Computationally efficient algorithm for sound speed imaging in pulse-echo ultrasound

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Computationally efficient algorithm for sound speed imaging in pulse-echo ultrasound

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Precise information about the spatial distribution of sound speed in tissue has diagnostic value in itself, and also enables effective aberration correction in standard ultrasonic imaging. An algorithm called Computed Ultrasound Tomography in Echo mode (CUTE) makes it possible to reconstruct quantitative sound speed images. However, the computational cost is high, which is an obstacle to CUTE implementation in real-time imaging systems. This paper presents an improved version of the CUTE algorithm called Quick-CUTE (Q-CUTE). The CUTE algorithm uses the inverse transformation matrix to reconstruct the sound speed spatial distribution. The Q-CUTE algorithm is based on simplified model with unified integration paths which enables solving the inverse problem without use of a large transformation matrix. The Q-CUTE algorithm was verified through numerical simulations. The obtained results differ from those of the CUTE algorithm but maintain the quantitative character of sound speed imaging. The computational complexity of the Q-CUTE algorithm is proportional to N while in case of the CUTE it is proportional to N squared (where N is a number of pixels in the sound speed image). This means that the Q-CUTE algorithm allows the quantitative sound speed imaging to operate in real time.

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1. INTRODUCTION

Information on the sound speed is required for correct reconstruction in ultrasound pulse-echo imaging. In practice, a sound speed value that is an average for soft tissues is used. Locally it may be far from reality and may cause aberrations. Development of a method for real-time sound speed reconstruction will enable efficient correction of aberrations in pulse-echo ultrasound imaging. Moreover, sound speed reflects mechanical properties of tissues and thus may provide additional diagnostic information.

Recent scientific reports introduced the Computed Ultrasound Tomography in Echo mode (CUTE)[1,2] – an algorithm for imaging of sound speed spatial distribution. Its frequency domain version is computationally optimized[1] but the resulting images are qualitative[2]. The spatial domain version of CUTE[2] offers quantitative sound speed imaging but at increased computational cost. This raises doubts concerning the application of the quantitative method in real-time imaging systems.

The objective of this work was to develop a quantitative sound speed imaging algorithm capable of operating in real-time at reasonable computing power. The said algorithm is called Q-CUTE (Quick CUTE).

2. METHODOLOGY

Both the reference method (spatial domain CUTE version) and the method being the subject of this work (Q-CUTE) utilize data obtained through the Compounded Plane Wave Imaging (CPWI) acquisition scheme. The CPWI sequence comprises a number of plane wave emissions at various transmit angles θ. Echoes acquired after each emission are used for reconstruction of radio frequency images. Due to lack of transmit focusing, the resolution of those images is low and therefore they are called Low Resolution Images (LRI).

The signal propagation path from the probe to a given pixel location P varies with the transmit angle θ while the back-propagation paths are independent on the transmit angle θ. Differences between two LRI's obtained at non-equal transmit angles θ are therefore a consequence of different transmit paths (for simplicity let us ignore noise, the interference character of the LRI signal, and refraction).

Any inconsistencies between the actual speed of sound c and its value c_0 used in the LRI reconstruction lead to application of incorrect delays. For notation simplicity, let us define $\Delta\sigma$ as an error in sound slowness (inverse of sound speed). The time delay error τ then equals:

$$\tau(z,x,\theta) = \frac{1}{\cos\theta}\int_0^z \left(\frac{1}{c(r)} - \frac{1}{c_0}\right) dz' = \frac{1}{\cos\theta}\int_0^z \Delta\sigma(r)\, dz' \tag{1}$$

where z and x are the Cartesian coordinates and r is the propagation path (Fig. 1a) such that:

$$r(z,x,\theta) = \{(z',x') : z' \in \langle 0,z\rangle, x' = x - (z - z')\tan\theta\} \tag{2}$$

The transmit time error difference $\Delta\tau_{m,n}$ for transmit angles θ_m and θ_n is as follows:

$$\Delta\tau_{m,n}(z,x) = \tau(z,x,\theta_n) - \tau(z,x,\theta_m) = \frac{1}{\cos\theta_n}\int_0^z \Delta\sigma(r_n)dz' - \frac{1}{\cos\theta_m}\int_0^z \Delta\sigma(r_m)dz' \tag{3}$$

The model described in Eq. (3) is the basis in the spatial domain version of the CUTE algorithm. It is used for determination of the $\Delta\sigma$ to $\Delta\tau$ transformation matrix which is next inverted and used for finding a solution of the inverse problem (estimation of $\Delta\sigma$ based on $\Delta\tau$). Due to ill-conditioning of the inversed problem, the Tikhonov regularization is used.

