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PRIFYSGOL CAERDYDD

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Annual Conference

Cardiff School of Engineering, 5th – 6th April 2004

ACME 2004

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Tuesday 6th April 2004

9:00 am – 10:40am Session 5a, Main Lecture Theatre (South 1.32)

9:00 – 9:20 A Thermodynamic Approach to Constitutive Modelling of Concrete
G.D. Nguyen and G.T. Houlsby.

9:20 – 9:40 Two-component contact for embedded planes in a 3D plastic-damage-contact constitutive model.
S. C. Hee and A.D. *Jefferson*

9:40 – 10:00 Size and Parabolic-Shape Optimization of Dome Structures with Buckling and Load Variation on the Joints Using Genetic Algorithm
M.R. Ghasemi and F. Azhdari

10:00 – 10:20 Constitutive Behaviour of a Pressure and Lode-Sensitive Material: Multiaxial Stiffness Change and Instabilities.
Roger Crouch and Mihail Petkovski

10:20 – 10:40 Recent developments in computational fracture mechanics at Cardiff
Q.Z. Xiao and B.L. Karihaloo.

10:40 – 11:10 Coffee Break, Room South 4.10(A)

9:00am – 10:40pm Session 5b, Lecture Theatre T4 (South 1.25)

9:00 – 9:20 Design Sensitivity of a Sequentially Coupled Problem: Casting.
R. Ahmad, D.T. Gethin, R.W. Lewis and E.W. Postek

9:20 – 9:40 Formulation of Lower Bound Limit Analysis as a Second Order Cone Programming (SOCP) Problem
A. Makrodimopoulos and C.M. Martin.

9:40 – 10:00 Finite element model of mould filling during squeeze forming processes.
E.W. Postek, R.W. Lewis, D.T. Gethin, R.S. Ransing.

10:00 – 10:20 Turbulence Modelling for Thermal Management of Electronic Systems
K. Dhinsa, C. Bailey, and K. Pericleous

10:20 – 10:40 Simulation of deformation of ductile pharmaceutical particles with finite element method.
L.L. Dong, R.W. Lewis, D.T. Gethin, and E.W. Postek.

10:40 – 11:10 Coffee Break, Room South 4.10(A)

Finite element model of mould filling during squeeze forming processes

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Introduction

During mould filling due to decreasing of liquidus temperature takes place solidification process. The paper deals mostly with a presentation of an application of a microstructural solidification model. The Navier Stokes equation describing the flow problem is solved using Taylor Galerkin method [1].

Thermal problem

The discretized heat transfer equation equation is of the form

$$\mathbf{K}\mathbf{T}_N + \mathbf{C}\dot{\mathbf{T}}_N = \mathbf{F} \tag{1}$$

where **K** and **C** are the conductivity and heat capacity matrices and **F** is the thermal loading vector. The thermal equation is integrated employing explicit time marching scheme.

Enthalpy method:

In the case of phase transformation, due to the existence of a strong discontinuity in the dependence of heat capacity with respect to time (Fig. 1), the enthalpy method is applied, [2]. The essence of the application of the enthalpy method is the involvment of a new variable (enthalpy). It allows to regularize the sharp change in heat capacity due to latent heat release during the phase transformation and leads to a faster convergence

