Available online at www.sciencedirect.com
SCIENCE DIRECT
Energy and Buildings 37 (2005) 301–310
ENERGY and BUILDINGS
www.elsevier.com/locate/enbuild

Three-dimensional conduction z-transfer function coefficients determined from the response factors

Elisabeth Kossecka[a,*], Jan Kosny[b]

[a]Department of Eco-Building Engineering, Polish Academy of Sciences, Swietokrzyska 21, 00 049 Warsaw, Poland
[b]Oak Ridge National Laboratory, Buildings and Materials Group, Building 3247, M.S. 6070, Oak Ridge, TN 37831-6070, USA

Received 20 January 2004; received in revised form 15 June 2004; accepted 30 June 2004

Abstract

A method of derivation of the conduction z-transfer function coefficients from response factors, for three-dimensional wall assemblies, is described.

Results of the conduction z-transfer function coefficients calculations are presented for clear walls and separated details which are listed in ASHRAE research project 1145-TRP: "Modeling Two- and Three-Dimensional Heat Transfer Through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs". Resistances, three-dimensional response factors and so-called structure factors, have been computed using the finite-difference computer code HEATING 7.2. The z-transfer function coefficients were then derived from a set of linear equations, constituting relationships with the response factors, which were solved using the minimum-error procedure.

Test simulations show perfect compatibility of the heat flux calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients, derived from the response factors.

Keywords: Heat transfer; Thermal response; z-transfer function; Simulation; Building envelope

1. Introduction

The z-transfer function method is used in the whole building simulation programs to model one-dimensional heat transfer through walls, roofs, and floors. Transfer function procedures, developed by Stephenson and Mitalas [1], pertain to one-dimensional structures made up of layers of homogeneous materials and have no allowance for walls with thermal bridges in which three-dimensional flow occurs.

Seem et al. [2,3] presented a method for calculating transfer functions for multidimensional heat transfer from a state space formulation. Spatial discretization of the problem results in the set of first-order differential equations. Exact solution to this set of equations is determined to represent response to the excitation modeled by a continuous piecewise linear curve.

Burch et al. [4,5] presented a numerical procedure for calculating first-order conduction-transfer function coefficients for complex building constructions containing two-dimensional thermal bridges. The heat-transfer response to the ramp excitation was predicted by the finite-difference model; the regression analysis was applied then to subtract the steady-state response, and to determine the first pole of the transfer function.

Brown and Stephenson [6] developed a method to determine transfer function coefficients from the surface-frequency response. This method, based on the Laplace and z-transfer function formalism, has been used to determine the z-transfer functions of the full-scale wall specimens with complex geometries, using the guarded hot-box procedures.

A method of derivation of the conduction z-transfer function coefficients from response factors, for three-dimensional wall assemblies, is presented here.

The response factors, which represent surface averaged heat flux due to triangular temperature excitations at discrete time instants, are calculated with the help of a finite-difference

* Corresponding author.
E-mail address: ekossec@ippt.gov.pl (E. Kossecka), kyo@ornl.gov (J. Kosny).

0378-7788/$ – see front matter
doi:10.1016/j.enbuild.2004.06.026

Nomenclature

b_n, c_n, d_n	dimensionless heat conduction z-transfer function coefficients
c_p	specific heat (J/(m^3 K)) [Btu/(lb °F)]
C	capacity per unit surface area of a wall or detail (kJ/(m^2 K)) [Btu/(ft^2 °F)]
E_b, E_c	relative errors of the z-transfer function calculations
N_b, N_c, N_d	maximum index of numerically significant coefficient b_n, c_n, d_n, respectively
$Q_{i,n\delta}$	heat flux at time $n\delta$ across the interior surface (W/m^2) [Btu/(h ft^2)]
R	thermal resistance per unit surface area of a wall or detail (m^2 K/W) [ft^2 °F h/Btu]
$T_{e,n\delta}$	exterior temperature at time $n\delta$ (°C) [°F]
$T_{i,n\delta}$	interior temperature at time $n\delta$ (°C) [°F]
V	volume of a wall element (m^3) [ft^3]
X_n, Y_n	response factors (W/(m^2 K)) [Btu/(h ft^2 °F)]
$Z\{Q\}$, $Z\{T\}$, $Z\{X\}$, $Z\{Y\}$, $B(z)$, $C(z)$, $D(z)$	z-transforms
Greek letters	
δ	time instant (h)
φ_{ii}, φ_{ie}	structure factors
θ	dimensionless temperature
ρ	density (kg/m^3) [lb/ft^3]

