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ON A MODEL FOR PREDICTION OF THE MOVEMENTS OF A CROWD IN NARROW EXITS

Z. Kotulski, W. Szczepiński

Institute of Fundamental Technological Research
Świętokrzyska 21, 00-049 Warsaw, Poland

In the paper a method of simulation of the movement of a crowd in narrow exits is presented. The stochastic model applied is based on the concepts proposed by J. Litwiniszyn concerning the analysis of movements of earth masses caused by underground mining works. The main part of the paper contains the results of mechanical simulation of the crowd movement in several geometrical configurations of the exits. The concluding sections of the paper present the diffusion interpretation of the obtained experimental results and a proposition of further research based on several practical models of movement of granular material.

1. Introduction

Below we present an attempt to apply a stochastic model to the simulation of the movement of a crowd in narrow exits. The model is based on the concept proposed by J. Litwiniszyn in his early works [1–3] concerning the analysis of movements of earth masses caused by underground mining works. In these papers he analyzed the movements of soil particles as a random walk due to gravity forces and random changes of mutual contacts between the particles. When the movement of a crowd is concerned, the displacement of particular persons is caused by their will to leave the gathering place through the particular exit and is influenced by random contacts with other persons. Thus, one can expect that there is a certain similarity between the random movements of particles in a bulk of granular body and random movements of persons in a crowd leaving the gathering place (theater, stadium, etc.), and that similar methods of analysis may be used in the two cases. Proposing such an approach to the analysis of movements of a crowd we do not claim that such a theoretical model strictly corresponds to reality or that better models could not be used. To verify the practical significances of the model, the observations of real movements of crowds in various situations would be needed.

As the introduction to Litwiniszyn's procedure, which constitutes the basis of the methodology used further in this paper, let us use the well-known demonstrating device known as the Galton's board (see, e.g., [4]), in which small metal balls falling down from a container strike numerous, regularly located pins and are randomly directed to the right and to the left with the same probability equal to 1/2. Finally, they fall at random into one of separate small containers at the bottom of the device. The distribution of the number of balls in consecutive containers is close to the normal distribution (see, e.g., [3–5]). This is demonstrated in Fig. 1, where the pins are shown as small circles and the field under consideration is divided into a set of rectangular cells.

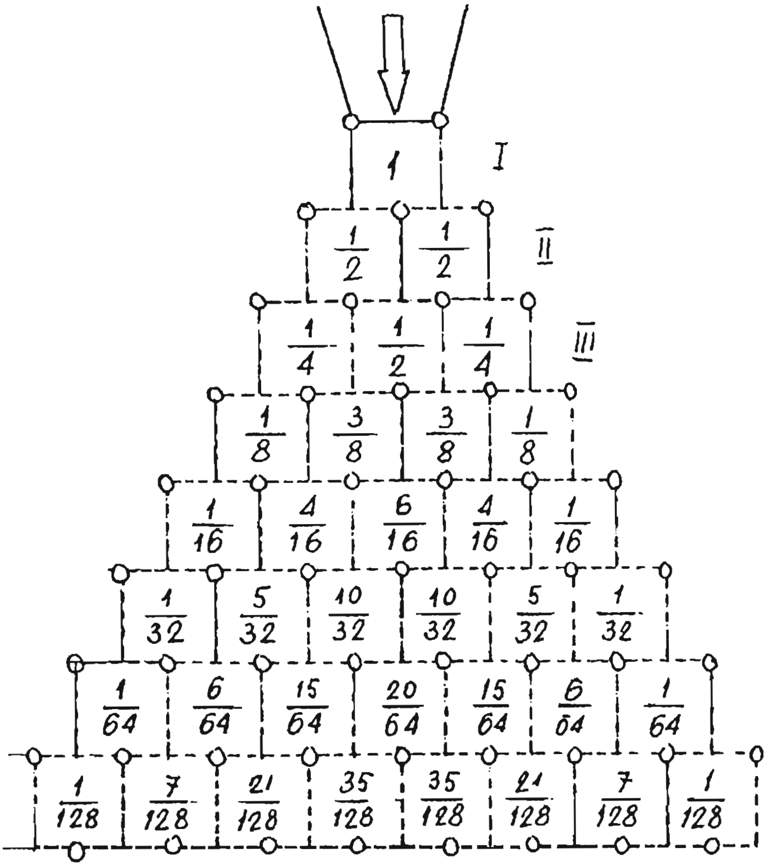


FIG. 1.

