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SIMULATION OF ELASTIC WAVE PROPAGATION USING THE DEFORMABLE DISCRETE ELEMENT METHOD

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1. Introduction

Elastic wave propagation is a fundamental phenomenon which is encountered frequently in different natural processes, for instance, earthquakes, and engineering problems such as structures subjected to impact loading. An elastic wave in a solid body can propagate in two modes, in the form of the longitudinal and shear waves. In the longitudinal wave, the motion of the material points is in the direction of propagation whereas in the shear wave, the motion of the material points is in a plane perpendicular to the direction of propagation. Wave propagation velocity is one of the main parameters characterizing waves. Wave velocity in elastic solids depends on elastic constants. The velocities of longitudinal and shear waves in elastic solids, c_l and c_s, respectively, are given as [1, 2]:

$$c_l = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}}, \qquad c_s = \sqrt{\frac{E}{2(1+\nu)\rho}} \tag{1}$$

where: ρ – bulk density, E – Young's modulus and ν – Poisson's ratio. The numerical methods such as the discrete element method (DEM), are commonly used for analysis of different problems of geomechanics or civil engineering involving wave propagation. Therefore, numerical models should reproduce accurately this phenomenon. The present work is aimed to investigate capability of the DEM to model properly wave propagation in solid materials, with special focus on possible enhancement of this capability yielded by the new formulation of the DEM, called Deformable Discrete Element Method or DDEM [3]. A numerical example has been presented in order to illustrate the wave propagation phenomenon in an elastic solid discretized with discs (2D discrete elements).

2. Deformable discrete element method formulation

The discrete element method assumes a material can be represented as a large assembly of particles, discs in 2D or spheres in 3D, interacting among one another by contact forces. The contact force between two elements, $\boldsymbol{F}_c$ in Eq. (2) is decomposed into normal and tangential components, $\boldsymbol{F}_n$ and $\boldsymbol{F}_t$ respectively and given as:

$$\boldsymbol{F}^c = \boldsymbol{F}_n + \boldsymbol{F}_t = F_n\mathbf{n} + \boldsymbol{F}_t, \tag{2}$$

where: $\mathbf{n}$ – the normal unit vector at the contact point. Contact models for the normal and tangential interaction can take into account different physical effects such as elasticity, plasticity, adhesion/cohesion, viscosity or friction. In the present work, the cohesive linear elastic model has been be used. The normal and tangential force components in this model are given by the linear relationships

$$F_n = k_n h, \qquad \boldsymbol{F}_t = k_t\mathbf{u}_t \tag{3}$$

where: k_n – normal contact stiffness, k_t – tangential contact stiffness, h – is the overlap or gap at the contact point and $\mathbf{u}_t$ – the relative displacement at the contact point in the tangential direction. In the standard DEM, the particle overlap is assumed to represent local particle deformation at the contact point, whereas the particles (discrete elements) are considered as rigid body. The Deformable Discrete Element Method (DDEM) developed by the authors enhances the standard DEM formulation [3]. In addition to the kinematics of standard DEM [3], the formulation of DDEM assumes uniform particle deformation under the internal particle stress induced by the contact forces.

Contact interaction models in the DEM and DDEM play a role of microscopic material models. Macroscopic material properties result from collective particle response governed by the contact model and its parameters. The micro-macro constitutive relationships are key issue in the use of the DEM. The dimensional analysis provides an appropriate framework to establish the micro-macro relationships. These relationships for the elastic constants in the standard DEM can be assumed in the following form:

$$E = k_n \Phi_E\left(\frac{k_t}{k_n}, n\right), \qquad \nu = \Phi_\nu\left(\frac{k_t}{k_n}, n\right) \tag{4}$$

where: E – Young's modulus, ν – Poisson's ratio, n – porosity. The corresponding relationships for the DDEM take the form:

$$E = k_n \Phi_E\left(\frac{k_t}{k_n}, \frac{k_n}{E_p}, \nu_p, n\right), \qquad \nu = \Phi_\nu\left(\frac{k_t}{k_n}, \frac{k_n}{E_p}, \nu_p, n\right) \tag{5}$$

where: E_p – particle Young's modulus, ν_p – particle Poisson's ratio.

