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SPECTRAL THEORY OF UNIDIRECTIONAL TRANSDUCERS

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A spectral theory of single-phase unidirectional transducers of the surface acoustic waves is presented. It helps to evaluate the stop-band width where the directionality takes place, and allows one to optimize the transducer structure in order to obtain its symmetric frequency response, with maximum directionality at the stop-band center.

Key words: surface acoustic waves, interdigital transducers.

1. Introduction

The single-phase unidirectional transducer (SPUDT) is a periodic system of especially designed groups of metal strips on piezoelectric substrate having the following properties: 1) it generates the surface acoustic waves (SAWs) in both directions, to the left and to the right from certain imaginary "generation centers" placed inside each group, 2) SAWs are partially reflected (due to the Bragg reflection) from the imaginary "reflection centers" in each group, and 3) both the generation and reflection centers are displaced by a quarter of SAW wavelength (in the best case). This results in the transducer unidirectionality, provided that the structure contains enough groups of strips to reflect the SAWs sufficiently [1]. Designing the group (the transducer's cell) including several strips of different width and spacings (in contrast to ordinary bidirectional interdigital transducers, IDTs, having two strips per cell) is not an easy task; there are only a couple of cells proposed in literature and applied in the SAW low-loss filters, the main area of applications of SPUDTs. This is due to the required evaluation of the positions of the generation and reflection centers based on the advanced electrostatic analysis of the electric charge distribution on strips, resulting either from the strip voltages or induced by the incident SAW.

The analysis presented here takes advantage of the recent theoretical solutions of both electrostatic problems mentioned above [2, 3]. The developed theory has this advantage over the earlier models [1, 4, 5] that it yields the dependence of the transducer directionality on frequency and particularly, it yields the stopband width where the directionality takes place, and also the relative SAW amplitudes generated in both the left and the right directions. The theory shows that the above mentioned displacement of the generation and reflection centers is not a correct characterization of the system directionality; some best systems have this displacement different from a quarter of wavelength. Moreover, although the presented theory neglects the mechanical interaction of the strips with the propagating SAWs, it can be easily generalized to account for this effect in the already presented manner [6].

2. Auxiliary electrostatic analysis

At center frequency, the propagating SAW wavelength equals the transducer period (the cell's length) $\Lambda = 2\pi/K$. The electric field excited at the plane of strips due to the substrate piezoelectricity is treated here as the preexisting (or "incident") field the strips are embedded in. This is the harmonic field $\exp(j\omega t - jIKx)$, where ω – angular frequency, and IK – the SAW wave-number for transducer working at its Ith overtone. The wave-field amplitudes are: D^I – the normal induction, and E^I – the tangential electric field. Both the propagating SAW and the voltage V applied to the transducer bus-bars (and thus to some strips in each cells) excite the electric field at the strips' plane represented by a series of spatial harmonics $\exp(-jnKx)$ with corresponding amplitudes D_n, E_n. The most interesting are the components coupled to the right and to the left propagating SAWs having wave-numbers $\pm IK$, respectively.

The electrostatic analysis presented in [3] yields the following relations for both $\pm I$ harmonic field amplitudes:

$$\begin{aligned} D_{-I} &= (1+t)D^{-I} + sD^I + p^*V, \\ D_I &= s^*D^{-I} + (1+t)D^I + pV, \\ J &= j\omega(d^*D^{-I} + dD^I + gV), \\ -j\epsilon E_{-I} &= (t-1)D^{-I} + sD^I + p^*V, \\ j\epsilon E_I &= s^*D^{-I} + (t-1)D^I + pV, \end{aligned} \tag{1}$$

where J is the cell contribution to the transducer current; the parameters t, g (real) and s, p, d (complex) can be evaluated from the electrostatic analysis taking into account that $D^{\pm I} = \mp j\epsilon E^{\pm I}$ for the "incident" field, where ϵ is the effective surface permittivity of the substrate. $D^{\pm I}$, having arbitrary value in this electrostatic analysis, will be evaluated later below using the dynamic properties of the substrate.

3. SAW propagation

The SAW wave-number $IK + \delta$ in the system may slightly differ from the value IK (δ assumed small). It results from the harmonic Green's function of piezoelectric media [7] that

$$\begin{aligned} \pm j\epsilon E_{\pm I} &= Z_{\pm} D_{\pm I}, \mathcal{Z} = \mathrm{diag}[Z_{-I}, Z_I], \\ Z_{\pm I} &= (IK \pm \delta - k_o)/(IK \pm \delta - k_v), \end{aligned} \tag{2}$$

where $k_{v,o} \sim \omega$ are the wave-numbers of SAWs propagating on free or metallized substrate. Solving Eqs. (1) with respect to $D^{\pm I}$ yields their dependence on the transducer voltage V, which applied to Eqs. (2) results in:

$$\begin{aligned} &(\mathcal{Z} - \mathcal{M})\mathcal{D} = (\mathbf{I} - \mathcal{M})\mathcal{P}V, \\ &\mathcal{M} = \frac{1}{(1+t)^2 - |s|^2} \begin{bmatrix} t^2 - 1 - |s|^2 & 2s \\ 2s^* & t^2 - 1 - |s|^2 \end{bmatrix}, \\ &\mathcal{D} = [D_{-I} \quad D_I]^T, \qquad \mathcal{P} = [p^* \quad p]^T \end{aligned} \tag{3}$$