The ball starting from the level I will be displaced to a cell on any other level below. The probability that it falls into the left-hand or into the right-hand cell on level II is equal to 1/2. The probabilities of falling the ball for cells on level III are 1/4, 1/2, 1/4, and similarly for other levels, as shown in the Fig. 1.

2. The method of finite cells

In the papers mentioned above, J. Litwiniszyn ingeniously analyzed the inverse problem in which the cavities in a bulk of a loose material move randomly upwards from the bottom. To illustrate his idea, let us consider a two-dimensional problem of a relatively wide container with an outlet at the middle of the bottom. Figure 2 shows the assumed system of finite cells.

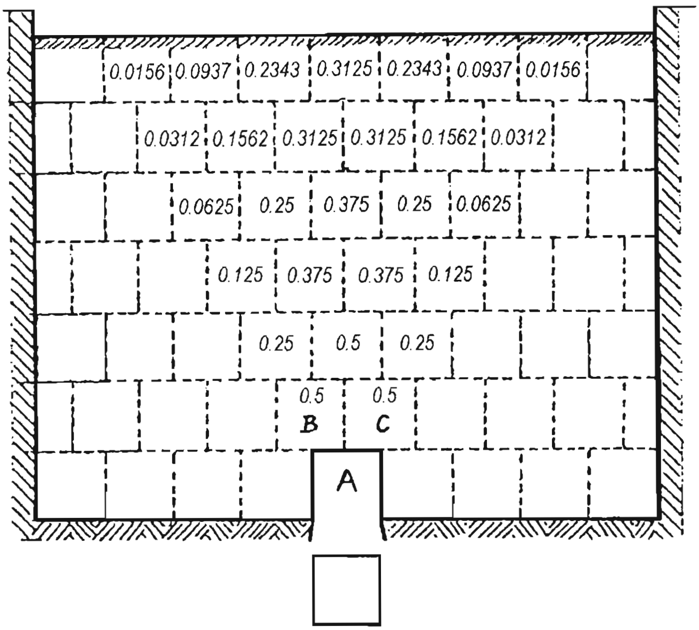


Fig. 2.

A portion of the loose medium has just now left cell A, leaving an empty space in it. The cavity in A formed in such a manner migrates upwards. We assume, as in the inverse problem shown in Fig. 1, that each time a portion of that cavity moves upwards, the probability of migrating into the right or into the left-hand cell lying just above is equal to 1/2. It means that at the beginning of the migration process, one half of the initial cavity A moves to the cell B and the other half is shifted to the cell C. If the volume of each cell is assumed to be a unit volume, the numbers in consecutive cells indicate how large portion of the initial unit volume A passed through the cell during the migration process. Since after migration each portion of empty space must be filled by the granular medium falling downwards, these numbers correspond to the average vertical displacement of the medium in particular cells. These vertical displacements are represented in Fig. 3. Thus, the approximate field of displacements is shown by the family of stepwise lines.

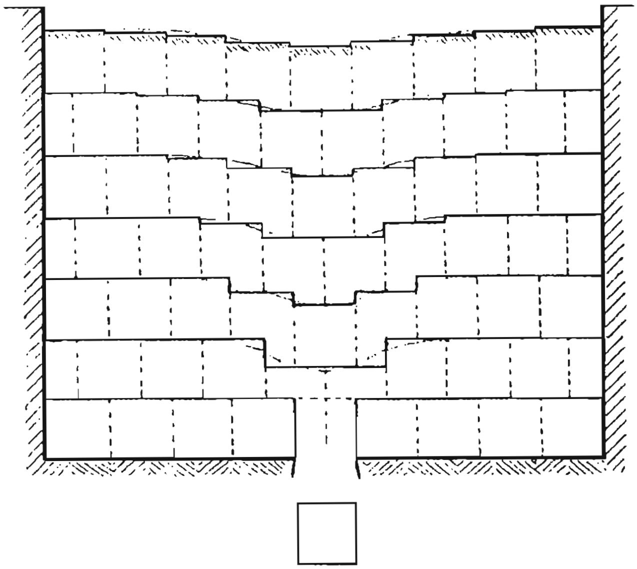


FIG. 3.