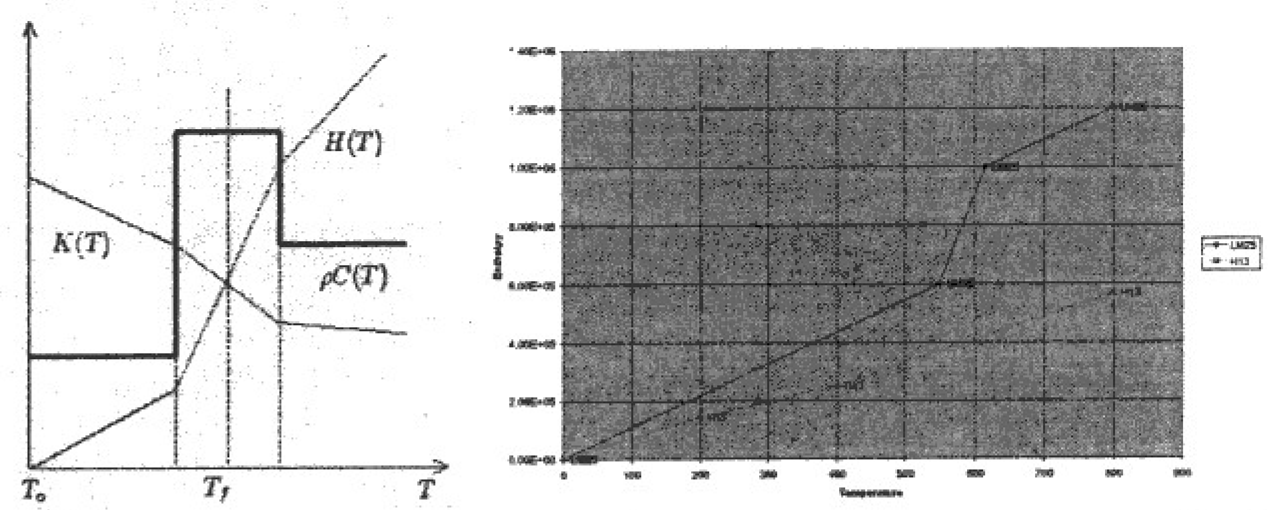


Figure 1. Illustration of the enthalpy method (left), enthalpy curves used in the example.

Introducing the new variable *H,* such as, $\rho c_p = dH/dT$ and employing the finite element approximation Eqn [1] is transformed to the following form

$$\mathbf{K}\mathbf{T}_N + \frac{d\mathbf{H}}{dT}\dot{\mathbf{T}}_N = \mathbf{F} \tag{2}$$

The definitions of the enthalpy variable for pure metals and alloys are given as follows

$$H = \begin{cases} \int c\,dT, & T \le T_m \\ \int c\,dT + (1-f_s)\Delta h_f, & T = T_m \\ \int c\,dT + \Delta h_f, & T > T_m \end{cases} \quad H = \begin{cases} \int c\,dT, & T \le T_{sol} \\ \int c\,dT + (1-f_s)\Delta h_f, & T_{sol} \le T \le T_{liq} \\ \int c\,dT + \Delta h_f, & T > T_{liq} \end{cases} \tag{3}$$

The following averaging formula [2] is used for the estimation of the enthalpy variable

$$(\rho c_p) \cong \left(\frac{\left(\partial H/\partial x\right)^2 + \left(\partial H/\partial y\right)^2 + \left(\partial H/\partial z\right)^2}{\left(\partial T/\partial x\right)^2 + \left(\partial T/\partial y\right)^2 + \left(\partial T/\partial z\right)^2} \right)^{1/2} \tag{3}$$

The averaging scheme is valid for mould filling and thermal stress analysis.

Mould filling

The flow of material is assumed to be Newtonian and incompressible [3]. The governing Navier Stokes equations is of the form

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (u \cdot \nabla)\mathbf{u}\right) = \nabla \cdot \mu\left[\nabla u + (\nabla u)^T\right] - \nabla p + \rho \mathbf{g} \tag{4}$$

where $\mathbf{u}$ is the velocity vector, p is pressure, μ is the dynamic viscosity and $\mathbf{g}$ is the gravitational acceleration vector. Mass conservation equation:

$$\nabla \cdot \mathbf{u} = 0 \tag{5}$$

Free surface tracking is governed by the first order advection equation

$$\frac{\partial F}{\partial t} + (\mathbf{u} \cdot \nabla)F = 0 \tag{6}$$

where F is the pseudo-concentration function varying from −1 to 1, $F<0$ indicates the empty region, $F>0$ indicates the fluid region, $F=0$ locates the free surface.

Microstructural solidification model

During entire process of forming of a part the solidification process takes place. To describe the process deeper a microstructure based solidification model is employed. The model stems from the assumptions given by Thevoz, Rappaz and Celentano [4, 5]. The basic assumptions are as follows: sum of solid and liquid fractions is equal one, solid fraction consists of dendritic and eutectic fractions.

$$f_l + f_s = 1,\ f_s = f_d + f_e \tag{7}$$

Further assumptions are connected with the fact of the existence interdendritic and intergranular eutectic fractions, the internal fraction consists of its dendritic and eutectic portions.

$$f_s = f_g^d f_i + f_g^e,\quad f_i = f_i^d + f_i^e \tag{8}$$

The last assumptions lead to final formulae for the dendritic and eutectic fractions (a spherical growth is assumed)

$$f_d = f_g^d f_i^d,\ \ f_e = f_g^d f_i^e + f_g^e,\ \ f_g^e = \frac{4}{3}\Pi N_d R_d^3,\quad f_i^e = \frac{4}{3}\Pi N_e R_e^3 \tag{9}$$