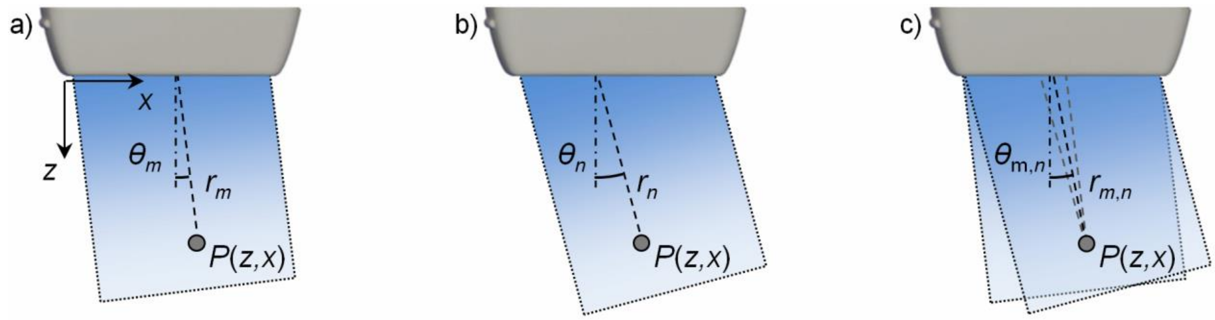

Figure 1. Illustration of transmit propagation paths to a given point (pixel) P for plane wave emission at various angles: a) path r_m for transmit angle θ_m, b) path r_n for transmit angle θ_n, and c) auxiliary path $r_{m,n}$ and a resulting angle $\theta_{m,n}$.

For the purpose of computational optimization, let us define a new path $r_{m,n}$ equally distant in the x-direction from paths r_m and r_n, routed at angle $\theta_{m,n}$ (Fig. 1c). The $\Delta\sigma$ values on paths r_m and r_n can be approximated linearly with use of $\Delta\sigma$ and $\Delta\sigma_x$ (partial derivative of $\Delta\sigma$ with respect to x-direction) values on path $r_{m,n}$:

$$\begin{aligned}\Delta\sigma(r_m) \approx \Delta\sigma(r_{m,n}) + \Delta\sigma_x(r_{m,n})(z - z')\left(\frac{\tan\theta_n - \tan\theta_m}{2}\right) \\ \Delta\sigma(r_n) \approx \Delta\sigma(r_{m,n}) - \Delta\sigma_x(r_{m,n})(z - z')\left(\frac{\tan\theta_n - \tan\theta_m}{2}\right)\end{aligned} \quad (4)$$

With this approximation, Eq. (3) takes the following form:

$$\Delta\tau_{m,n}(z,x) = a_{m,n}\int_0^z \Delta\sigma(r_{m,n})dz' + b_{m,n}\int_0^z (z' - z)\Delta\sigma_x(r_{m,n})dz' \quad (5)$$

where terms $a_{m,n}$ and $b_{m,n}$ are:

$$\begin{aligned}a_{m,n} = \left(\frac{1}{\cos\theta_n} - \frac{1}{\cos\theta_m}\right) \\ b_{m,n} = \left(\frac{1}{\cos\theta_n} + \frac{1}{\cos\theta_m}\right)\left(\frac{\tan\theta_n - \tan\theta_m}{2}\right)\end{aligned} \quad (6)$$