The idea of finite cells technique shown in Fig. 3 for the basic configuration with a single empty cell A at the bottom may be extended for more advanced situations. An example is shown in Fig. 4 where at the bottom of a system of cells there are several empty cells.

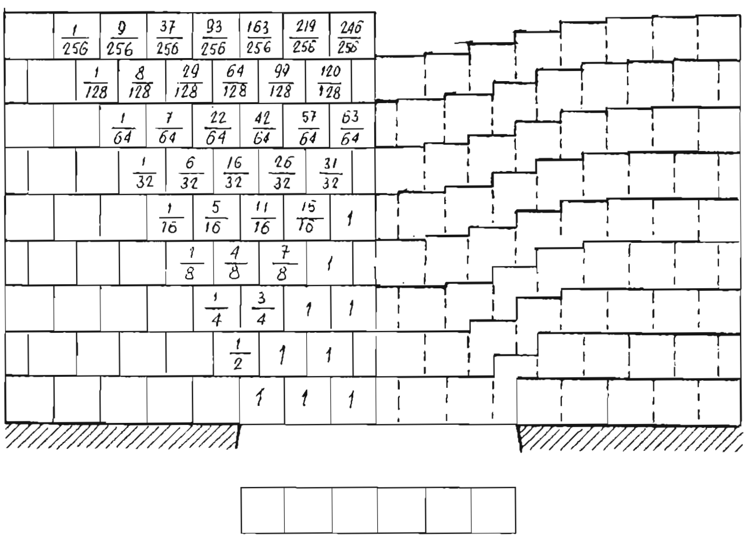


FIG. 4.

These cavities move upwards in a process of movement of individuals attributed to particular cells towards the exit. The numbers in consecutive cells shown on the left-hand side of the figure indicate how large portion of the initial empty unit volume passed through the cell in question during the migration process. On the right-hand side of the figure the stepwise lines represent the averaged vertical displacement of individuals in the crowd moving towards the exit.

Finally let us remark that in the both cases presented in Figs. 3, 4, the exit should be considered as narrow, in spite of the fact that its width in both the cases is quite different. In the paper the exit will be considered as narrow, if the movement of the cells in the direction parallel to the symmetry axis of the exit excites an additional intensive movement in the perpendicular direction. Its intensity could be expressed quantitatively (in terms of the dimensions of the container, the width of the exit and the dimensions of the cells), but this exceeds the scope of the paper.

3. Analysis of displacements

In Sec. 2 was described the numerical procedure allowing to calculate vertical components of displacements vectors. However, each individual in the crowd may be displaced also horizontally. Below is presented a simple approximate method based on the finite cells technique described in Sec. 2. Let us analyze any arbitrary set of adjacent cells, for example taken from the system of cells shown in Fig. 2. They are represented in Fig. 5a. The numbers in them correspond to the fraction of the initial volume of the cavity A, which passed through the cell during migration towards the free surface of the bulk of the medium. According to the finite cells methodology, only one half of these fractions migrates from each cell A and B to the cell C. It is assumed that this migration takes place along the respective lines $A-C$ or $B-C$ joining central points of the cells. Directions and magnitudes of these migrating portions of the cavity may be represented by vectors $\mathbf{w}_{BC}$ and $\mathbf{w}_{AC}$ as shown in Fig. 5b. They may be treated as components of the resulting vector $\mathbf{w}_{\mathrm{cav}}$ representing the direction and the magnitude of the averaged momentary flux of the cavity into cell C during the migration process. The opposite vector $\mathbf{w}_{\mathrm{mat}}$ may be treated as representation of the flux of persons in the crowd filling the space left by cavities moving upwards.