computer code, to simulate three-dimensional heat conduction. They are used as ‘‘input data’’, to determine z-transfer function coefficients from the set of linear equations, which includes relationships with the response factors and compatibility conditions. This primarily infinite set of equations is reduced to a finite one and solved using the minimum-error procedure. The method gives very good results, in the sense that heat-flux courses obtained from simulations, using the response factors and the z-transfer function coefficients, in most cases almost coincide.

2. Relationships between response factors and z-transfer function coefficients

In terms of response factors, heat flux across the interior surface of a wall element at time instant $n\delta$, $Q_{i,n\delta}$, can be represented as follows [7,8]:

$$Q_{i,n\delta} = \sum_{k=0}^{n} \left[X_k T_{i,(n-k)\delta} - Y_k T_{e,(n-k)\delta}\right], \tag{1}$$

where $\{T_{i,n\delta}\}$ and $\{T_{e,n\delta}\}$ are sequences of the ambient (or surface) temperature values, and $\{X_n\}$ and $\{Y_n\}$ are sequences of the response factors.

As far as three-dimensional problems are concerned, the heat-flux values in Eq. (1), as well as the response factors, are to be understood as averages over the surfaces of a wall element, separated from rest of the wall by an adiabatic lateral surface. Driving temperatures are functions of time only, and do not depend on spatial coordinates, which is also the case when boundary conditions of the first kind are assumed. Dimensions of the element and location of the lateral cut surface are to be established while developing a three-dimensional model, in order to determine its thermal characteristics.

The z-transform of the interior heat flux, $Z[Q_i]$ is related to the z-transforms of the interior and exterior temperature, $Z[T_i]$ and $Z[T_e]$, by the following equation (see [9]):

$$Z[Q_i] = Z\{X_n\} \cdot Z[T_i] - Z\{Y_n\} \cdot Z[T_e], \tag{2}$$

where $Z\{X_n\}$ and $Z\{Y_n\}$ are the z-transforms of the sequences of the response factors, $\{X_n\}$ and $\{Y_n\}$:

$$Z[Q_i] = \sum_{n=0}^{\infty} Q_{i,n\delta} z^{-n}, \qquad Z\{X_n\} = \sum_{n=0}^{\infty} X_n z^{-n}, \qquad Z\{Y_n\} = \sum_{n=0}^{\infty} Y_n z^{-n}. \tag{3}$$

The compatibility condition which the response factors, X_n and Y_n, should satisfy:

$$\sum_{n=0}^{\infty} X_n = \sum_{n=0}^{\infty} Y_n = \frac{1}{R} \tag{4}$$

is equivalent to the following condition for the z-transforms $Z\{X_n\}$ and $Z\{Y_n\}$:

$$\lim_{z \to 1} Z\{X_n\} = \lim_{z \to 1} Z\{Y_n\} = \frac{1}{R}. \tag{5}$$

Here, R denotes the resistance per unit surface area, determined from the average heat flux in the steady-state conditions.

Now let $Z\{X_n\}$ and $Z\{Y_n\}$ be given as the quotients:

$$Z\{X_n\} = \frac{1}{R}\frac{C(z)}{D(z)}, \qquad Z\{Y_n\} = \frac{1}{R}\frac{B(z)}{D(z)}, \tag{6}$$

where

$$B(z) = \sum_{n=0}^{\infty} b_n z^{-n}, \qquad C(z) = \sum_{n=0}^{\infty} c_n z^{-n}, \qquad D(z) = \sum_{n=0}^{\infty} d_n z^{-n}. \tag{7}$$

Eq. (2) can be rewritten in the following form:

$$D(z)\cdot Z[Q_\mathrm{i}] = \frac{1}{R}\{C(z)\cdot Z[T_\mathrm{i}] - B(z)\cdot Z[T_\mathrm{e}]\}. \tag{8}$$

