# Method of Averages to Determine Insulation Conductivity under Transient Conditions

**ELISABETH KOSSECKA**

*Institute of Fundamental Technological Research*
*Polish Academy of Sciences*
*ul.Swietokrzyska 21, 00-049 Warsaw, Poland*

**ABSTRACT:** A simple method of data analysis, called *the averaging technique*, is presented which may be useful in long-term field measurements of temperature dependent thermal conductivity of insulating materials, is presented. Its mathematical basis is the integral statement, which follows from the heat conduction equation. For linear dependence of conductivity on temperature, on neglecting capacity terms, it yields simple algorithms to determine thermal conductivity in transient heat flow, for the weighted average temperature, when the heat flux and surface temperatures are measured simultaneously, and also when the *heat flux comparator* method is employed. The problem of proper control of the temperature inside the exposure test box, in order to minimize the error due to neglecting capacity terms, is discussed.

**KEY WORDS:** temperature dependent conductivity, nonlinear heat conduction, Kirchoff's potential, field measurements, heat comparator method, method of averages

## INTRODUCTION

THERMAL PERFORMANCE OF insulating materials under natural condition is affected by factors which cannot be reproduced and accounted for in laboratory tests. This is the reason for field measurements, which need a special data analysis technique, appropriate for transient heat flow conditions.

E-mail: ekossec@ippt.gov.pl



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Measurements of the thermal resistance of insulating materials are usually performed with a heat flow transducer. This method can also be used under transient conditions if the transducer is calibrated as a function of temperature and if its response period is much shorter than that of the tested system.

A method developed for in situ testing, called the *heat flow comparator method*, is presented in Bomberg et al. [1,2]. The purpose of the work of Bomberg et al. was evaluation of the long term thermal performance of cellular plastics and verifying the model of aging. Different foam products, built into the roof of an exposure box, were exposed to the environment and examined for almost three years.

The method consists of testing two material slabs placed in contact with one another, one of them being a reference material with known dependence of thermal conductivity and heat capacity on temperature, and the other being a test specimen whose thermal properties are to be determined. Both the thermal conductivity and heat capacity of the reference material must be known as functions of temperature. The measurements required for application of the heat flux comparator technique involve only the temperatures of the free surfaces of the test specimen and the reference material and at the interface of the two.

In data analysis the particular *courses matching technique* is employed. The heat flux across the boundary surface between the reference and the tested specimen is calculated using a numerical algorithm to solve the heat transfer equation through the reference specimen. By imposing the requirement of heat flux continuity at the contact boundary between test and reference specimens, corresponding values of thermal conductivity and heat capacity of the tested specimen are found with an iterative technique. The basis for selection of a given thermal conductivity as representative of the tested material is matching the interface heat fluxes, represented by the correlation coefficient value.

An alternative, very simple method of data analysis, which may be useful in experiments conducted for a long time, is presented below. It is the averaging technique [5-7], which follows from the integral formulae for the heat flow through the surfaces of a plane slab. The need to determine heat capacity of the material is avoided; its approximate value is necessary only in error analysis. Averaging procedures eliminate random measurement errors, therefore this technique gives good results in determining thermal resistances of building elements from in-situ measurements even for low values of the heat flux; this was demonstrated by Kossecka et al. [6].

## INTEGRAL FORMULAE FOR THE HEAT FLOW IN THE CASE OF TEMPERATURE DEPENDENT CONDUCTIVITY

In case of conductivity and specific heat of the material dependent on temperature the heat conduction equation

$$\rho\, c_p(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right] \tag{1}$$

for the temperature $T(x,t)$ is nonlinear. Equation simplifies on introducing the new temperature function, determined by the Kirchoff's transformation [3]:

$$\eta(T) = \int_{T_0}^{T} \lambda(T')dT' , \tag{2}$$

where $T_0$ is an arbitrary reference temperature. Equation (1), written in terms of $\eta$, takes the form:

$$\frac{\rho c_p}{\lambda}\frac{\partial \eta}{\partial t} = \frac{\partial^2 \eta}{\partial x^2} . \tag{3}$$

It is now quasilinear equation, as temperature dependent $c_p$ and $\lambda$ are now functions of the Kirchoff's potential $\eta$.

