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STRUCTURAL DESIGN SENSITIVITY ANALYSIS BY INCREMENTAL PROCEDURES

M. Kleiber, T. D. Hien, E. Postek

ABSTRACT

An incremental formulation based on the Taylor's expansion for nonlinear design sensitivity problems is developed. Computational aspects are discussed. A few numerical benchmark problems illustrate the paper.

1. INTRODUCTION

Design sensitivity analysis of linear structures has been by now well established in the literature. One of the first papers considering this type of analysis was that of Zienkiewicz, [1]. The next important contribution was due to Haug and Arora [2] where the adjoint variable and direct differentiation methods were presented. The variational approach to design sensitivity was contained in the papers by Dems and Mróz [3,4] and Haug, Choi, Komkov [5]. The latter paper discussed examples which have became valuable benchmarks for sensitivity computation of complex structures using the finite element method.

Nonlinear sensitivity was considered by Haftka and Mróz [6] and Mróz, Kamat, Plant [7]. A comprehensive discussion of computer implementation aspects was presented by Arora, Cardoso [8] who introduced algorithms for the design sensitivity analysis employed within the ADINA system. However, the approach is not in fact incremental and thus not general enough to deal with any nonlinear behaviour.

Many results considering computational aspects of static, dynamic and stochastic design sensitivity contain papers by Hien and Kleiber [9,10,11]. In the present work the incremental approach based on the first order expansion is given. This is the most general and consistent method which involves tangent stiffness matrix and thus it is suitable for both geometric and material nonlinear problems.

2. PROBLEM STATEMENT.

Consider the structural response functional at time t+Δt for a spatially discretized system with N degrees of freedom given by

$$\overset{(t+\Delta t)}{\Psi} = \overset{(t+\Delta t)}{G}\,[\overset{(t+\Delta t)}{q}(b),\, b]. \tag{1}$$

The equilibrium equation at time t+Δt is expressed in the incremental form as

$$\overset{(t)}{K}_{\alpha\beta}\,[q\,(b),\, b]\,\Delta q_{\beta}(b) = \Delta Q_{\alpha}(b), \tag{2}$$

where $b = \{b_e\}$, $e = 1,\ldots,E$; $\overset{(t)}{q} = \{q_\alpha\}$, and $\Delta q = \{\Delta q_\alpha\}$, $\alpha = 1,\ldots,N$ denote the vectors of design variables, nodal displacements and displacement increments, respectively. The objective is to evaluate the sensitivity gradient coefficients of the response functional with respect to design variables at t+Δt assuming $\partial\Phi/\partial b$ at time t to be given. We introduce the following notation for partial derivatives

$$(.)_{e} = \partial(.)/\partial b_e,\quad (.)._{\alpha} = \partial(.)/\partial q_\alpha. \tag{3}$$

Differentiation of Eq. (1) with respect to b_e using the chain rule leads to

$$\overset{(t+\Delta t)}{\psi}._{e} = \overset{(t+\Delta t)}{G}._{e} + \overset{(t+\Delta t)}{G}._{\alpha}\, q_{\alpha}._{e}\,. \tag{4}$$

To express the foregoing equation explicitly in terms of the design variable variation let us first make the first order expansion of the function ψ about $\overset{(t)}{q}_\beta$ to obtain

$$\begin{aligned} \overset{(t+\Delta t)}{\psi} &= \overset{(t)}{\psi}._{e} + (\overset{(t)}{G}._{\alpha e} + \overset{(t)}{G}._{\alpha\beta}\, \overset{(t)}{q}_{\beta}._{e})\Delta q_\alpha \;+ \\ &+ (\overset{(t)}{G}._{\alpha} + \overset{(t)}{G}._{\alpha\beta}\Delta q_\beta)\,\Delta q_{\alpha}._{e}\,. \end{aligned} \tag{5}$$

Since the configuration, displacement field and sensitivity gradient coefficients are all known at t, the only term which is necessary to determine is the derivative of the displacement increment with respect to design variables, i.e. $\Delta q_{\alpha}._{e}$. This is done by differentiating eq. (2) with respect to b_e to get

$$\Delta q_{\alpha}._{e} = \overset{(t)}{K}{}^{-1}_{\alpha\beta}\,[\Delta Q_{\beta}._{e} - (\overset{(t)}{K}_{\beta\eta}._{e} + \overset{(t)}{K}_{\beta\eta}._{\gamma}\,\overset{(t)}{q}_{\gamma}._{e})\,\Delta q_\eta]. \tag{6}$$

Substituting the above equation into Eq. (5) yields the expression for sensitivity at t+Δt. This is known as the direct differentiation technique.