3. Numerical example

Wave propagation phenomenon has been simulated using a rectangular sample of length ~ 16.5 mm, width ~ 2 mm, discretized with 682 bonded disc elements with non-uniform size varying between maximum radius $r_{max.} = 1.449 \cdot 10^{-4}$ m and minimum radius $r_{min.} = 1.003 \cdot 10^{-4}$ m, particle density, $\rho_P = 2000$ kg/m^3 and normal contact stiffness, $k_n = 10$ GPa. Taking into account the porosity of the DEM sample, an average bulk density $\rho = 1797.36$ kg/m^3 is obtained. Simulations are performed for the ratio k_t/k_n ranging between 0.0 to 1.0 in steps of 0.1. For signal input, initial displacements of the particles have been defined using following function:

$$u_y^0 = A\left(\cos\frac{2\pi x}{L} + 1\right) \qquad (6)$$

with $0 \le x \le L/2$, where amplitude $A = 0.01$ mm and wavelength $L = 10$ mm is assumed, x – coordinate of the particles in x-direction within length $L/2$ in reference to the left edge of the sample. Periodic boundary conditions are applied on the bottom and top layers of particles while particles forming the right edge of the sample are fixed. Figure 1 shows the propagation of shear wave pulse in the discrete sample using the standard DEM model for the ratio $k_t/k_n = 0$ in terms of *y*-displacement vectors of the elements at different time steps. Dependence of shear wave velocity on the ratio of tangential to normal contact stiffness, k_t/k_n is presented in Figure 2. Analytical wave velocities are determined as a function of the ratio k_t/k_n using constitutive relationships corresponding to Eq. (4) for standard DEM [3] in Eq. (1). Wave velocity is determined numerically as an average of wave velocities between four different pairs of nodes with varying horizontal distance between a pair. Nodal pairs were chosen arbitrarily at different vertical positions within the sample. Peak-to-peak method is used to evaluate time taken by wave to reach from one node to another within a pair with a known distance between them. It can been seen that a good agreement between numerical and analytical results has been obtained.

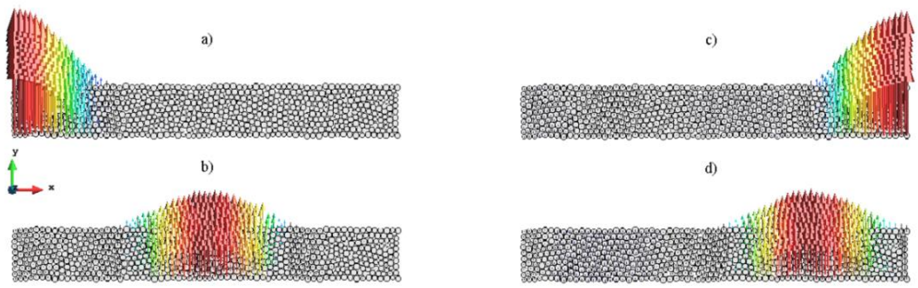

Figure 1. Shear wave propagation: *y*-displacement vectors at, a) $t = 0$ s, b) $t = 6.04 \cdot 10^{-6}$ s, c) $t = 1.175 \cdot 10^{-5}$ s, d) $t = 1.47 \cdot 10^{-5}$ s

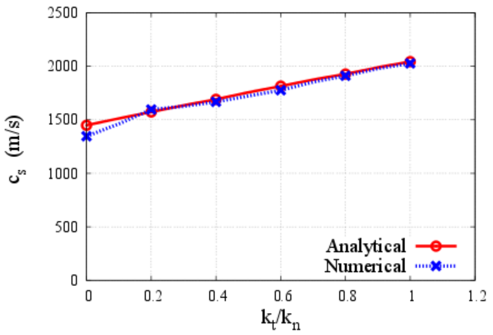

Figure 2. Shear wave velocity w.r.t. ratio k_t/k_n – comparison between numerical and analytical results.

4. Conclusion

The discrete element method proves to be a robust numerical tool to properly reproduce the wave propagation phenomenon in solid materials. Furthermore, results for the longitudinal wave propagation using standard DEM will be presented. It will be shown that numerical results obtained with the DDEM induces flexibility over standard DEM to correctly reproduce elastic waves.

References

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