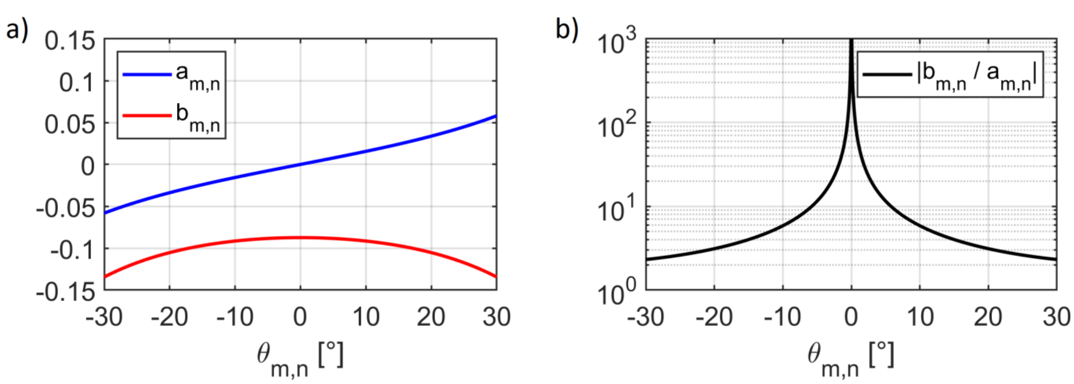

Figure 2. Graphs of: a) $a_{m,n}$ and $b_{m,n}$ terms and b) absolute values of $b_{m,n}$ to $a_{m,n}$ ratio.

Equation (5), after integration by parts of its second integral, takes the form:

$$\Delta\tau_{m,n}(z,x) = a_{m,n}\int_0^z \Delta\sigma\big(r_{m,n}\big)dz' - b_{m,n}\int_0^z \left(\int \Delta\sigma_x\big(r_{m,n}\big)dz'\right)dz' \quad (7)$$

The above integral equation is the basis for the Q-CUTE method. The unified integration paths allow solving the inverse problem without using large transformation matrices. The proposed solution includes two steps. First, Eq. (7) is simplified by neglecting the first term on the right side of the equation. This is justified i.a. by the $b_{m,n}$ to $a_{m,n}$ ratio, especially for $\theta_{m,n}$ close to zero (Fig. 2b). Solving the simplified form of Eq. (7) yields:

$$\Delta\sigma_x\big(z,x,\theta_{m,n}\big) \approx \frac{-1}{b_{m,n}}\left[\frac{d^2}{dz'^2}\Delta\tau_{m,n}\big(r_{m,n}\big)\right]_{z'=z} \quad (8)$$

$$\Delta\sigma(z,x) = \int_{-\infty}^{x} \overline{\Delta\sigma_x}\,(z,x')dx' + C(z) \quad (9)$$

where $\overline{\Delta\sigma_x}$ is the weighted average of $\Delta\sigma_x$ over $\theta_{m,n}$ angles, and C is the integration constant depending on z. In the second step of the Q-CUTE algorithm the missing constant C is calculated. Incorporation of Eq. (9) and $\overline{\Delta\sigma_x}$ into the unmodified form of Eq. (7), after rearrangement, yields:

$$C_{m,n}(z,x) = \frac{1}{a_{m,n}}\left[\frac{d}{dz'}\Delta\tau_{m,n}\big(r_{m,n}\big)\right]_{z'=z} - \int_{-\infty}^{x} \overline{\Delta\sigma_x}\,(z,x')dx' + \frac{b_{m,n}}{a_{m,n}}\int_0^z \overline{\Delta\sigma_x}\,\big(r_{m,n}\big)dz' \quad (10)$$

Finally, the integration constant C in Eq. (9) is substituted with $C_{m,n}$ values averaged over x-dimension and $\theta_{m,n}$ angles. Knowing the $\Delta\sigma$ one can calculate the speed of sound c:

$$c(z,x) = \frac{c_0}{1 + c_0\Delta\sigma(z,x)} \quad (11)$$

As in case of the CUTE algorithm, Q-CUTE needs regularization. It is realized by replacing the derivative operators in Eq. (8,10) with regularized ones. These, in turn, are precisely approximated using computationally efficient infinite impulse response filters.

At the end it should be explained how the transmit time error differences $\Delta\tau_{m,n}$ are obtained. They translate into phase differences $\Delta\varphi_{m,n}$ which can be estimated as:

$$\Delta\varphi_{m,n} = arg[filt(LRI_n \circ LRI_m^*)] \quad (12)$$

The *LRI*s in Eq. (12) denote complex signals obtained from real valued *LRI*s using the Hilbert transform. The * and ◦ operators are the complex conjugate and Hadamard product respectively. The filt and arg functions denote spatial smoothing filtration (implemented with use of a finite impulse response filter) and calculation of the argument of complex numbers. The phase differences $\Delta\varphi_{m,n}$ and mean signal frequency f are sufficient information to estimate $\Delta\tau_{m,n}$.

$$\Delta\tau_{m,n} = \frac{\Delta\varphi_{m,n}}{2\pi f} \quad (13)$$

3. RESULTS AND DISCUSSION

Sound speed imaging quality with use of CUTE (spatial domain version) and Q-CUTE methods is compared in Fig. 3. The raw echo signals were simulated using a numerical tissue model containing the information on sound speed spatial distribution (Fig. 3a). The value of the sound speed in the model (1520 m/s and 1530 m/s for the background and for the circular inclusion, respectively) was lower than that assumed in the reconstruction (1540 m/s). The CPWI method was used for acquisition of LRIs which were next used as inputs for the CUTE and Q-CUTE algorithms. The regularization factor for each algorithm was optimized to reach a compromise between imaging contrast and distortions level. The Q-CUTE gives slightly lower contrast and introduces horizontal distortions. The inclusion, however, is still clearly visible, and there is less distortion at the bottom of the image.