Let us now introduce the adjoint technique. Consider the last term in Eq. (5). Introducing Eq. (6) into this term we define an adjoint vector $\lambda = \{\lambda_\beta\}$ which is independent of b_e as follows

$$\overset{(t)}{\lambda_\gamma} = \overset{(t)}{K}{}_{\alpha\gamma}^{-1}(\overset{(t)}{G}_{.\alpha} + \overset{(t)}{G}_{.\alpha\beta}\Delta q_\beta). \tag{7}$$

If the system stiffness matrix is symmetric the equlibrium equation of the adjoint system can be written as

$$\overset{(t)}{K}_{\alpha\beta}\lambda_\beta = \overset{(t)}{G}_{.\alpha} + \overset{(t)}{G}_{.\alpha\beta}\Delta q_\beta . \tag{8}$$

Consequently, the expression for design sensitivity at time instant $t + \Delta t$ takes the form

$$\begin{aligned}\overset{(t+\Delta t)}{\psi}_{.e} = \overset{(t)}{\psi}_{.e} + (\overset{(t)}{G}_{.\alpha e} + \overset{(t)}{G}_{.\alpha\beta}\,\overset{(t)}{q}_{\beta .e})\Delta q_\alpha + \lambda_\alpha[\Delta Q_{\alpha .e} - \\ (\overset{(t)}{K}_{\alpha\beta .\gamma}\overset{(t)}{q}_{\gamma .e} + \overset{(t)}{K}_{\alpha\beta .e})\Delta q_\beta].\end{aligned} \tag{9}$$

In contrast to the incremental equations of the primary system, the adjoint equation Eq.(8) is linear.

3. COMPUTER IMPLEMENTATION.

The adjoint variable technique is choosen here for computing the design derivatives. There are two sets of equations to be solved for primary for adjoint structures, respectively. The adjoint set is linear with respect to the adjoint variable at time t; it is thus possible to solve the equations for both systems in parallel.

The incremental set of Eqs. (2) may be solved for displacements by any nonlinear algorithm, such as Newton - Raphson [12, 13] schemes, for instance, and then the adjoint vector may be calculated exploiting the triangularized tangent stiffness matrix of the primary system at time t. The next step is to evaluate the design sensitivity gradient coefficients according to Eq. (9). For simple response functions frequently used in engineering practice such as

$$\psi = |q/q_a| - 1 < 0, \tag{10}$$

in which the quantity q_a may be understood as an allowable displacement, Eq. (9) is reduced to

$$\overset{(t+\Delta t)}{\psi}_{.e} = \overset{(t)}{\psi}_{.e} + \overset{(t+\Delta t)}{\lambda}_\alpha\ [\Delta Q_{\alpha .e} - (\overset{(t)}{K}_{\alpha\beta .\gamma}\overset{(t)}{q}_{\gamma .e} + \overset{(t)}{K}_{\alpha\beta .e})\Delta q_\beta\ ]. \tag{11}$$

An important considerationis to choose an effective method of determining the derivatives of the stiffness matrix with respect to displacements and to adjoint variables. The derivatives with respect to design variables can be calculated explicitly by direct differentiation, or implicitly by using a finite difference technique [14] or by the least square fit method. In the finite element context the derivatives with respect to displacemnts can be preferably evaluated by an implicit technique since the stiffness matrix is generally an implicit function of displacements. Both methods have been employed in our study. The derivatives are calculated explicitly for truss, beam and quadrilateral flat plate-shell elements and implicitly for isoparametric Ahmad-type shell element.

4. *FINITE ELEMENT PACKAGE CODE POLSAP AND NUMERICAL EXAMPLES.*

The finite element system POLSAP is a considerable extended version of the well known program SAP - IV [15]. The current linear version of POLSAP provides 14 types of analysis including static and dynamic sensitivity of deterministic and stochastic response [11]. Any arbitrarily complex beam-plate-shell structures can be analyzed. The structural response and constraints can be assumed as functions of displacements and stresses. Design variables may be taken as cross-sectional areas, Young modula, lengths (for beams and bars), thickness (for plates and shells) and material densities of structural members.

The program is operative on IBM-PC and compatible computers under DOS or UNIX systems. The work to extend POLSAP into nonlinear sensitivity range based on the approach presented here is underway.

The numerical results shown below have been computed for linear structures. The nonlinear counterparts will be discussed during the conference presentation. The response functions are expressed by Eq. (10). The design derivatives are calculated with respect to cross-sectional areas for bars and with respect to thickness for plate. All of the design constraints considered are imposed on nodal displacements along z-axes and have the same value of 0.01 m.

Example 1.

The classical example is the von Mises two-bar truss shown in Fig. 1.

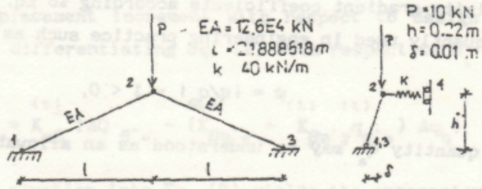

Fig. 1. *Two-bar truss.*

The nonlinear static results are presented in the work of Kotula and Kleiber [16].