For linear dependence of conductivity on temperature

$$\lambda(T) = \lambda_0 + \beta(T - T_0) \tag{4}$$

the relation between $T$ and $\eta$ is given by:

$$\eta(T) = \lambda_0(T - T_0) + \beta\,\frac{(T - T_0)^2}{2} . \tag{5}$$

It is assumed that $T$ and $T_0$ belong to the linearity region of the function $\lambda(T)$. For example, for rigid, cellular polystyrene, depending on its type, the ratio $\beta/\lambda_0$ varies within the range of 0.004 - 0.005 [$^{\circ}C^{-1}$] approximately, what means the conductivity variation of 10% at temperature variation of 20 - 25ºC [4].

The method of conductivity determination, used by Bomberg et al. [1,2], relies on numerical solution of Equation (3), for given surface temperature courses, assuming default values of $\lambda_0$, $c_p$, $\beta$, and then applying the optimization technique to find the values of these parameters matching given heat flux courses. The independence of $c_p$ on temperature was assumed, what is reasonable in the range normally occurring under field conditions. In the method of averages the only quantities to be determined are $\lambda_0$ and $\beta$.

Consider a homogeneous plane slab of thickness $L$. Its surfaces correspond to the planes $x = 0$ and $x = L$. On multiplying Equation (1) by the linear function $(1- x/L)$ and integrating its both sides over the slab thickness, after performing integration by parts, we have:

$$\int_0^L \rho c_p \frac{\partial T}{\partial t}\left(1-\frac{x}{L}\right)dx = \int_0^L \frac{\partial}{\partial x}\left[\lambda\frac{\partial T}{\partial x}\right]\left(1-\frac{x}{L}\right)dx$$

$$= \lambda\frac{\partial T}{\partial x}\left(1-\frac{x}{L}\right)\Bigg|_{x=0}^{x=L} + \frac{1}{L}\int_0^L \lambda\frac{\partial T}{\partial x}dx$$

$$= -\lambda\frac{\partial T(0)}{\partial x} + \frac{1}{L}\int_0^L \frac{\partial \eta}{\partial x} = q(0) + \frac{1}{L}\left[\eta(L) - \eta(0)\right] \qquad (6)$$

where $q(0)$ denotes the heat flux at $x = 0$. Equation analogous to (6), in which $q(L)$ appears, is to be obtained by multiplying Equation (1) by $x/L$; their sum gives the heat balance equation for the slab. Equation (6) is in fact a consequence of the reciprocity principle in one dimension, as the linear function is a solution of Equation (3) for the steady state of heat flow.

We integrate now Equation (6) with respect to time, over the interval $[t_1, t_2]$. The following integral:

$$\int_{t_1}^{t_2} \rho c_p(T)\frac{\partial T}{\partial t}dt = \int_{t_1}^{t_2}\frac{\partial}{\partial t}\left[\int_0^T \rho c_p(T')dT'\right]dt = \int_{T(t_1)}^{T(t_2)} \rho c_p(T')dT' \qquad (7)$$

represents the variation of heat content per unit volume, between $t_1$ and $t_2$, whereas the quantity $\Delta Q_C(0)$

$$\Delta Q_C(0,t_1,t_2) = \int_{t_1}^{t_2}\int_0^L \rho c_p(T)\frac{\partial T}{\partial t}\left(1-\frac{x}{L}\right)dx\,dt$$

$$= \int_0^L\left(1-\frac{x}{L}\right)\int_{T(x,t_1)}^{T(x,t_2)}\rho c_p(T')dT'dx \qquad (8)$$

represents the variation of heat content in a slab element of unit cross-sectional area, transferred across the plane $x = 0$. It depends only on the initial and final thermal state of the slab. For $c_p$ independent of temperature

$$\Delta Q_C(0,t_1,t_2) = \int_0^L \rho c_p \left[T(x,t_2) - T(x,t_1)\right]\left(1 - \frac{x}{L}\right) dx \,. \tag{9}$$