In order to calculate the magnitude of the averaged displacement vector $\mathbf{u}$ of persons in the crowd, it is assumed that its direction coincides with the direction of the vector $\mathbf{v}_{\mathrm{mat}}$. To make this procedure consistent with that described in Sec. 2, it is assumed that the vertical component of the displacement vector $\mathbf{u}$ is equal to the vertical displacement of the respective sector of the stepwise-deformed boundary between the rows of cells (see Fig. 3). Using this approximate numerical procedure, the vectors of displacements have been calculated for the problem shown in Fig. 4 the results are shown in Fig. 6, in which vectors represent averaged theoretical displacements of individuals situated in the particular cell after all persons from the first row of cells have left the chamber. It is seen that the movement of persons takes place mainly in the central part of wedge-shaped area.

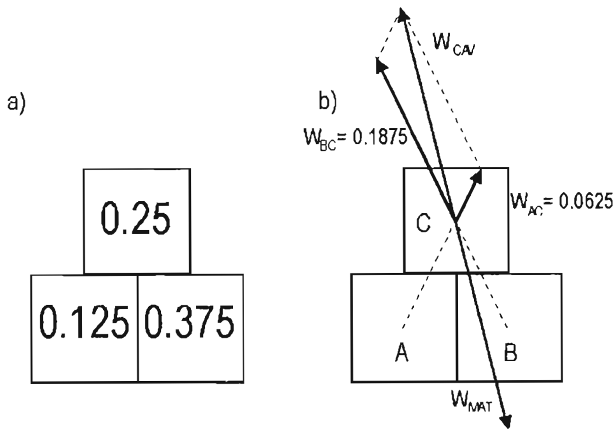


Fig. 5.

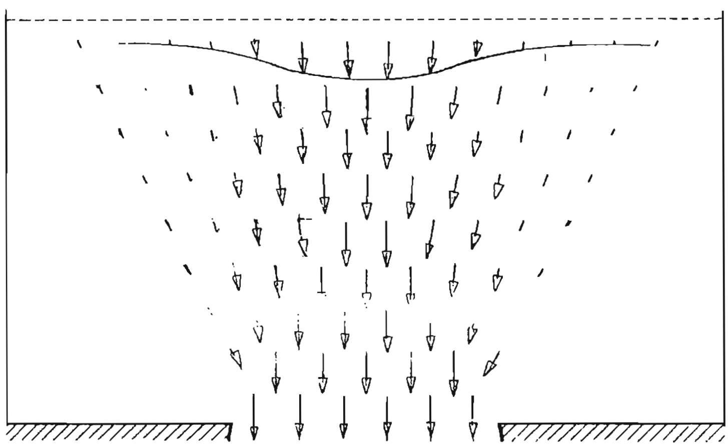
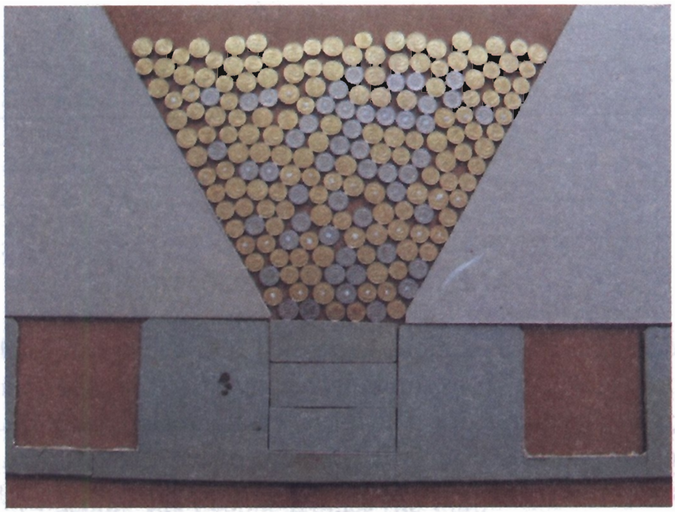


Fig. 6.