The constraint is imposed at node 2. The sensitivity gradient coefficient for both bars $d\psi/db$ is equal to -7.374043.

Example 2.

The second example is that of a space bar structure(Fig. 2), for which nonlinear static results where given in [17].

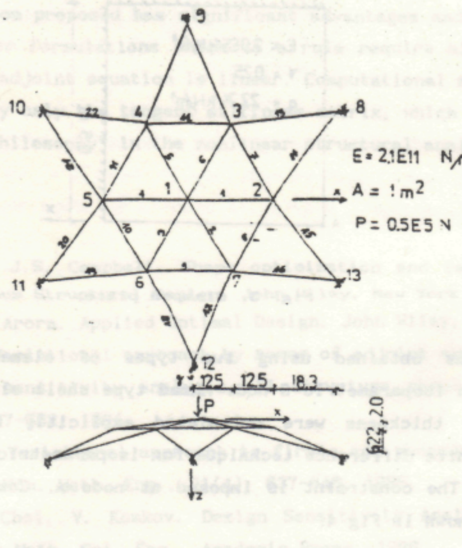

Fig. 2. Space truss.

The results of the sensitivity analysis are assembled in Tab. 1 which reflects the change of design sensitivity for ideal and slightly imperfect structure. An error of 0.2 m along x-axis in position of the apex node is assumed. The design constraint is imposed at node 1.

Table 1.

Design sensitivity gradients coefficients for space truss

element number	Perfect structure	Imperfect structure
1	-2.6082303117E-03	-2.6292336493E-03
2	-2.6082236449E-03	-2.6185512616E-03
3	-2.6082236449E-03	-2.5972868848E-03
4	-2.6082303117E-03	-2.5867117128E-03
5	-2.6082236449E-03	-2.5972868848E-03
6	-2.6082236449E-03	-2.6185512616E-03

Example no 3.

A quater of rectangular clamped plate is considered next (Fig. 3).

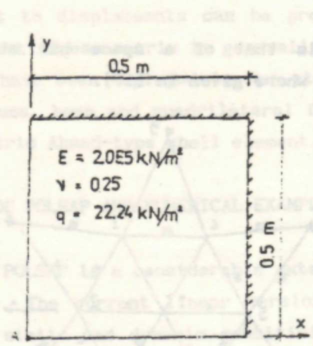

Fig. 3. Clamped plate.

The solution was obtained using two types of elements. Flat plate-shell quadrilateral and isoparametric 9-node Ahmad type shell elements. The derivatives with respect to thickness were calculated explicitly for quadrilaterals and implicitly by finite difference technique for isoparametric elements. 100 elements for both cases. The constraint is imposed at node A. Comparison of results for linear case is shown in Fig 4.

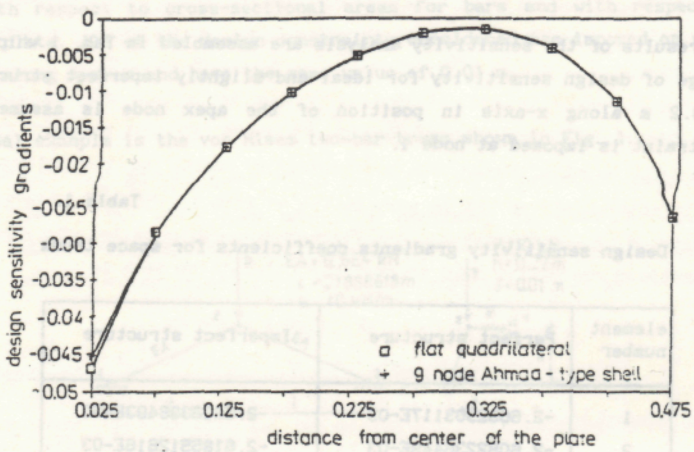

Fig. 4. Design sensitivity gradients coefficients for clamped plate calculated explicitly and implicitly.

5. FINAL REMARKS.

The application of the first order expansion only for all the functions involved seems to be consistent with the first order sensitivity analysis. The incremental formulation proposed has significant advantages and no drawbacks when compared against other formulations which as a rule require also finding of the internal forces. The adjoint equation is linear. Computational algorithms based on the formulation employ only the tangent stiffness matrix, which is consistent with the general "rate - philosophy" in the nonlinear structural analysis.

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STRESZCZENIE

W artykule przedstawiono podejście przyrostowe do analizy wrażliwości konstrukcji. Metoda oparta jest na rozwinięciu funkcji w szereg Taylora. Rozważane są jej aspekty obliczeniowe. W artykule zawarte są przykłady numeryczne.