Eq. (1) for $Q_{\mathrm{i},n\delta}$, assuming $d_0 = 1$, is now replaced by (see [1]):

$$Q_{\mathrm{i},n\delta} = \frac{1}{R}\left[\sum_{m=0}^{n} c_m T_{\mathrm{i},(n-m)\delta} - \sum_{m=0}^{n} b_m T_{\mathrm{e},(n-m)\delta}\right] - \sum_{m=1}^{n} d_m Q_{\mathrm{i},(n-m)\delta}. \tag{9}$$

The dimensionless conduction z-transfer function coefficients b_n and c_n correspond to the coefficients b_n and c_n from [10], multiplied by R. For the purpose of simulations, only numerically significant coefficients are important.

Formulae (6) for the z-transforms, when rewritten in the form:

$$C(z) = R\cdot Z\{X_n\}\cdot D(z),\quad B(z) = R\cdot Z\{Y_n\}\cdot D(z) \tag{10}$$

are equivalent to the convolution type relationships between the response factors X_n, Y_n and the conduction z-transfer function coefficients b_n, c_n, and d_n:

$$b_n = R\sum_{k=0}^{n} Y_{n-k} d_k, \qquad c_n = R\sum_{k=0}^{n} X_{n-k} d_k. \tag{11}$$

Eq. (5) for the z-transforms $Z\{Y_n\}$ and $\mathrm{Z}\{X\}$ now has the following form:

$$\left.\frac{B(z)}{D(z)}\right|_{z=1} = \left.\frac{C(z)}{D(z)}\right|_{z=1} = 1. \tag{12}$$

Eq. (12) yields the following compatibility condition for the dimensionless z-transfer function coefficients:

$$\sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty} c_n = \sum_{n=0}^{\infty} d_n. \tag{13}$$

3. Determining the z-transfer function coefficients from the response factors

On the basis of Eqs. (11) and (13), one may try to determine z-transfer function coefficients from sequences of the response factors, $\{Y_n\}$ and $\{X_n\}$. This is the most straightforward method; the z-transfer functions obtained in this way are expected to ‘‘exactly’’ reproduce the output for any input function composed of straight-line segments, joining the points which represent its values at $t = n\delta$.

Assuming that z-transfer function coefficients with indices above some n are negligibly small, and $d_0 = 1$, we obtain the following set of linear equations:

$$b_0 + b_1 + b_2 + b_3 + \cdots + b_n = 1 + d_1 + d_2 + d_3 + \cdots + d_n \tag{14.1}$$

$$b_0 = RY_0 \tag{14.2}$$

$$b_1 = R(Y_1 + Y_0 d_1) \tag{14.3}$$

$$b_2 = R(Y_2 + Y_1 d_1 + Y_0 d_2) \tag{14.4}$$

$$b_n = R(Y_n + Y_{n-1}d_1 + Y_{n-2}d_2 + Y_{n-3}d_3 + \cdots + Y_0 d_n) \tag{14.n}$$

$$0 = R(Y_{n+1} + Y_n d_1 + Y_{n-1}d_2 + Y_{n-2}d_3 + \cdots + Y_1 d_n) \tag{14.n+1}$$

$$c_0+c_1+c_2+c_3+\cdots+c_n = 1+d_1+d_2+d_3+\cdots+d_n \tag{15.1}$$

$$c_0 = RX_0 \tag{15.2}$$

$$c_1 = R(X_1 + X_0 d_1) \tag{15.3}$$

$$c_2 = R(X_2 + X_1 d_1 + X_0 d_2) \tag{15.4}$$

$$c_n = R(X_n + X_{n-1}d_1 + X_{n-2}d_2 + X_{n-3}d_3 + \cdots + X_0 d_n) \tag{15.n}$$

$$0 = R(X_{n+1} + X_n d_1 + X_{n-1}d_2 + X_{n-2}d_3 + \cdots + X_1 d_n) \tag{15.n+1}$$

If structure factors are calculated together with resistance and response factors, one may use conditions imposed by the structure factors on z-transfer function coefficients as subsidiary equations [11]:

$$\sum_{n=1}^{\infty} nb_n - \sum_{n=1}^{\infty} nd_n = \frac{RC}{\delta}\varphi_{ie}\sum_{n=0}^{\infty} d_n, \tag{16}$$

$$\sum_{n=1}^{\infty} nc_n - \sum_{n=1}^{\infty} nd_n = -\frac{RC}{\delta}\varphi_{ii}\sum_{n=0}^{\infty} d_n. \tag{17}$$