The integral statement, which follows from the governing Equation (1), written in terms of time averages denoted here by overbars, takes thus the form:

$$\bar{q}(0) = \frac{1}{L}\left[\bar{\eta}(0) - \bar{\eta}(L)\right] + \frac{\Delta Q_C(0)}{\Delta t}, \qquad \Delta t = t_2 - t_1 \,. \tag{10}$$

For sufficiently long time interval [$t_1$,$t_2$], the ratio of the capacity term $\Delta Q_C(0)$ and $\Delta t$ is in general small as compared with the remaining terms in Equation (11).

## ALGORITHMS TO DETERMINE THERMAL CONDUCTIVITY EMPLOYING THE AVERAGING TECHNIQUE

### Heat flux and temperature measurements

Let us assume now that the surface heat flux $q(0)$ is measured simultaneously with the surface temperature of the slab, $T(0)$ and $T(L)$. At linear dependence of conductivity on temperature, Equation (10) takes the form:

$$\bar{q}(0) = \frac{\lambda_0}{L}\left[\bar{T}(0) - \bar{T}(L)\right]$$

$$+ \frac{\beta}{L}\left\{\frac{\overline{T(0)^2} - \overline{T(L)^2}}{2} - T_0\left[\bar{T}(0) - \bar{T}(L)\right]\right\} + \frac{\Delta Q_C(0)}{\Delta t} \tag{11}$$

For given courses of $q(0)$, $T(0)$ and $T(L)$, and prescribed value of the reference temperature $T_0$, which may be particularly zero, neglecting the capacity term, we obtain from Equation (12) the linear equation with two unknowns: the intercept $\lambda_0$ and the temperature coefficient of thermal conductivity $\beta$. To determine both quantities we need at least one more equation, for another heat transfer process, with different values of the surface heat flux and temperatures.

We may proceed, however, in a different way. In this case, when the term proportional to $\beta$ in Equation (11) is zero, it requires

$$\beta\{\cdots\}=0 \quad \Leftrightarrow \quad T_0\left[\overline{T}(0)-\overline{T}(L)\right]=\frac{\overline{T(0)^2}-\overline{T(L)^2}}{2}\,. \tag{12}$$

Equation (11) has the same form as that for a linear medium:

$$\overline{q}(0)=\frac{\lambda(T)}{L}\left[\overline{T}(0)-\overline{T}(L)\right]+\frac{\Delta Q_C(0)}{\Delta t}\,. \tag{13}$$

For a given heat transfer process Equation (12) determines some reference temperature, $T_0$, for which Equation (13) is satisfied. However the value of the reference temperature in Equation (4) was set to some extent arbitrarily; we may assume therefore that for the process considered it is just the temperature, called now $T^*$, determined by Equation (12).

Neglecting the capacity term $\Delta Q_C(0)$ in Equation (13), we get the following formula to determine the conductivity $\lambda(T^*)$ at the temperature $T^*$ from the surface temperature and heat flux data:

$$\lambda(T^*)=L\,\frac{\overline{q}(0)}{\overline{T}(0)-\overline{T}(L)} \tag{14}$$

$$T^*(t_1,t_2)=\frac{1}{2}\,\frac{\overline{T(0)^2}-\overline{T(L)^2}}{\overline{T}(0)-\overline{T}(L)} \tag{15}$$

The temperature $T^*$ may be represented as:

$$T^*(t_1,t_2)=\frac{1}{\Delta t}\int_{t_1}^{t_2}\frac{T(0)-T(L)}{\overline{T}(0)-\overline{T}(L)}\,\frac{T(0)+T(L)}{2}\,dt \tag{16}$$

In steady state of heat flow, $T^*$ is the average of the slab surface temperatures, $T(0)$ and $T(L)$; whereas in unsteady conditions it is the weighted average, with ratio of current and mean surface temperature differences as the weighting function.