N_d, N_e are the grain densities and R_d, R_e are the grains radii. The grain densities and grains sizes are governed by nucleation and growth evolution laws. The rate of growth of the dendritic and eutectic nuclei is given below. It depends on the undercooling and there is assumed the Gaussian distribution of the nuclei.

$$\dot{N}_{(d,e)} = N_{\max(d,e)} \frac{1}{2\Pi} \exp\left(-\frac{\Delta T - \Delta T_{N(d,e)}}{2\Delta T_{\sigma(d,e)}} \right) \left\langle -\dot{T} \right\rangle, \qquad \Delta T_{(d,e)} = T_{(d,e)} - T \tag{10}$$

The rates of the dendritic and eutectic grains radii are established being an experimental dependence. Finally, internal dendritic fraction depends on melting temperature and k' is the partition coefficient.

$$\dot{R}_{(d,e)} = f_{R(d,e)}, \quad f_i^d = 1 - \left(\frac{T_m - T}{T_m - T_l} \right)^{\frac{1}{k'-1}} \tag{11}$$

A numerical example concerning mould filling is provided.

Example

Simulation of mould filling of an aluminium part (valve) employing the presented above solidification model is performed. The material of the mould is steel H13. The initial temperature of the cast is 650 deg and the 200 deg. Heat capacity and conductivity are the functions of temperature, radii rates (eutectic and dendritic) are given in Fig. 2.

