Plastoshell. Program for static elastic-plastic analysis of plates and ahells

Zbigniew Kacprzyk
Warsaw University of Technology

Eligiusz Postek
Polish Academy of Sciences

Metody Komputerowe
w Inżynierii Lądowej
Tom 3, Nr 3-4, 1993, s.89–94

*analiza i synteza konstrukcji,
mechanika materiałów i konstrukcji,
notka informacyjna*

Computer Methods
in Civil Engineering
Vol. 3, No 3-4, 1993, p.89–94

*analysis and synthesis of structures,
mechanics of materials and structures,
information note*

PROGRAM ANALIZY STATYCZNEJ SPRĘŻYSTO-PLASTYCZNEJ PŁYT I POWŁOK PLASTOSHELL

PLASTOSHELL PROGRAM FOR STATIC ELASTIC-PLASTIC ANALYSIS OF PLATES AND SHELLS

Zbigniew Kacprzyk, Eligiusz Postek

*Politechnika Warszawska
Wydział Inżynierii Lądowej
Ośrodek Metod Komputerowych
Al. Armii Ludowej 16
00-637 Warszawa*

*Warsaw University of Technology
Faculty of Civil Engineering
Computer Method Center
Al. Armii Ludowej 16
00-637 Warszawa, Poland*

Summary

The modern structural mechanics is strictly computer oriented. To prepare a part of a course considering nonlinear analysis of shells a published finite element programm PLASTOSHELL of Figueiras and Owen is used. The following finite elements are implemented: 8-node Serendipity, 9-node Lagrangian and 9-node Heterosis. The program was tested to check if it may be used in an accurate analysis of shells.

Streszczenie

Nowoczesna mechanika konstrukcji jest ściśle zoreintowana na obliczenia komputerowe. W pracy analizuje się program komputerowy PLASTOSHELL opublikowany przez Figueirasa i Owena w pracy [1]. Program ma oprogramowane następujące elementy powłokowe: 8-węzłowy serendipa, 9-węzłowy lagrange'a i 9-węzłowy heterogeniczny. Testowano program pod względem jakości wyników i możliwości użycia w zajęciach dydaktycznych.

Modern structural mechanics is strictly computer oriented. So, a significant effort is paid to develop efficient tools for teaching of mechanics using computers. One of the most difficult structures to analyze are shells of of arbitrary shape. There are not so many published codes which enable such analysis.

To prepare a part of a course considering nonlinear analysis of shells a published finite element program PLASTOSHELL of Figueiras and Owen [1] is used.

The theoretical bases of the program are given below. They are implemented the following finite elements: 8-node Serendipity, 9 node Lagrangian and 9-node heterosis (Fig. 1). The geometry nad the displacement field of the 8-node element are described by the serendipity shape functions. A good reference may be found in the work of Ahmad [2]. Good results for thin shells may be obtained using the reduced numerical integration. The Lagrangian element is presented in the work of Zienkiewicz et al. [3]. The thin shells should be calculated using elements integrated selectively. The "heterosis" element originally constructed by Hughes and Cohen [4] for plates is recognized to be the best. There is used a specific "hierarchical" formulation. The shape function for the 9th 44 middle" node is assumed as "bubble function"

$$N_9 = (1-\xi^2)(1-\eta^2) \tag{0.1}$$

and the displacements associated with the degrees of freedom connected with the 9th node are relative to the remaining displacements. To obtain the stiffness for thin shells there is not necessary to use reduced or selective intergration rules.

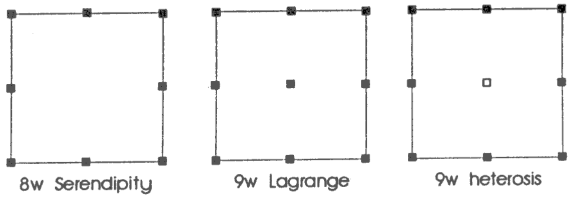

Fig. 1.

The situation of integration points when related to the local curvi-linear coordinate system for 3 and 2 point integration rules is given in Fig. 2.