In order to verify experimentally such a theoretical motion pattern, a preliminary simple experimental simulation model composed of an assembly of coins of three different diameters has been used. The initial configuration of the assembly corresponding to the theoretical problem shown in Fig. 4 is presented in Fig. 7a. The coins are located on a glass plate in the initial horizontal position. Then the plate is inclined with respect to the horizontal plane and the coins begin to slide downwards due to the gravity forces. This movement is disturbed by random mutual contacts between neighbors. The final configuration of displaced coins is shown in Fig. 7b. By comparing both figures a and b, the displacements of several coins have been measured. Such measured displacement vectors are shown in Fig. 8. It is seen that the result largest displacements take place in the central part, similarly to the result predicted by a theoretical model, see Fig. 6.

a)



b)

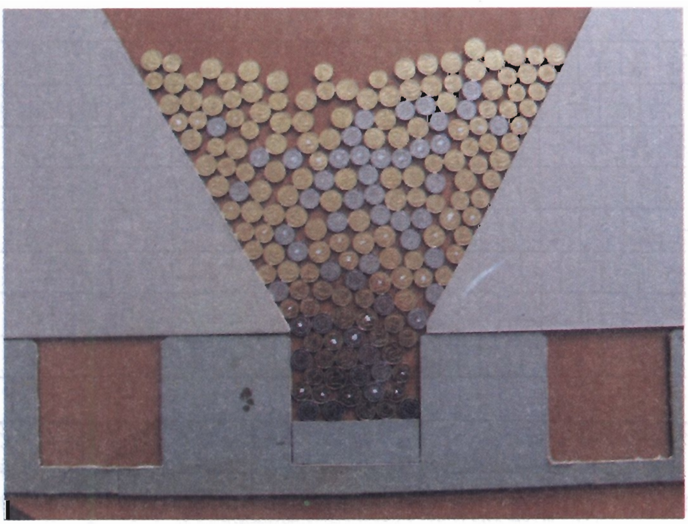


FIG. 7a, b.

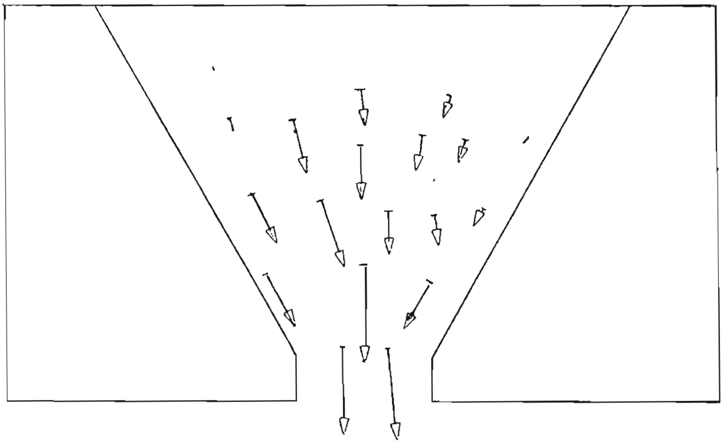


FIG. 8.

4. EXAMPLE OF APPLICATION

Using the numerical technique described above one can analyze numerous particular problems. As an example let us consider the movements of a crowd through regularly located exits. The initial configuration along with the assumed system of cells is shown in Fig. 9. It is assumed that persons occupying the first row of cells adjacent to exits have just left the region, leaving these unit cells empty, as shown in the figure. These empty unit spaces move then upwards according to the stochastic numerical procedure discussed in previous sections.

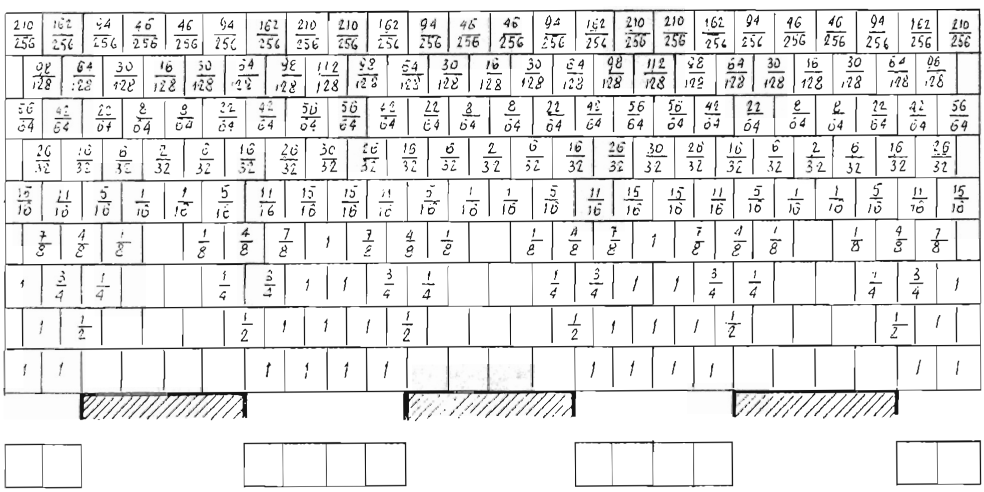


FIG. 9.