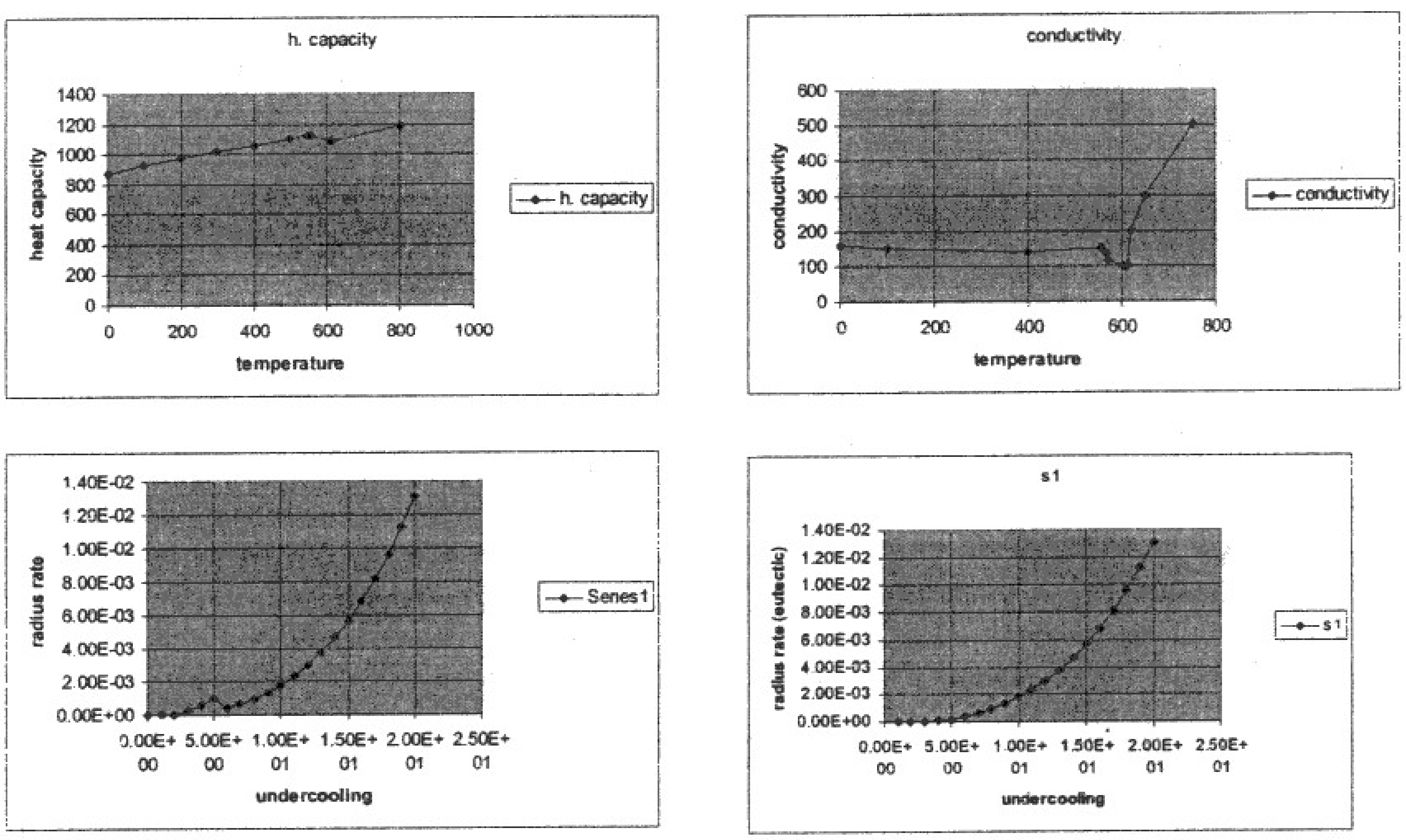
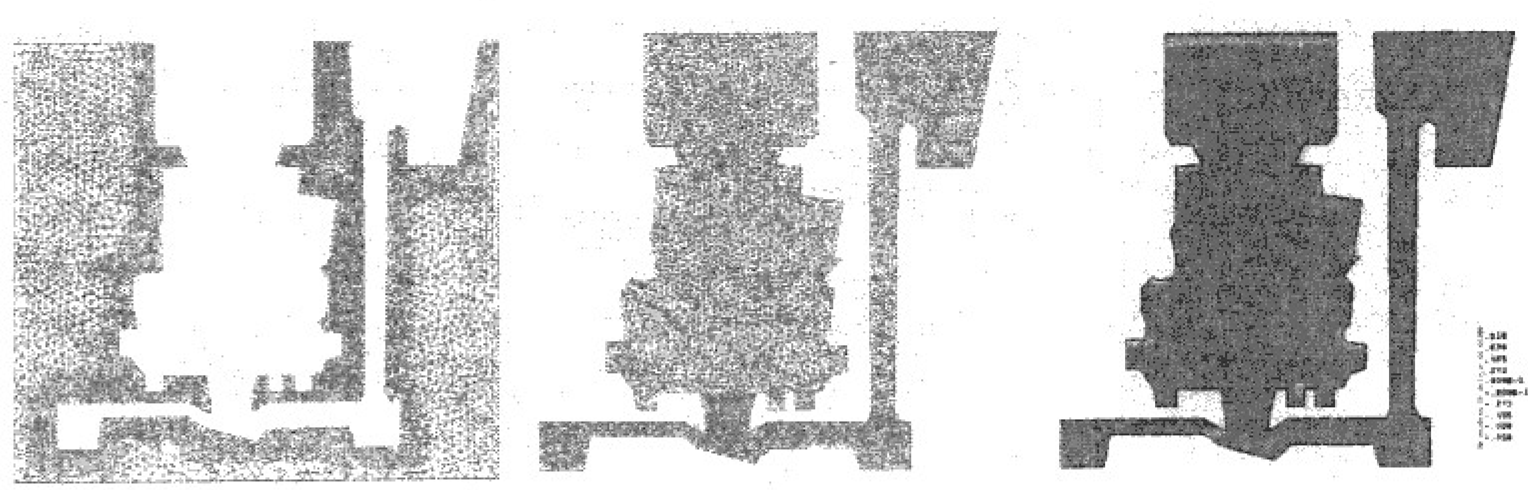


Figure 2. Material properties (from top, left, clockwise): heat capacity and conductivity vs temperature, eutectic and dendritic radius rates vs undercooling.



The cast and mould are modelled using 10422 nodes. The meshes of the mould and cast are given in Fig. 15. The process of the mould filling is followed until the form is almost filled (95 sec.) what is shown also in Fig. 3 (distribution of the pseudo-concentration function). The left, lower branch of the mould is still unfilled. The temperature distribution and velocity field is presented in Fig. 3.

