FINITE DEFORMATION BASED MODEL OF CYTOSKELETON

Eligiusz Postek (1), Rod Smallwood (2), Rod Hose (3)

1. University of Sheffield, Department of Computer Science, UK; 2. University of Sheffield, Department of Computer Science; 3. University of Sheffield, Medical Physics and Clinical Engineering, UK

Introduction

The cytoskeleton is modelled as a tensegrity structure. It consists of tendons and struts [Stamenovic, 2005]. The basic tensegrity structures are developed from icosahedron.

The cells undergo finite deformations behaviour and their response to loading conditions is visco-elastic.

These types of structures are sensitive to initial prestressing what is intuitively natural since without the initial stress state they would not exist. The other factors affecting the mechanical behaviour of the cell are the constitutive parameters and positions of the nodes, in fact, the shape of the cell.

Methods

The generic incremental FE equation is formulated in the Updated Lagrangian frame [Kleiber, 1997] and reads

$$\left(\int_{\Omega'} \mathbf{B}'^{T}_{L}\, {}^{t}_{t}\bar{\boldsymbol{\tau}}\, \mathbf{B}'_{L}\, d\Omega' \right) \Delta\mathbf{q} + \int_{\Omega'} \mathbf{B}^{T}_{L} \Delta\mathbf{S}\, d\Omega' = \int_{\Omega'} \mathbf{N}^{T} \Delta\mathbf{f}\, d\Omega' + \int_{\partial\Omega'_{\sigma}} \mathbf{N}^{T} \Delta\mathbf{t}\, d(\partial\Omega'_{\sigma}) \qquad (1)$$

where $\mathbf{B}'^{T}_{L}$ and $\mathbf{B}^{T}_{L}$ are the nonlinear and linear operators, $\mathbf{N}$ is the shape functions matrix, $\Delta\mathbf{S}$ is the stress increment, $\bar{\boldsymbol{\tau}}$ is the Cauchy stress matrix, $\Delta\mathbf{q}$ is the displacement increment, $\Delta\mathbf{f}$ and $\Delta\mathbf{t}$ are the body forces and the boundary tractions increments. The integration is done over the domain Ω and its boundary Ω_{σ}.

The constitutive model is visco-elastic such as the stress increment depends on total stress $\mathbf{S}$, the shear modulus (G), the bulk modulus (K) and the strain increment $\Delta\mathbf{E}$ as follows

$$\Delta\mathbf{S} = \mathbf{D}^{const}\left({}^{t}_{t}\mathbf{S}, G, K\right)\Delta\mathbf{E} \qquad (2)$$

with the relaxation function

$$G(t) = G_o + \sum_{i=1}^{n} G_i \exp\left(\frac{-t}{\lambda_i}\right) \qquad (3)$$

where t is the time and λ_i are the relaxation times of the particular parallel dampers. These above describe the generalized Maxwell model.

Exemplary results and discussion

The icosahedral tensegrity structure is presented in Fig. 1. The CSK is fixed at the surface and is tested for the uni-axial extension. We adopted the following data, namely, height of the cell 64 μm, cross-sectional areas of the tendons (filaments) $10 nm^2$, cross-sectional areas of the struts (microtubules) 190 nm^2, Young's moduli of the tendons 2.6GPa and the struts 1.2GPa, initial prestressing forces 20 nN, maximum loading 0.1N, relaxation time 1.0 sec, G_i/G_o ratio 0.91 (case A) and 0.1 (case B).

We investigated the stiffening effect during extension and up to buckling and postbuckling equilibrium paths in compression. Our interests were focused on visco-elasticity effects and geometrically imperfect system.

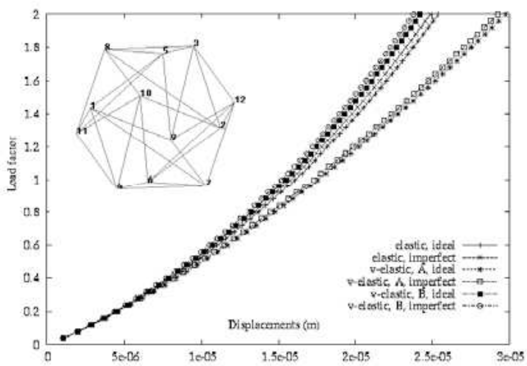

Figure 1: Icosahedron based tensegrity structure, tension test, materials elastic and visco-elastic, perfect and imperfect geometries.

The tension tests demonstrate stiffening of the structure. We can see that the considered imperfection pattern (moving the node under the force in the plane perpendicular to it) slightly stiffens the CSK. Considering the visco-elastic cases the low value of the shear ratio stiffens the structure while the high value of the shear ratio softens the CSK.

The cell model is an element of a matrix of cells. We observe the effects of imperfections, in this case different materials in the cell on the behaviour of the cell assembly under tension and compression tests.

References

Stamenovic D., Acta Biomaterialia, 1:255-262, 2005.

Kleiber M. et al, Parameter Sensitivity in Nonlinear Mechanics: Theory and Finite Elements, Wiley & Sons, 1997.