To determine the temperature coefficient of thermal conductivity, $\beta$, still at least one more value of $\lambda$, corresponding to different value of the weighted average temperature, $T^*$, is necessary. The need to determine thermal capacity of the material is here avoided, however its approximate value is necessary to estimate the error due to neglecting the unknown capacity term $\Delta Q_C(0)$.

Theoretical relative error of $\lambda(T^*)$, determined using Equation (14), is given by:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta Q_C(0)}{\Delta t \overline{q}(0) - \Delta Q_C(0)} \tag{17}$$

To estimate $\Delta Q_C(0)$ we assume $c_p$ independent of temperature, and temperature in a slab at the beginning and end of the measurement period, $T(x,t_1)$ and $T(x,t_2)$, given by the steady state solution:

$$T(x) = \left(1 - \frac{x}{L}\right)T(0) + \frac{x}{L}T(L) \tag{18}$$

Evaluating the integral in Equation (9) gives for $\Delta Q_C(0)$

$$\Delta Q_C(0) \approx \frac{C}{3}\Delta T(0) + \frac{C}{6}\Delta T(L); \qquad C = \rho c_p L \tag{19}$$

where $\Delta T(0) = T(0,t_2) - T(0,t_1)$ and $\Delta T(L) = T(L,t_2) - T(L,t_1)$.

The approximate expression for the relative error of the conductivity $\lambda(T^*)$, determined using Equation (14), is thus as follows:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{RC}{\Delta t}\frac{\frac{1}{3}\Delta T(0) + \frac{1}{6}\Delta T(L)}{\overline{T}(0) - \overline{T}(L)}; \qquad R = \frac{L}{\lambda(T^*)} \tag{20}$$

The error is twice as sensitive to the variation of temperature at this surface ($x = 0$) at which the heat flux is being measured, as to variation of temperature at the opposite surface; however it may be zero if they are of different sign. When the average difference of surface temperatures approaches zero the error tends to infinity.

Figure 1 illustrates the dependence of the percentage relative error of $\lambda$ on measurement time, for the extruded polystyrene slabs of thickness 25 - 75 mm, assuming the following material properties (at 24°C [4]) and temperature courses characteristics:

$$\lambda = 0.029 \text{ W/(mK)}, \quad \rho = 26 \text{ kg/m}^3, \quad c_p = 1.22 \text{ kJ/(kgK)}$$

$$\overline{T}(0) - \overline{T}(L) = 10°C, \quad \Delta T(0) = 0°\text{C}, \quad \Delta T(L) = 5°\text{C}$$

The plots in Figure 1 indicate, that two days long test period should, in general, secure good enough accuracy, expressed by the relative conductivity error below 1%.