The numbers in particular cells indicate how large portion of the unit empty space passed through the cells during the migration process. Using these numbers one can calculate the stepwise approximation of the movement within the region, see Fig. 10. Formation of the dead zones with no movement within them can be seen in Figs. 9 and 10.

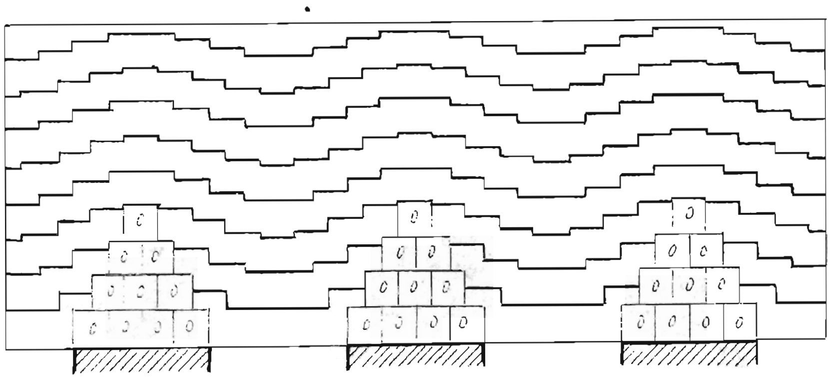


FIG. 10.

In Fig. 11 are shown the displacement vectors calculated according to the procedure described in Sec. 3. In place of the deadzones appearing in theoretical solutions, the wedge-shaped structures near the exits have been introduced in order to prevent pushing the people against the wall.

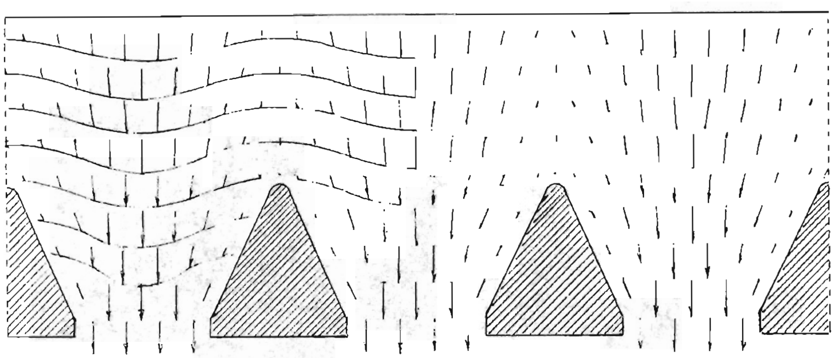


FIG. 11.

The theoretical analysis of the movement presented here has been confronted with the experimental simulation in which a large assembly of coins of three different diameters slides down on a glass plate inclined to the horizontal plane. The initial experimental configuration is shown in Fig. 12. The experiment was performed in three stages. In stage 1 one strip from each hole at the bottom simulating the exit has been removed. In stage 2 two strips have been removed and finally in stage 3, three strips have been removed. Figure 13 presents the configuration assembly of displaced coins corresponding to stage 2.

FIG. 12.

FIG. 13.

Comparing the positions of coins in Figs. 12 and 13 one can find their displacements between the initial stage and stage 2. The displacement vectors are shown in Fig. 14. The concentration of movement along the extended axes of exits is visible. Also dead zones with no remarkable movements can be observed. In such an experimental modeling and also in real movement of a crowd, one can hardly expect such a regularity of movement as predicted by the idealized theoretical solution, compare Fig. 11. However, main features of the movement are close one to the other. The disturbances in the movements are caused mainly by random mutual contacts between the coins. As a result, the coins are rambling in their way towards the exits. Such rambling in paths of a number of coins are shown in Fig. 15. Consecutive vectors of these paths correspond to the displacements of a coin between the stages 0-1-2-3 as indicated in the figure.