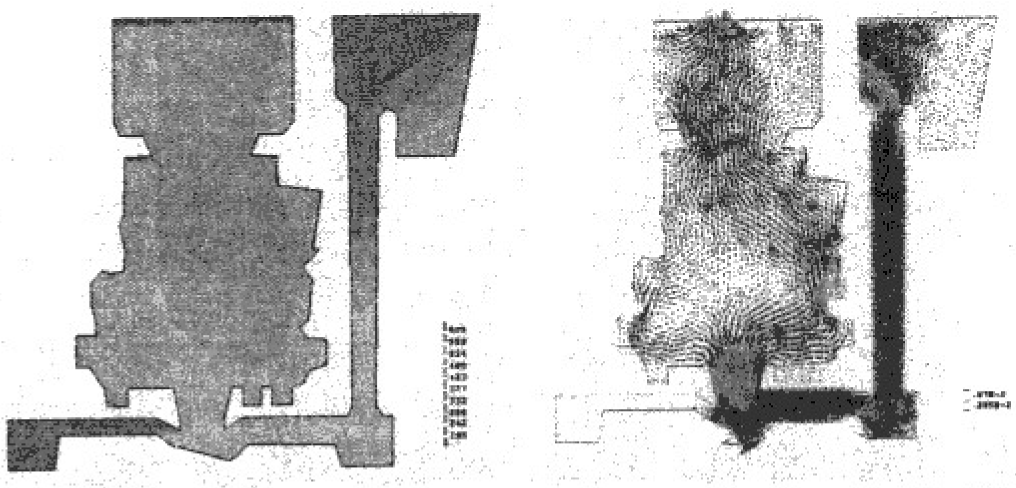


Figure 4. Temperature distribution, velocity field, 95 sec

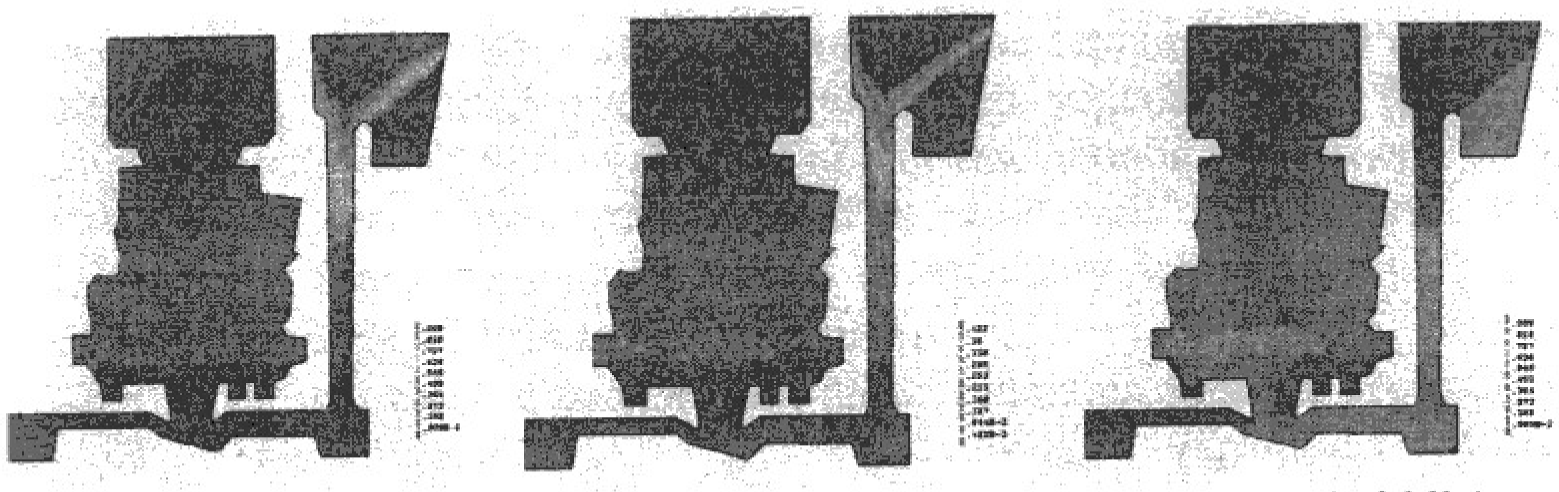


Figure 5. Liquidus distribution (left), dendritic fraction distribution (middle), eutectic fraction (right)

The distribution of the main microstructural variables, namely, distribution of the liquidus, dendritic and eutectic fractions is given in Fig. 5. It can be noticed that the dendritic fraction is concentrated in the thinner part of the section (inlet and lower part of the valve) while the eutectic one is concentrated in the main body of the part.

Acknowledgment
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