The structure factors, φ_{ii} and φ_{ie}, are given by (see [11,18,19]):

$$\varphi_{ie} = \frac{1}{C}\int_V \rho c_p \theta(1-\theta)\,\mathrm{d}v, \tag{18}$$

$$\varphi_{ii} = \frac{1}{C}\int_V \rho c_p (1-\theta)^2\,\mathrm{d}v, \tag{19}$$

where C is the total thermal capacity of a wall element of volume V:

$$C = \int_V \rho c_p\,\mathrm{d}v, \tag{20}$$

and θ is the dimensionless temperature for a problem of steady-state heat transfer through the wall element with an adiabatic lateral surface, for ambient temperatures $T_\mathrm{i} = 0$ and $T_\mathrm{e} = 1$. For plane walls, the products $C\cdot\varphi_{ii}$ and $C\cdot\varphi_{ie}$ are equivalent to the thermal mass factors, introduced by [12] (see also [13]).

One may use more equations than the number of unknowns and apply minimizing procedures to get the solution. Maximum indices N_b, N_c, and N_d of the coefficients b_n, c_n, and d_n, which should be included, depend on the specific dynamic thermal properties of a given wall assembly. In general, the total number of numerically significant z-transfer function coefficients increases with resistance and mass of wall; however, it is not the rule. Trying different kinds of cut off of the sequences $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$, one should control the following quantities:

$$E_b = \frac{\sum_{n=0}^{N_c} b_n}{\sum_{n=0}^{N_d} d_n} - 1, \qquad E_c = \frac{\sum_{n=0}^{N_b} c_n}{\sum_{n=0}^{N_d} d_n} - 1. \tag{21}$$

E_b and E_c represent the resultant errors of the z-transfer function coefficients calculations.

The z-transfer function coefficients determined in this way correspond to the selected time step—here 1 h. If a smaller time step is to be used in simulations, say 1/2 h or 15 min, the whole procedure must be repeated.

For plane walls, response factors with sufficiently high indices, above some M, satisfy the condition:

$$\frac{Y_{m+1}}{Y_m} = \frac{X_{m+1}}{X_m} = \text{const} = \alpha, \quad m > M \tag{22}$$

$$\alpha = e^{-1/\tau_1}, \tag{23}$$

where τ_1 is the first, of largest value, time constant of a wall. Therefore, the set of equations (14.1, ..., 14.n+k), (15.1, ..., 15.n+k) for b_n, c_n, and d_n, practically is not infinite in the sense that for sufficiently high indices, successive equations are just the preceding ones multiplied by α. Because α is the root of $D(z)$, the condition $d_1 < -\alpha$ is always satisfied.

Numerically calculated response factors for three-dimensional wall assemblies have, in general, similar properties; however, their ratios show small variations even for large indices, where they drop several orders of magnitude as compared with the first ones.

Solving Eq. (11) for Y_n and X_n, with $d_0 = 1$, gives the recurrence formulae which may be used to additionally verify a solution obtained for z-transfer function coefficients:

$$Y_0 = \frac{b_0}{R}, \qquad X_0 = \frac{c_0}{R}, \tag{24}$$

$$Y_n = \frac{b_n}{R} - \sum_{k=1}^{n} Y_{n-k} d_k, \qquad X_n = \frac{c_n}{R} - \sum_{k=1}^{n} X_{n-k} d_k; \quad n \geq 1. \tag{25}$$

Results of such verification, for a wood-stud wall, are presented in Appendix A.

4. Conduction z-transfer function coefficients for common-wall assemblies

Dynamic thermal properties of 20 common wall assemblies were analyzed within the ASHRAE research project ASHRAE 1145-TRP: “Modeling Two- and Three-dimensional Heat Transfer Through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs” [14,15,17]. The list of wall assemblies considered includes clear walls and details of the wood- and steel-framed wall systems, insulated concrete forms (ICF wall), sandwich walls with metal and plastic ties, and two-core block masonry walls, uninsulated and with insulation inserts. A representative sample of details includes: corners, intersections between above-grade wall, floor, foundation, wall/roof interfaces, and framing-in of walls around windows.