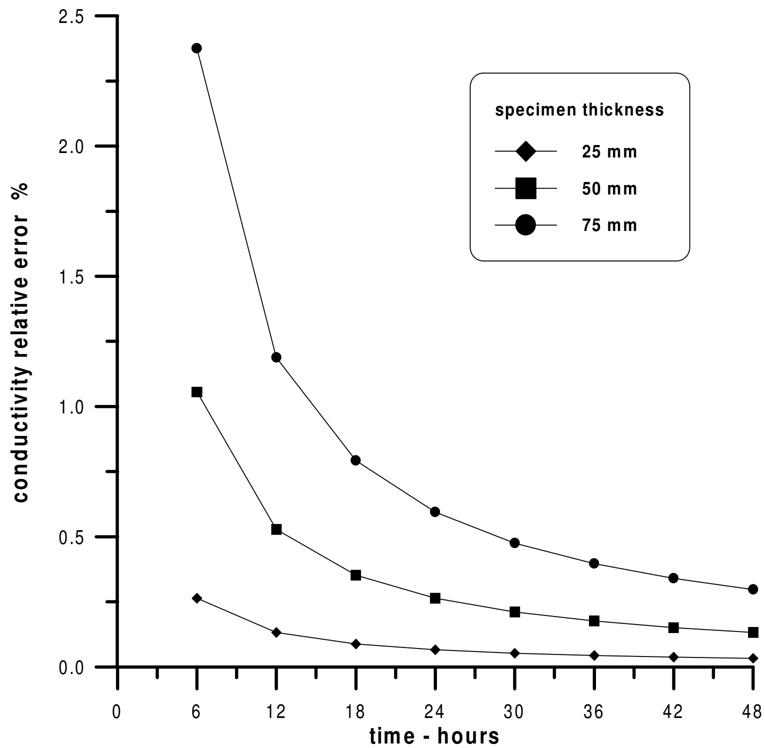


**FIGURE 1.** Dependence of the relative conductivity error on measurement time

## Heat flux comparator technique

When the „heat flux comparator" technique, to determine conductivity of a tested material assuming known properties of the reference material is employed, the formulae developed above can be applied to both specimens in thermal contact.

The measurement set scheme is represented in Figure 2. The surface temperatures of the reference specimen are $T_1$ and $T_2$, whereas of the tested specimen are $T_2$ and $T_3$.

The heat flux continuity condition, at the interface between two materials, by virtue of Equation (14), yields the relation:

$$\frac{\lambda_2\left(T^*_{(2)}\right)}{L_2}\left[\overline{T}_2-\overline{T}_3\right]+\frac{\Delta Q_{C_2}(0)}{\Delta t}=-\left[\frac{\lambda_1\left(T^*_{(1)}\right)}{L_1}\left[\overline{T}_2-\overline{T}_1\right]+\frac{\Delta Q_{C_1}(0)}{\Delta t}\right] \tag{21}$$

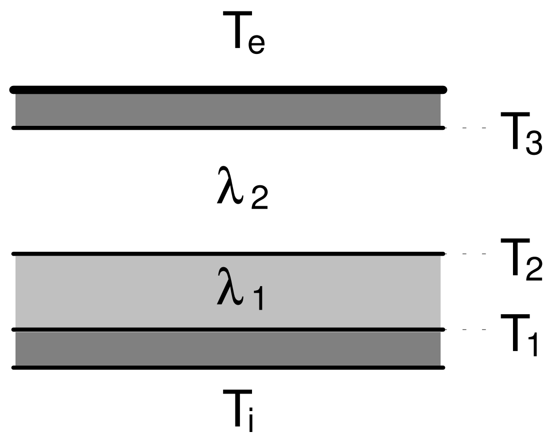


**FIGURE 2.** Measurement set scheme in the *heat flow comparator* method

Symbols with index 1: $L_1$, $\lambda_1$, $\Delta Q_{C1}(0)$, $T_{(1)}{}^*$ denote here the thickness, conductivity, capacity term and weighted average temperature for the reference specimen, respectively, whereas those with index 2: $L_2$, $\lambda_2$, $T_{(2)}{}^*$, $\Delta Q_{C2}(0)$, the same quantities for the tested specimen. Neglecting capacity terms gives the relation between conductivities $\lambda_2$ at $T_{(2)}{}^*$ and $\lambda_1$ at $T_{(1)}{}^*$:

$$\lambda_2\left(T^*_{(2)}\right)=\lambda_1\left(T^*_{(1)}\right)\frac{L_2}{L_1}\,\frac{\overline{T}_1-\overline{T}_2}{\overline{T}_2-\overline{T}_3} \tag{22}$$

where $T_{(1)}{}^*$ and $T_{(2)}{}^*$ for both specimens are determined by Equation (15). The theoretical relative error of the conductivity $\lambda_2$, calculated from Equation (22), by virtue of Equation (21) is given by:

$$\frac{\Delta\lambda_2}{\lambda_2}=\frac{R_1}{\Delta t\left(\overline{T}_1-\overline{T}_2\right)}\left[\Delta Q_{C_1}(0)+\Delta Q_{C_2}(0)\right], \qquad R_1=\frac{L_1}{\lambda_1\left(T_1^*\right)} \tag{23}$$

