switches or circuit breakers could be possibly implemented.

The proposed methodology of damage identification has been based on the steepest descent approach, which is slowly convergent and requires many

[1]Polish patent application no. P-390193

iterations. The heaviest computational effort in VDM-based procedures is related with calculation of gradient of objective function, which needs to be done at every iteration. In order to accelerate the convergence, it is planned to implement procedures with optimized or adaptive step length, or to apply other optimization approaches (e.g conjugate gradients or quasi-Newton methods). The methodology has been formulated to identify **arbitrary** modifications of parameters within the network, which is the most general case and allows to assume any damage scenario, but usually results in minor inaccuracies of the obtained solution (e.g. residual deviations of parameters in non-modified elements). In practical implementation in the monitoring system, where particular types of damages can be expected (e.g. breaks), introduction of additional constraints or penalty functions on optimized parameters should lead to more precise results of identification.

Another step will be formulation of algorithms for automatic generation of optimal network layouts. This will be considered as a single- or multi-criteria optimization problems involving topological conditions of diagnosability but also the issues of selection of optimal test stimuli and reference responses. From practical reasons, an important aspect will be minimization of the number of required test stimuli supply and response measurement, which is however opposed to the problem of diagnosability. In the case of switchable networks, a new problem is to determine the optimal decomposition of the base layout into subnetworks, which will enable to identify any combination of defects within the assumed global set of distortion locations, for the fixed locations of test stimuli and response measurements.

Ultimately, the developed methodology is intended to be applied in the ELGRID monitoring system, particularly in the implementation based on the concept switchable networks. The ELGRID system is developed within two Polish research projects which involve the problem of health monitoring of aerospace materials and civil engineering infrastructure. Namely, the system will be applied to detection of cracks and delamination in laminated composite panels and detection of cracks in concrete structures.

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Other resources

[53] http://bwrc.eecs.berkeley.edu/Classes/IcBook/SPICE/

[54] http://www.gnu.org/software/gnucap/

[55] http://qucs.sourceforge.net/

[56] http://www.scilab.org/