Drawings of all those wall assemblies, with dimensioned simulation areas, are included in the final report of ASHRAE 1145-RP project [14]. They are also available at ORNL Internet site (http://www.ornl.gov/sci/roofs+walls/research/detailed_papers/whole_bldg/index.html). Drawings of the most common structures may be found in [10], chapter 24.

Response factors, resistances, and structure factors were calculated using the finite-difference computer code HEATING 7.2 [16], for boundary conditions of the first kind. Dimensionless, normalized response factors, given as $R{\cdot}X_n$ and $R{\cdot}Y_n$, for the 2 × 4 steel-stud system wall assemblies, are depicted in Figs. 1 and 2. They represent relations of the responses to unit triangular temperature excitations after time $n\delta$, to the steady-state heat flux, due to a unit boundary temperature difference, equal to $1/R$.

The conduction z-transfer function coefficients were determined as approximate solutions of finite sets of equations, generated by Eqs. (14–17). The resultant errors,

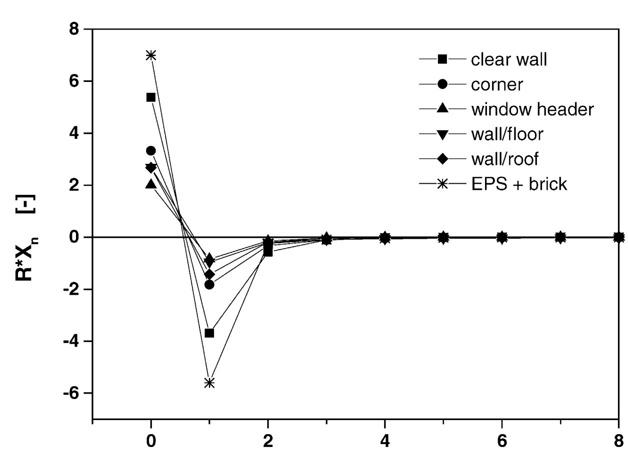

Fig. 1. Dimensionless response factors $R{\cdot}X_n$ for the 2 × 4 steel-stud system wall assemblies.

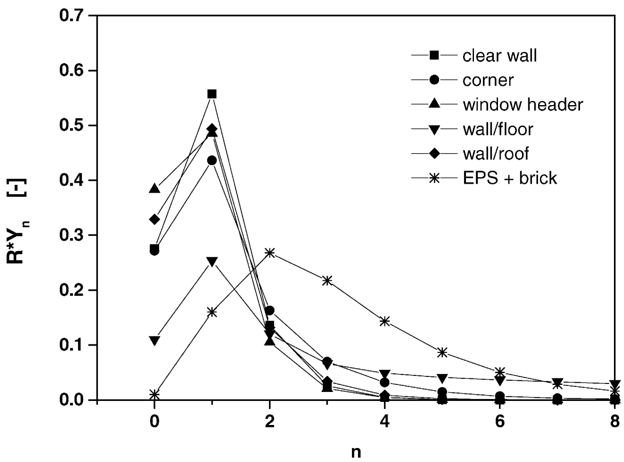

Fig. 2. Dimensionless response factors $R{\cdot}Y_n$ for the 2×4 steel-stud system wall assemblies.

E_b and E_c, were observed, while trying different kinds of cut off of the sequences $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$, and also different kinds of reduction of the number of equations to satisfy compatibility Eq. (13) as well as possible. Modern professional calculation software allows one to easily examine different solutions of a problem, modifying the numbers of unknowns and using minimum-error procedure to find the solution of a set of N linear equations with M variables ($N \geq M$).