Assuming that $\Delta Q_{C1}(0)$ and $\Delta Q_{C2}(0)$ are given by the approximate expression (19), which means constant specific heat capacity, and temperature distribution for the initial and final state in both specimens as for the steady state of heat flow, yields:

$$\frac{\Delta\lambda_2}{\lambda_2}\approx\frac{R_1}{\Delta t\left(\overline{T}_1-\overline{T}_2\right)}\left[\frac{C_1}{6}\Delta T_1+\frac{\left(C_1+C_2\right)}{3}\Delta T_2+\frac{C_2}{6}\Delta T_3\right] \tag{24}$$

$$C_1=L_1\,\rho_1\,c_{p1}, \qquad C_2=L_2\,\rho_2\,c_{p2}$$

The error appears to be most sensitive to variations of the contact surface temperature, $T_2$.

Employing again the assumption of steady state at the beginning and end of the measurement period, we may eliminate $\Delta T_2$ expressing it in terms of $\Delta T_1$ and $\Delta T_3$:

$$\Delta T_2 = \frac{R_2}{R}\Delta T_1 + \frac{R_1}{R}\Delta T_3 , \qquad R = R_1 + R_2 \tag{25}$$

Moreover, for sufficiently long measurement time $\Delta t$, with the accuracy up to terms linear in $1/\Delta t$, we may replace the difference of time averages $\overline{T}_1 - \overline{T}_2$ by the difference of time averages $\overline{T}_1 - \overline{T}_3$, according to

$$\frac{\overline{T}_1 - \overline{T}_2}{R_1} \approx \frac{\overline{T}_1 - \overline{T}_3}{R} \tag{26}$$

which gives:

$$\frac{\Delta\lambda_2}{\lambda_2} = \frac{RC}{\Delta t}\,\frac{a_1\,\Delta T_1 + a_3\,\Delta T_3}{\overline{T}_1 - \overline{T}_3} \tag{27}$$

$$a_1 = \frac{C_1}{6C} + \frac{R_2}{3R}, \qquad a_3 = \frac{C_2}{6C} + \frac{R_1}{3R}; \qquad C = C_1 + C_2 \tag{28}$$

The error is proportional to the ratio of the time constant $RC$ of the whole two-layer slab, composed of the test and reference specimen, to the measurement time $\Delta t$. The sensitivity coefficients, $a_1$ and $a_3$, satisfy the condition:

$$a_1 + a_3 = \frac{1}{2} \tag{29}$$

For $R_2 > R_1$ and $C_1 > C_2$, $a_1 > a_3$, what means that the relative error is more sensitive to variations of $T_1$ than to variations of $T_3$.

In the experiments described in References [1,2] the tested foams and reference materials, built in the roof of the exposure box, were sandwiched between the plywood substrate and fiberboard overlay. Thermocouples were placed on each surface of both specimens, to measure temperatures that then were used as boundary conditions in the heat flux calculations. Thin black membrane was used to cover the test assembly. Driving temperatures in such an experiment are, however, not the temperatures of free surfaces of the specimens but rather the interior temperature within the exposure test box, $T_i$, and the roof assembly upper surface temperature, which depends on the outdoor temperature, solar radiation and other weather factors.