(the superscript T means the matrix transposition). Neglecting V, one obtains the dispersive equation for SAWs, the solution of which is:

$$\begin{aligned} &\delta = \pm\sqrt{(\varepsilon - \varepsilon_1)(\varepsilon - \varepsilon_2)}, \qquad \epsilon = (k_v + k_o)/2 - IK, \\ &\varepsilon_i = (k_o - k_v)[1 - 2/(1 - \lambda_i)]/(k_o + k_v), \end{aligned} \tag{4}$$

where $\lambda_{1,2}$ are the eigenvalues of $\mathcal{M}$ associated with eigenvectors $[\pm F_1; F_2]$. The sign of δ is chosen such that the forward component of the right-propagating SAW have the wave-number $IK + \delta$. The values of $\varepsilon_{1,2}$ determine the stopband where the SAW wave-number is complex due to the Bragg scattering in the considered periodic system. [In the case of mechanical interaction [6] of strips with SAW characterized by the coefficient χ, the generalized result for the stopband width is $2|\chi - s^*(k_o - k_v)/(k_o - k_v)|$.]

4. Transduction

The spectral theory [6] of SPUDTs can now be developed. The finite, L-long system generates SAWs of different amplitudes to the right, a_L^-, and to the left, a_R^+, depending on the transducer voltage V (with certain proportionality coefficient A depending on ω, p, δ, L and the piezoelectric coupling coefficient of the substrate):

$$\begin{aligned} a_L^- &= VA[p^*(o + \delta + \lambda\chi) - p(o - \delta + \lambda/\chi)e^{-j\delta L}], \\ a_R^+ &= VA[p(o + \delta + \lambda/\chi) - p^*(o - \delta + \lambda\chi)e^{-j\delta L}], \\ o &= (k_o + k_v)(\varepsilon - \overline{\varepsilon})(1 - \lambda_1)(1 - \lambda_2)/(k_o - k_v), \\ \lambda &= \lambda_2 - \lambda_1, \quad \overline{\varepsilon} = (\varepsilon_1 + \varepsilon_2)/2), \end{aligned} \tag{5}$$

where $\chi = (F_1 p)/(F_2 p^*)$ is the directionality factor depending on the cell geometry, the best value of which is $\pm j$. The above relations allow one to evaluate the relative amplitudes of the left and the right-generated SAWs with respect to $a = (|a_L^-|^2 + |a_R^+|^2)^{1/2}$, resulting in the transducer directionality as a function of frequency deviation from the stopband center, proportional to o.

5. Discussion and conclusions

The structure directionality is best seen for $L \to \infty$. This removes the second-order effect of the SAWs reflections from the transducer edges. Its dependence on frequency (represented by o) clearly shows whether the strip cell is correctly designed to obtain symmetric dependence of the SAWs relative amplitudes $\overline{a}^{\pm}_{L,R} = a^{\pm}_{L,R}/a$ on frequency. One of these amplitudes should take zero value at the stop-band center and the other – its maximum unitary value, as shown in Fig. 1a). The well-known structure proposed in [4] exhibits slightly asymmetric frequency-dependent directionality, Fig. 1b). To improve this, one may try different widths and positions of strips in the cell and choose one of many solutions, usually yielding different stop-bands. This can be done with the help of the simple numerical code [8] (the figures present the output graphics). Two curves, upper with maximum unitary value, and lower with minimum zero value, joining into the common horizontal line at the $1/\sqrt{2}$ level, present the relative values of the left and the right-propagating SAW amplitudes (vertical axis). They are different only in the stop-band; horizontal axis o represents frequency. The complex number at the top of the figures is the evaluated directionality factor χ (its best values are $\pm j$), while the two large numbers at the bottom are $\lambda_{1,2}$ determining the stop-band edges; the small center number is I – the overtone number.

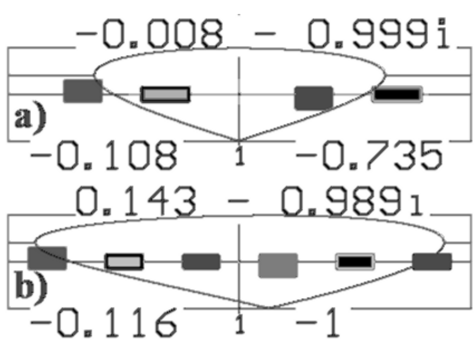

Fig. 1. Directionality of SPUDTs: thin curves represent the left and the right generated SAW amplitudes dependent on frequency (horizontal axis). a) Novel SPUDT cell having four strips with edges at $\Lambda[-55, 4$; 8.5,13.7; 23.45,28; 32.5,37.7$]/48$; the first and third strips are connected to the transducer bus-bars and two other are interconnected. b) Almost ideal characteristics of the frequently applied SPUDT [4]. Gray boxes represent cell strips: the raised and lowered ones are strips connected to the transducer bus-bars, the interconnected strips are marked by frames. The complex numbers at the top of the figures are the directionality factor χ; at the bottom the real values of eigenvalues $\lambda_{1,2}$ are presented on both sides of small number at the center which is the overtone number I.

Investigation of several other applied SPUDT structures show that the displacement of the generation and reflection centers does not characterize the structure directionality satisfactorily. Some best structures with symmetric frequency characteristics have this displacements different from $\Lambda/4$, like the novel one presented in Fig. 1a, while the others with the $\Lambda/4$ displacements, may have asymmetric characteristics. For finite L, the Eqs. (5) yield only partial directionality of SPUDTs because $|\overline{a}^{\pm}_{L,R}|$ never reach the limit values: 0 or 1. The above equations can help one to choose proper L to obtain the required SPUDT's directionality at the stop-band center.

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