The results are collected in Tables 1–3. The maximum index of a coefficient does not exceed five; accuracy is within five decimal digits. Negative values of the coefficients b_n with higher indices, which appear for almost all lightweight wood- and steel-framed wall assemblies, and empty-core concrete masonry blocks, seem questionable at first sight. It was necessary to admit them to satisfy, with sufficient accuracy, the compatibility condition (Eq. (13)). For steel-framed walls with heavy layers, brick or stucco, all b_n are positive. For the coefficients c_n and d_n, the sign sequence is always + and −, alternately. One should take into account, however, that a negative value of some b_n does not mean that impact of some temperature value may be "negative", as temperatures enter into the expression for the current heat flux (Eq. (9)) not only through b_n and c_n, but also through preceding values of the heat flux itself. Comparatively high values of the coefficients c_n are due to the fact that surface-film resistance were not included while calculating response factors.

As an example, let us consider 2×4 wood-stud clear wall (see Table 1). Summing up the z-transfer function coefficients gives:

$$\sum_{n=0}^{4} b_n = 0.30363, \qquad \sum_{n=0}^{4} c_n = 0.30365,$$

$$\sum_{n=0}^{4} d_n = 0.30364, \qquad E_b = -0.00003, \qquad E_c = 0.00003.$$

The errors E_b and E_c (see Eq. (21)) here are very small. (For most cases the errors are below10^{-2}, for several cases below 10^{-5}, maximum error appears for the wood-stud wall/roof intersection, E_b = 0.036). Results of the "reversibility test", using Eqs. (24) and (25), show very good compatibility of the response factors recalculated from z-transfer function coefficients with those originally calculated for three-dimensional models of wall assemblies (see Appendix A). Heat flux test calculations (Fig. 4) show an excellent

Table 1
Wood-stud system—clear walls and details (z-transfer function coefficients for three-dimensional models)

Wall assembly	R value (m^2 K/W) [ft^2 °F h/Btu]	n	0	1	2	3	4
2 × 4 Clear wall	2.00486 [11.39127]	b_n	0.19337	0.24476	−0.15501	0.01954	0.00097
		c_n	7.64880	−12.33863	5.81307	−0.84437	0.02478
		d_n	1.00000	−0.91447	0.23694	−0.01887	0.00004
2 × 6 Clear wall	3.07767 [17.48678]	b_n	0.15322	0.251223	−0.28426	0.06269	0.00765
		c_n	10.64983	−20.93661	12.97188	−2.59789	0.06460
		d_n	1.00000	−1.25323	0.45295	−0.04808	0.00018
2 × 4 Corner	1.84391 [10.47675]	b_n	0.19209	0.20133	−0.14086	0.00992	
		c_n	6.43935	−9.92949	4.08246	−0.32742	
		d_n	1.00000	−0.93234	0.20719	−0.01098	
2 × 4 Window header	1.65508 [9.40385]	b_n	0.10141	0.16607	−0.01517	0.01302	
		c_n	5.05305	−7.80490	3.47950	−0.46234	
		d_n	1.00000	−0.99046	0.27744	−0.02166	
2 × 4 Wall/floor	1.72387 [9.79471]	b_n	0.08635	0.14395	−0.00347	0.00869	
		c_n	5.34251	−7.49327	2.44246	−0.05226	
		d_n	1.00000	−0.89680	0.13624		
2 × 4 Wall/roof	1.65447 [9.40040]	b_n	0.09812	0.14358	−0.01307	0.00500	
		c_n	7.09777	−9.93976	3.20196	−0.11769	
		d_n	1.00000	−0.92645	0.16872		

Table 2
Steel-stud system–clear walls and details (z-transfer function coefficients for three-dimensional models)

