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# DESIGN SENSITIVITY OF REINFORCED CONCRETE PLATE/SHELL STRUCTURES

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## Introduction

Design sensitivity gradients are believed to have great significance in realistic assesment of structural response. Sensitivity techniques form a tool to follow changes in structural behaviour caused by variations of design parameters. The sensitivity gradients make it also possible to investigate the influence of different type of material or geometrical imperfections on the structural response.

One of the first papers considering the design sensitivity analysis (DSA) was presented by Zienkiewicz and Campbell [1]. Recently, efforts of the investigators are mostly focused on the nonlinear performance of structures. The total Lagrangian formulation with the computer implementation in Adina was presented in [2]. A formulation considering path-dependent problems was given in [3] and the updated Lagrangian formulation of the design sensitivity problems for arbitrary nonlinearities was given in [4]. Applications of the DSA to the state estimation of complex structures were presented in [5]. The response of reinforced concrete structures should always be considered nonlinear and thus it may be used as an example illustrating the theory of design sensitivity for nonlinear systems. This paper deals with the design sensitivity of reinforced plate and shell structures.

## Problem statement

The up-to-date DSA of reinforced concrete structures requires the formulation typical of path-dependent problems. The updated Lagrangian formulation presented in [6] is used in this paper.

The following incremental static equilibrium equation is considered

$$\Delta F = \Delta Q \tag{1}$$

where $\Delta Q$ is the external nodal force increment and $\Delta F$ is the internal nodal force increment. To deal with the sensitivity problem the direct differentiation method is applied which implies the following linear equation for the increments of displacement design derivatives

$$\left(K^{(t-p)} + K_{\sigma}\right) \frac{d\Delta q}{dh} = \frac{d\Delta Q}{dh} - \left.\frac{d\Delta F}{dh}\right|_{\Delta q(h)=const} \tag{2}$$

where $K^{t-p}$ is the elasto-plastic tangent matrix, $K_{\sigma}$ is the initial stress matrix and on the r.h.s. we have the design derivatives of the external and internal equivalent incremental nodal forces, the latter differentiation carried out at the incremental process assumed independent of design.

The problem is solved with the constraint function

$$\Phi = \frac{|q(\mathrm{h})|}{q^a} - 1 \leq 0$$

where $q$ is the total displacement at a selected point and $q^a$ is a design constraint.

**Constitutive models**

Concrete is modelled using a plasticity based constitutive model [7] with the concept of uniaxial equivalent strains. The curve stress–strain in compression is expressed by the so called Saenz's equation. The plastic strains are assumed to appear already at the beginning of the process. During strain softening the stress–strain relationship is linear until crushing. The material is unloaded assuming the initial tangent modulus. To model the reinforcement the elasto-plastic work hardening model is applied.

**Solution method**

The nonlinear incremental equilibrium equation (1) is solved using Newton–Raphson method. Simultaneously, the linear DDM equation (2) is solved for the increments of the displacement design derivatives. The displacement design derivatives are accumulated and in the "material" procedures at integration points the increments of the derivatives of uniaxial equivalent plastic strains are calculated and accumulated.

To model plate/shell structures a 3D shell, 9-node, layered element is used and the method to obtain the stress design derivatives with isoparametric elements is similar to that applied in [8]. The information which is necessary for the design sensitivity gradients estimation is calculated and saved for every specific integration point in each element.

Numerical results for large-scale structural mechanics problems will be provided at the conference.

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