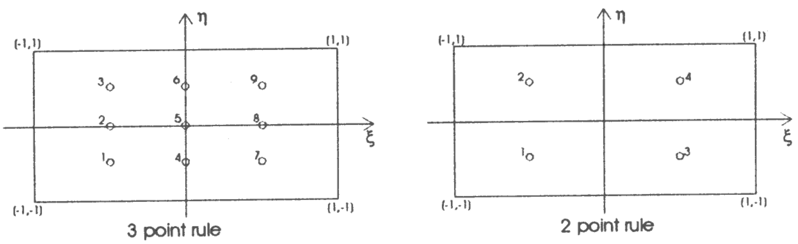

Fig. 2.

The elements are layered, so they may be integrated through the thickness what in consequence, allows to associate with each layer a different type of an anisotropic material.

The elastic-plastic materials with isotopic hardening may be analyzed. The elastic-plastic stiffness matrix is of the form:

$$\boldsymbol{K} = \int_V \boldsymbol{B}^T \boldsymbol{D}_{ep} \boldsymbol{B}\, dV, \tag{0.2}$$

where

$$\boldsymbol{D}_{ep} = \boldsymbol{D} - \frac{\boldsymbol{D a a}^T \boldsymbol{D}}{\boldsymbol{A} + \boldsymbol{a}^T \boldsymbol{D a}} \tag{0.3}$$

and $\boldsymbol{A}$ is the hardening parameter

$$\boldsymbol{A} = \boldsymbol{H}' = \frac{d\overline{\sigma}}{d\overline{\epsilon}_p l} \tag{0.4}$$

which is expressed in terms of the effective stress $\overline{\sigma}$ and effective strain $\overline{\epsilon}_p$, and $\boldsymbol{a}^T$ is the plastic flow vector

$$\boldsymbol{a}^T = \frac{\partial F}{\partial \sigma}, \tag{0.5}$$

F is the plasticity function. The plasticity condition may be written as follows:

$$F(\sigma, \xi) = f(\sigma) - Y(\xi) = 0, \tag{0.6}$$

where f is a function of the stress deviator invariants, Y is a function of the hardening parameter ξ. The generalized Huber-Mises plasticity condition is used.

To analyze the large dispalcements the Total Lagrangian approach is used, so the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor at time t are referred to the original configuration. The large displacements and moderate rotations analysis in the sense of von Karman hypothesis is possible. Summing up, the large displacements elastic-plastic layered plate and shells analysis may be carried out.

To solve the sets of equations the program uses the frontal procedure. The nonlinear equilibrium equations are solved by the following procedures: initial stiffness, differential stiffness, (there is no iteration and the tangent stiffness n is updated at each time step), modified Newton-Raphson (the stiffness is updated at the beginning of the time step and only the equilibrium iterations with constant stiffness are performed) and a certain full Newton-Raphson method where the stiffness is always updated after the second iteration and always when the yield occurs even in one integration point.

The program was tested in Engineering Software Division at the Computer Methods Center in the Warsaw University of Technology. The program is run on a AT386 computer with Weitek coprocessor, under UNIX. A series of calculations of test examples has been carried and as well as noe example with more realistic number of unknows.

The scheme of a plate is given in Fig. 3. This is a 12×12 m square plate, the assumed thickness is 0.4 m. It is hinged on the two opposite edges and free on the remaining two. The plate is loaded with one point force applied at the free edge at the distance 2.4 m from the hinge. The material data are as follows: Young modulus $E = 3 \times 10^4\,\mathrm{kN/m^2}$, Poisson's ratio $\nu = 0.3$ and the hardening module $H' = 300\,\mathrm{kN/m^2}$, uniaxial yield stress in tension $\sigma_{pl} = 30\,\mathrm{kN/m^2}$, shear yield stress $\tau_o = 40\,\mathrm{kN/m^2}$.

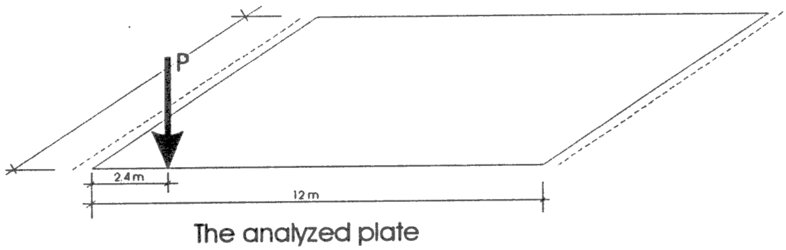

Fig. 3.