Let $R_{si}$ denote the resistance for heat transfer between the bottom surface of the reference specimen and the interior of the exposure test box, and $R_{se}$ the resistance between the upper surface of the test specimen and the plane where the temperature $T_e$ is measured (which may be $T_3$ in particular case); $R_T$ is the sum:

$$R_T = R_{si} + R_1 + R_2 + R_{se} \tag{30}$$

Employing once more the assumption of steady state at $t_1$ and $t_2$, and sufficiently long measurement time, we may express $\Delta T_1$ and $\Delta T_3$ in terms of $\Delta T_i$ and $\Delta T_e$, and $\overline{T}_1 - \overline{T}_3$ in terms of $\overline{T}_i - \overline{T}_e$:

$$\Delta T_1 = \frac{R + R_{se}}{R_T} \Delta T_i + \frac{R_{si}}{R_T} \Delta T_e \,, \qquad \Delta T_3 = \frac{R_{se}}{R_T} \Delta T_i + \frac{R + R_{si}}{R_T} \Delta T_e \tag{31}$$

$$\frac{\overline{T}_1 - \overline{T}_3}{R} \approx \frac{\overline{T}_i - \overline{T}_e}{R_T} \tag{32}$$

In terms of $\Delta T_i$ and $\Delta T_e$ the relative error of $\lambda_2$ is now given by:

$$\frac{\Delta \lambda_2}{\lambda_2} = \frac{R_T C}{\Delta t} \frac{a_i \Delta T_i + a_e \Delta T_e}{\overline{T}_i - \overline{T}_e} \tag{33}$$

with modified sensitivity coefficients, which are now $a_i$ and $a_e$

$$a_i = \frac{C_1}{6C} \frac{R}{R_T} + \frac{R_2}{3R_T} + \frac{R_{se}}{2R_T} \,, \qquad a_e = \frac{C_2}{6C} \frac{R}{R_T} + \frac{R_1}{3R_T} + \frac{R_{si}}{2R_T} \tag{34}$$

satisfying the condition $a_i + a_e = 1/2$.

At comparable values of $R_{se}$ and $R_{si}$, and $R_2 > R_1$, $C_1 > C_2$, $a_i > a_e$ , which means that the error is more sensitive to $T_i$ variations than to $T_e$ variations. Stabilization of the temperature inside the exposure box during the experiment is thus advisable; this was reported by Bomberg et al. [1]. However, when the thermal comfort in an exposure box is not a goal, but rather the measurement accuracy, the proper control of $T_i$, to minimize expression (33), may give even better results. This would require:

$$a_i \Delta T_i = - a_e \Delta T_e \qquad \Rightarrow \qquad T_i + \frac{a_e}{a_i} T_e = const \tag{35}$$

which could be possible to attain within limited range of $T_e$ variations.

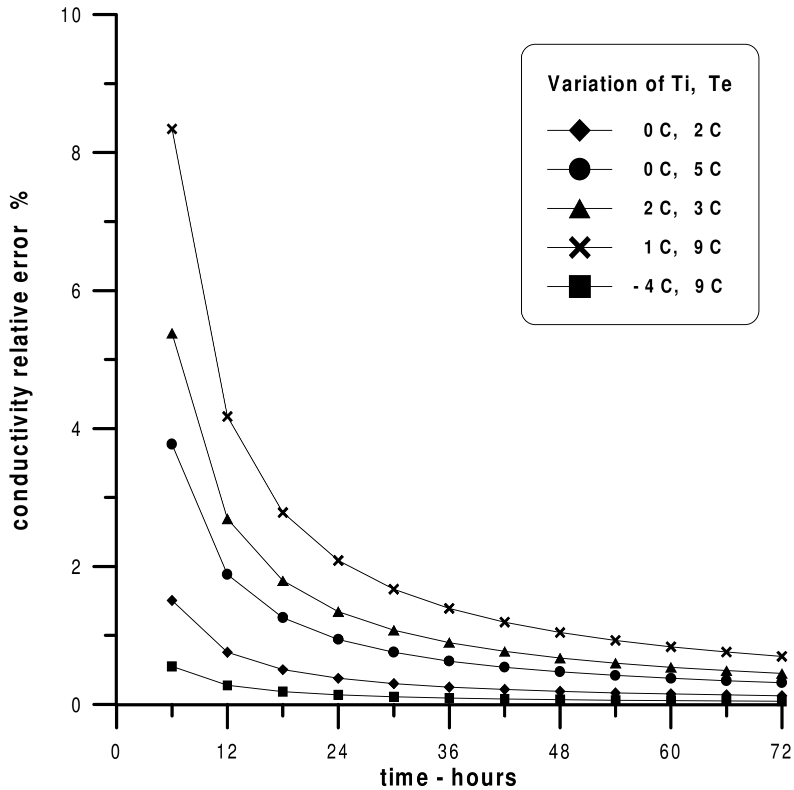


**FIGURE 3.** Effect of time on relative conductivity error for different ambient temperature variations during the test period.

