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Review of the doctoral dissertation of Mr. Christopher I Trombley: Dynamics of Systems of Charged Particles Sedimenting in a Viscous Fluid

Mr. Christopher Trombley dissertation thesis deals with the phenomenon of sedimentation of particles suspended in a viscous fluid. The thesis is purely theoretical not supported by experiments. The sedimentation phenomenon itself has been known since the beginning of our civilization. It occurs in the production of wine, beer, varnishes, etc. Due to its practical applications in technology, food industry, biology, and medicine, it is also the subject of intense theoretical research starting from the work of the Irish scientist George Stokes in the mid-19th century. The majority of papers concern the fall of electrically uncharged particles in an incompressible viscous fluid. For example, it has been found that such particles can coherently sink forming geometrical structures, generally not asymptotically stable. It is known that often, during falling, the particles can acquire electric charges, as is in the case of water droplets forming a thundercloud. However, the literature on charged particle sedimentation is rather scarce.

As the author of the paper emphasizes, the long-known theorem, proved at the time of G. Stokes by Samuel Earnshaw and concerning the behavior of charged particles in a vacuum, could suggest that the presence of an electric charge should rather destabilize the system. As shown, however in the reviewed doctoral dissertation, the opposite is true! This makes the thesis exceptionally interesting.

Thus, **the aim of the doctoral dissertation** of Mr. Trombley was to investigate the influence of the presence of the electric charge of the suspended particles on the possibility of formation of stable configurations falling as a whole. In the dissertation **only two particles falling** in a viscous fluid are considered. If the viscous forces dominate the inertia forces, then with a good approximation, the linear system of Stokes equations can be used to describe the fluid motion caused by falling particles. It has the advantage (linearity) that the sum of solutions is again a solution, which to some extent allows the results obtained for two particles to be extended to sufficiently diluted suspensions of a larger number of particles. We should keep in mind that in the case of particles with smaller density than the fluid density the particles will move up, against the gravitational force.

The following **simplifications were adopted** in the analysis carried out in the dissertation

1. *There are only two particles in the fluid*
2. *the particles have spherical shapes and generally different radii*
3. *The electric charge is uniformly distributed over the surface of the sphere. This is equivalent to the assumption that it is placed in the center of the sphere.*
4. *Although the movement of the particle under consideration is the resultant of the force of gravity and the forces resulting from the remaining hydrodynamic interactions of other spherical particle, the particle under consideration is treated as a point particle. In other words, its center falls in the flow generated by the remaining particle. Thus, the possible rotation of such a particle which is generated by the curl of the velocity field is not taken into account.*
5. *The possible Debye screening (as in liquid electrolytes) is not taken into account. Consequently, the electric forces between the particles are given by Coulomb law.*

Although, these assumptions are somewhat restrictive, they permit to simplify the problem making it analytically tractable. Taking into account possible rotation of particles would lead to much more complicated analysis possible probably only by the numerical approach. Debye screening may be

relevant for electrolytic fluids. Assumptions 3 and 4 can be justified if the distance between particles is much larger then their diameters.

RESULTS

The obtained results are contained in two co-authored papers:

1. In paper B. Trombley, C.I. and Ekiel-Jeżewska, M.L., 2018. Stable Configurations of Charged Sedimenting Particles (Physical Review Letters, 121(25), p.254502) it is shown that in the case of opposite charged particles and particles aligned in such a way that the line connecting their centers is parallel to the gravitational field, stationary, asymptotically stable configurations of two particles falling together are possible for a wide class of parameters (i.e., charges and diameters). Moreover, the stability basins of these configurations have been determined. It has been shown also that their measure is generally infinite.

2. In paper: Chris I Trombley and Maria L Ekiel-Jeżewska 2021 J. Phys. Commun. 5 075005, the results of the first paper were significantly extended.

a) it is shown that there are also inclined configurations where the angle between the gravitational field and the line joining the centers of the spheres may have values different from zero.
b) The results obtained for charged particles were compared with the results for uncharged particles.
c) The basins of attraction of stable attracting configurations of charged particles, were analyzed.
d) Some numerical examples are given for illustration.

The analysis was performed in a coordinate system associated with one of the moving spheres. In this way, it was possible to reduce the problem to two dimensions, which significantly simplified the analysis. For example, it allowed to use the results known in the case of two dimensions, such as the Poincaré–Bendixson theorem in analyzing the global behavior of trajectories.

The dissertation consists of three works co-authored by the thesis supervisor, Professor Ekiel-Jeżewska. The first of these papers: A. Trombley, C.I. and Ekiel-Jezewska, M.L., 2019. Basic Concepts of Stokes Flows, published in Flowing Matter (pp. 35-50). Editors: Toschi, F. and Sega, M., Springer, Cham. is introductory and does not contain original results. Basic facts concerning the Stokes system of equations are given there, such as **principle of minimal energy dissipation**, the **Stokes paradox**, and the **fundamental solution** of the Stokes equations. These notions and facts are used later in the analysis of the motion of sedimenting particles and thus may be very helpful for the reader.

Mr. Trombley's input, particularly in papers 2 and 3, **was dominant** as it follows from the written opinion of prof. Ekiel-Jeżewska.

Together with 3 publications of the dissertation Mr. Trombley is a coauthor of 8 publications, covering variety of research subjects, from economy to magnetic levitation and fluid mechanics. He also had 5 presentations on various international conferences.

SUMMARIZING. The results obtained by Mr. Trombley are not only interesting and valuable from the scientific point of view, but they can (and certainly will be) be useful in practice, as evidenced by the publication in Phys. Rev. Lett.

Therefore, I highly appreciate Mr. Trombley's doctoral dissertation. In my opinion it deserves a distinction and consequently I am asking for his admission to the further procedure of awarding him the doctor degree in technical sciences.