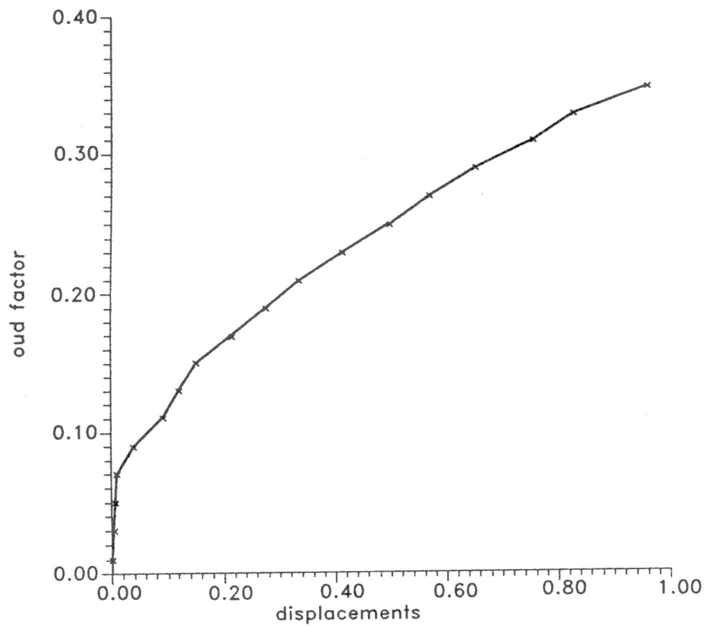

Fig. 4.

The loading process is performed until the whole plate becomes plastic. The plate was discretized into 100 elements and 4 layers. 9-node „heterosis" elements have been used. The solution is performed in 30 incremental steps. During the solution the stiffness matrix is reformed at each time step always after the second iteration and when the yield occurs even in one integration point. The user time was about 48 hours.

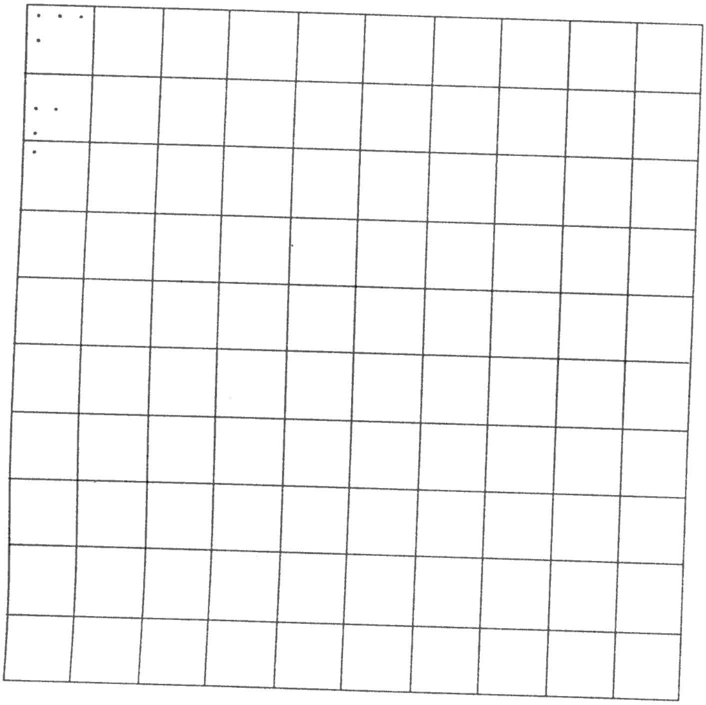

Fig. 5.