More accurate control algorithm can be obtained using the response factors method (assuming linear heat transfer). This would give the current temperature $T_i$ as a function of current $T_e$ and also the past values of $T_i$ and $T_e$.

Figure 3 illustrates the effect of measurement time on the theoretical relative error of the test specimen conductivity, for different $\Delta T_i$ and $\Delta T_e$ values, at mean $T_i$ and $T_e$ difference 10ºC. Calculations were performed for the measurement set as that described in the article of Bomberg et al. [1]. The test specimen was extruded polystyrene slab of thickness $L_2 = 50$ mm, whereas the reference specimen was glass fiber board of thickness $L_1 = 25$ mm. $R_{si}$ is the resistance of the inner substrate, plywood of 12 mm, plus the inner surface film resistance of 0.12 m$^2$K/W, whereas $R_{se}$ the resistance of the outer overlay, 12 mm fiberboard, plus the outer surface film resistance of 0.05 m$^2$K/W. The resistance of a thin membrane, used to cover the test assembly, was neglected. Thermophysical properties of the materials: rigid cellular polystyrene - as in the previous example, glass fiberboard: $\rho = 112\ \mathrm{kg/m^3}$, $\lambda = 0.050\ \mathrm{W/(mK)}$, $c_p = 0.84\ \mathrm{kJ/(kgK)}$, plywood: $\rho = 600\ \mathrm{kg/m^3}$, $\lambda = 0.16\ \mathrm{W/(mK)}$, $c_p = 2.51\ \mathrm{kJ/(kgK)}$, fiberboard: $\rho = 1000\ \mathrm{kg/m^3}$, $\lambda = 0.18\ \mathrm{W/(mK)}$, $c_p = 2.51\ \mathrm{kJ/(kgK)}$. This gives: $R_T C/2 = 1.39$ h, $a_i = 0.67$, $a_e = 0.33$, $a_i/a_e \approx 2$.

The plots in Figure 3 indicate again that, similarly as when the heat flux is measured, two days long test period, with not extremely large changes of the indoor and outdoor conditions between the beginning and the end, should, in general, secure good enough accuracy, represented by the relative conductivity error below 1%.

## CONCLUSIONS

An integral statement, which follows from the heat conduction equation, constitutes mathematical basis for the method of averages, to determine temperature dependent conductivity of a material under transient conditions from surface heat flux and temperature measurements.

For linear dependence of conductivity on temperature, on neglecting capacity terms, which is reasonable for sufficiently long test period, from the integral statement one obtains the linear equation with two unknowns: the intercept and the temperature coefficient of thermal conductivity, to be determined from average values of the surface heat flux and surface temperatures and their squares. This gives simple algorithms to determine thermal conductivity in transient heat flow, for the weighted average temperature, when the heat flux and surface temperatures are measured simultaneously and also when the *heat flux comparator* method is employed. The need to determine heat capacity of the material is avoided.

The relative conductivity error, due to neglecting capacity terms, depends on ratios of surface temperature variations between the beginning and end of the test period to the average difference of these temperatures. The error is particularly sensible to temperature variations on this surface, on which the heat flux is measured or compared.

When the special exposure box is used, for the long-term testing of insulation materials resistances under field conditions, the precise control of the inner temperature is necessary. The stabilization of this temperature is reasonable; however the proper control to minimize approximately known value of the capacity term, may give even better results.

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