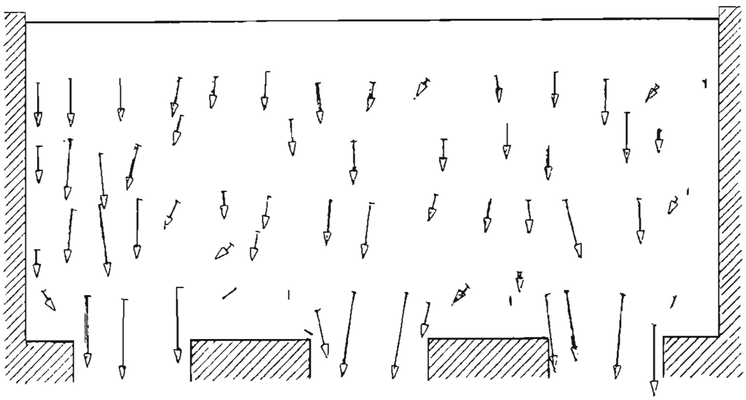


FIG. 14.

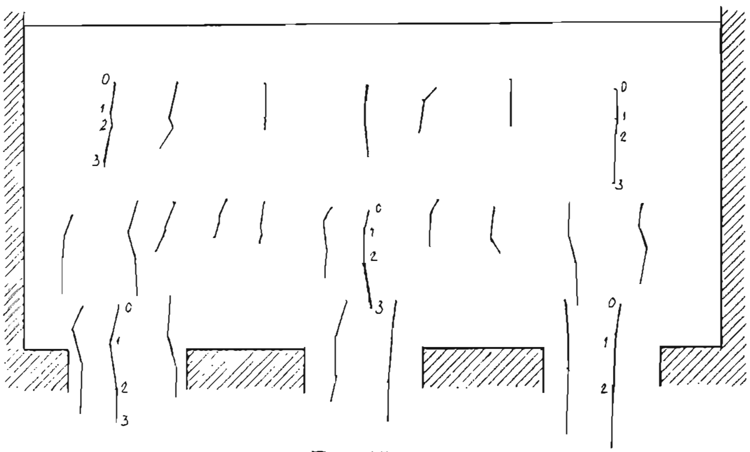


FIG. 15.

Using this stochastic numerical procedure one can analyze numerous other problems. As an example, the displacements of individuals in the crowd leaving the region through a sequence of exits in the presence of repeating obstacles is shown in Fig. 16.

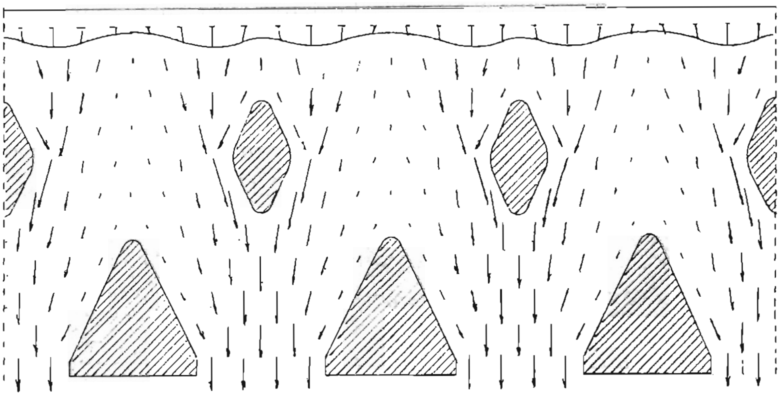


FIG. 16.