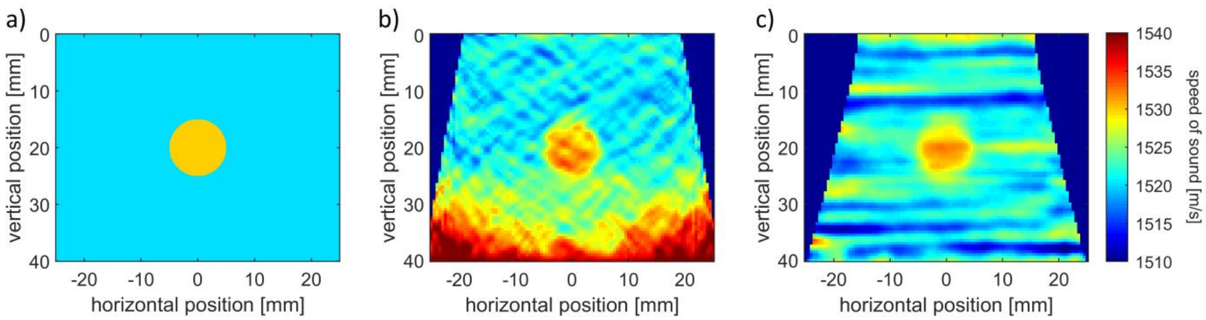

Figure 3. Sound speed maps: a) used in input data simulation, b) obtained with the reference algorithm (spatial domain CUTE version), and c) obtained with the presented algorithm (Q-CUTE).

The key feature of the Q-CUTE algorithm is its low computational complexity *O(N)*, where *N* denotes a number of reconstructed sound speed image pixels. This is a significant improvement when compared to the spatial domain CUTE algorithm whose computational complexity is $O(N^2)$ (Fig. 4). In practical application, e.g. in a mobile device with a NVIDIA TEGRA X1 processor, the CUTE provides 256x256 images with a frame-rate of 3 fps while Q-CUTE reaches nearly 2000 fps (provided that the processor uses its entire computing power for this one task, and each output frame is calculated with use of 20 LRI pairs).

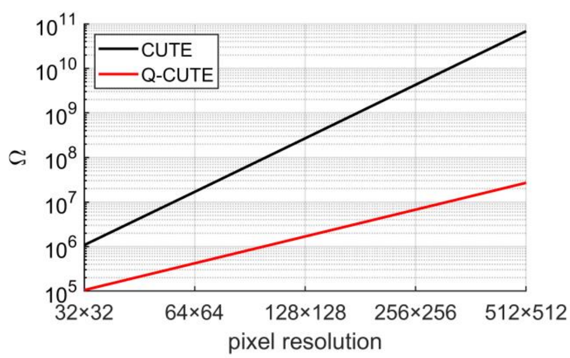

Figure 4. Computational complexity of spatial domain CUTE and Q-CUTE algorithms.

4. CONCLUSION

The presented algorithm (Q-CUTE) maintains the quantitative character of sound speed imaging while being much less computationally demanding than the reference algorithm (spatial domain version of CUTE). This advantage may be decisive in the implementation of the sound speed imaging algorithm in ultrasound systems, especially in mobile devices where power consumption, and with it also the computing power are limited.

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REFERENCES

[1] M. Jaeger, G. Held, S. Preisser, S. Peeters, M. Grunig, M. Frenz, "Computed Ultrasound Tomography in Echo mode (CUTE) of speed of sound for diagnosis and for aberration correction in pulse-echo sonography," Proc. of SPIE, **9040**, 90400A-1 - 90400A-12 (2014).

[2] M. Jaeger, M. Frenz, "Quantitative imaging of speed of sound in pulse-echo sonography," 2015 IEEE International Ultrasonics Symposium, url: www.youtube.com/watch?v=Ck75XbfLQtY (2015).