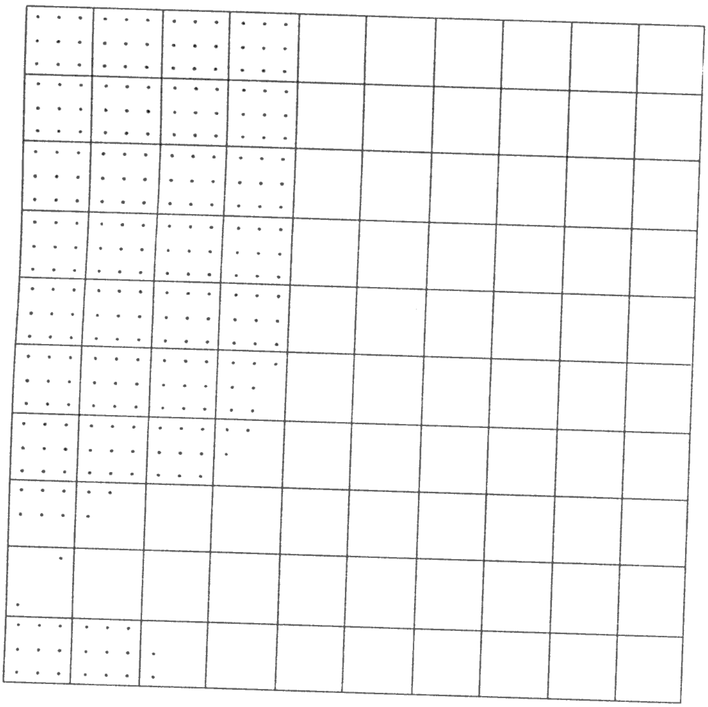

Fig. 6.

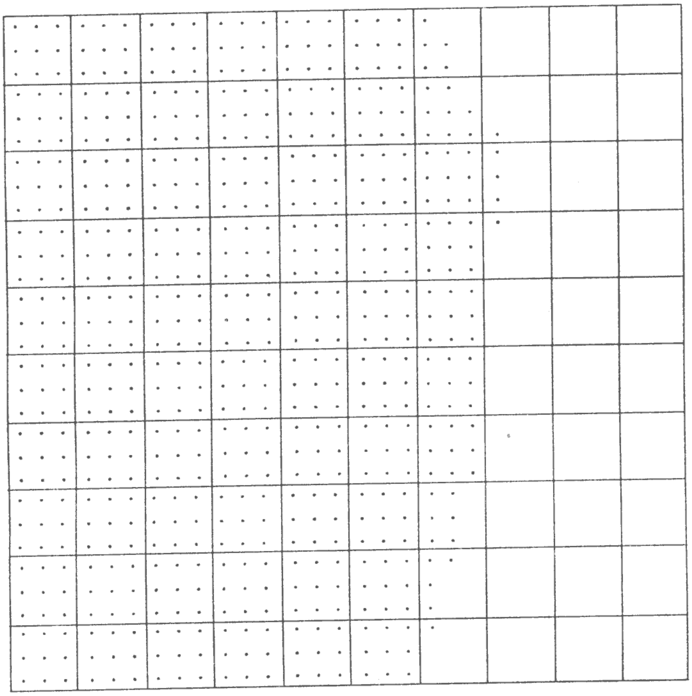

Fig. 7.

The plot of the load factor versus displacement at the point under the force is given in Fig. 4. Figs. 5,6 and 7 give a picture of the development of plastic zones in the upper layer of the plate for the incremental steps 2,4 and 6, respectively.

The program was tested to check if it may be used in student groups dealing with shells. It appears that small tests (about 10 – 15 elements) may be performed by the students. However, larger nonlinear shell examples which are usually found in engineering practice cannot be calculated using the computers on our disposal. The situation may change if the planned new UNIX network will be installed.

References

[1] Hinton E., Owen D.R.J., *Finite Elements in Plasticity – Theory and Practice*, Pineridge Press, Swansea, 1980.

[2] Ahmad S., Irons B.M., Zienkiewicz O.C., *Analysis of thick and thin shell structures by curved finite elements*, in Int. J. Num. Meth. Eng., Vol. 2, 1970.

[3] Pugh E.D.L, Hinton E., Zienkiewicz O.C, *A study of quadrilateral plate bending elements with reduced integration*, Int. J. Num. Meth. in Eng., Vol. 12, 1978.

[4] Hughes T.J.R., Cohen M., *The "heterosis" finite element for plate bending*, Com uters and Structures Vol. 9 1978.