5. Discussion

In the paper we have presented an elementary probabilistic model of crowd movements based on the adapted random walk process (the superposition of a constant velocity unidirectional movement and the one-dimensional random walk). As it is seen, such an approach has a very good mechanical analogy: the gravitational flow of granular media. In both cases we have applied one conservation law: the conservation of probability (in probabilistic interpretation) and the mass conservation law (continuity equation) in mechanical interpretation. Following the reasoning, we could transform the persons' movements into the discrete (finite differences) description to the continuous model governed by partial parabolic-type differential equations. The continuous model leads to analytical representation of the continuous approximations of the boundary surfaces of the region occupied by the crowd. As in the case of Galton's board cf., e.g., [4], in the limit we obtain the heat (diffusion) equation of the form:

$$\frac{\partial P(x,y)}{\partial y} - D\frac{\partial^2 P(x,y)}{\partial x^2} = 0, \tag{5.1}$$

where $P(x,y)$ represents the probability that a person will cross the level y at point x. In the model, the independent variable y represents the direction along "gravitation" and the variable x – the perpendicular direction. Solving the equation under the condition:

$$\int_{-\infty}^{\infty} P(x,y)dx = 1 \tag{5.2}$$

and substituting $\sigma = \sqrt{2Dy}$, we obtain:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2}\right]. \tag{5.3}$$

This function, for some value σ and under appropriate interpretation, describes the shape of the top surface of the granular medium leaving a container presented, e.g., in Figs. 3, 4 or 6, in the continuous limiting case.

Also some superposition of such horizontally shifted functions could describe analogous surfaces at Figs. 11 and 16. The situation at Fig. 16 requires several functions with two different values of the parameter σ.

So far, we did not give any interpretation of the parameter D in Eq. (5.1). In the model of a discrete random walk applied to obtain the diffusion equation, the parameter D is:

$$D = \frac{a^2}{2b}, \tag{5.4}$$

where a and b are the dimensions of unit cells in the direction of x and y, respectively. These dimensions in our experiment and in the model of crowd movements were equal to each other. However, there is no problem to consider a more complicated model with cells having the two dimensions different.

6. Further research

Following the reasoning of the authors considering mechanical flow problems of granular media, we could propose other mathematical descriptions of movements of a crowd. Thus, extending the random walk description in the cellular model, we could use cellular automata to model jumps between adjacent cells (cf., [6, 7]). As it is known, the cellular automata are discrete dynamical systems with a simple cellular structure (e.g., similar to the structure of the cells presented in figures shown in this paper), but with sufficiently complex behavior describing movements from cell to cell, depending on the global state of the cells in the previous step. The cellular automata can have several states per cell (in the random walk model we have only two states: the cell can be empty or occupied) and complicated rules of evolution: the probabilities of jumps (or rather changes of states) can depend on the previous states of the adjacent cells or more distant cells (lying in several layers around a cell). In such a way we could describe non-local interactions of persons in the crowd and some non-homogeneous disturbances of the movement.

Let us remark that the cellular automata description of the crowd movement is strictly phenomenological. In this case we observe the behavior of the crowd over some period of time and then identify the laws of transformation in the corresponding cellular model. Certainly, such an approach implicitly takes into account any mechanical constraints of the movement as well as some social conditions in the moving groups of people. In the literature one can also find papers where the mechanical laws are explicitly included into the cellular description of the movement, e.g., the discrete mechanical models of rigid bodies presented in ([8]).

Trying to make a deeper analysis of the movement of crowd one can extend the elementary model considered in this paper, where only the law of conservation of mass (the continuity equation) was taken into account, and additionally consider the other fundamental laws of mechanics: the law of conservation of linear momentum and the energy conservation law. For example, in the paper [9] the authors tried to describe the road traffic using mechanical analogy. The obtained one-dimensional model was formulated as a partial differential equation of the hyperbolic type, what finally resulted in the finite speed of transportation (what is more realistic than in the parabolic type of the governing equation).

Finally, let us remark that the governing hyperbolic equation could be obtained not only through deep mechanical analysis of the motion of individuals in the crowd. Restricting the considerations only to the random walk over a cellular medium, we can also use the hyperbolic telegrapher's equation in a one-dimensional case (see [10]), or the system of hyperbolic equations in two and more dimensions (see [11]). According to a certain interpretation, the equations describe the probability of finding a particle (in our case: a person) in the place with given spatial coordinates. Certainly, to model a realistic situation of the crowd movements, the equations must have appropriately identified parameters.

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