Wall assembly	R value (m^2 K/W) [ft^2 °F h/Btu]	n	0	1	2	3	4	5
2 × 4 Steel-stud clear wall	1.54809 [8.79595]	b_n	0.27544	0.46780	−0.03740	−0.00348		
		c_n	5.38342	−5.43867	0.77712	−0.01951		
		d_n	1.00000	−0.32450	0.02704	−0.00018		
2 × 4 Corner	0.92953 [5.28141]	b_n	0.27182	0.23733	−0.11860	0.00796	0.00019	
		c_n	3.32708	−4.25094	1.46144	−0.14221	0.00333	
		d_n	1.00000	−0.73120	0.13649	−0.00664	0.00004	
2 × 4 Window header	1.54924 [8.80252]	b_n	0.38340	0.40350	0.00274			
		c_n	2.00592	−1.25812	0.04204			
		d_n	1.00000	−0.21374	0.00358			
2 × 4 Wall/floor	0.66806 [3.79581]	b_n	0.10985	0.07647	−0.20620	0.05227	0.00743	
		c_n	2.69068	−5.29400	3.35848	−0.74658	0.03217	
		d_n	1.00000	−1.61234	0.75266	−0.10020	0.00062	
2 × 4 Wall/roof	0.43190 [2.45400]	b_n	0.32878	0.33525	−0.08624	−0.00082		
		c_n	2.67848	−2.71613	0.65392	−0.03925		
		d_n	1.00000	−0.48144	0.05880	−0.00035		
2 × 4 Steel stud + 1-in EPS + brick	2.25132 [12.79160]	b_n	0.01014	0.15042	0.11515	0.00060		
		c_n	6.99557	−12.38192	7.07266	−1.50306	0.09593	
		d_n	1.00000	−0.96989	0.26922	−0.02015		
2 × 6 Steel-stud clear wall	1.99120 [11.31363]	b_n	0.20008	0.48490	−0.02034	−0.00691		
		c_n	6.90969	−7.48203	1.26283	−0.03276		
		d_n	1.00000	−0.37815	0.03605	−0.00017		
2 × 6 steel-stud + EPS + stucco	2.66764 [15.15703]	b_n	0.10508	0.34909	0.01933			
		c_n	8.12097	−11.41045	4.10657	−0.33939	0.00108	
		d_n	1.00000	−0.57984	0.05862			
2 × 6 steel stud + EPS + brick	2.72190 [15.46532]	b_n	0.00820	0.14244	0.12483	0.00222		
		c_n	8.28614	−14.88340	8.59382	−1.82738	0.10926	−0.00005
		d_n	1.00000	−0.97089	0.26764	−0.01836		

agreement of the results obtained using three-dimensional response factors and three-dimensional z-transfer function coefficients.

Some general conclusions concerning dynamic thermal properties represented by the response factors and the z-transfer function coefficients for subsequent types of walls are as follows:

Wood-framed wall system. Differences in dynamic thermal properties of clear walls and separated details are rather small; resistance differences are more significant.

Table 3
Wall assemblies constructed of concrete and insulation (z-transfer function coefficients for three-dimensional models)

Wall assembly	R value (m^2 K/W) [ft^2 °F h/Btu]	n	0	1	2	3	4
ICF-wall	1.97768 [11.23044]	b_n	0.00333	0.01647	0.00388	0.00022	
		c_n	12.44518	−24.90756	14.40928	−1.92311	0.00012
		d_n	1.00000	−1.16082	0.18473		
Sandwich wall with metal ties	1.34800 [7.65912]	b_n	0.02377	0.24051	0.14861	0.00778	
		c_n	35.22708	−49.09367	15.08613	−0.79797	
		d_n	1.00000	−0.70113	0.12272	−0.00002	
Sandwich wall with plastic ties	1.86246 [10.58216]	b_n	0.01576	0.22377	0.16632	0.00908	
		c_n	50.74143	−71.97368	23.01010	−1.36291	
		d_n	1.00000	−0.71157	0.12653	−0.00002	
Empty concrete blocks	0.23857 [1.35549]	b_n	0.19225	0.14069	−0.16884	0.04993	0.00318
		c_n	5.98628	−10.21773	5.26507	−0.84659	0.03018
		d_n	1.00000	−1.00519	0.23533	−0.01301	0.00009
Insulated concrete blocks	0.40328 [2.29137]	b_n	0.04707	0.13063	0.01402	0.03217	0.00389
		c_n	9.72126	−16.99955	8.85120	−1.39633	0.05121
		d_n	1.00000	−0.98904	0.23027	−0.01356	0.00012

Steel-framed wall system. Differences in dynamic thermal properties of clear walls and separated details are more significant here. The effect of an additional layer of brick or stucco, together with an EPS foam layer, is substantial.

ICF wall, with the complex internal three-dimensional concrete core. With limited amount of concrete, it shows dynamic thermal properties, which are specific for very heavy structures; Y_n response factors (and also X_n response factors with high indices) decay very slowly, and b_n coefficients are very small. For the heat flow simulations (to secure sufficient accuracy) one should use much more than 40 response factors; at the same time, maximum index of b_n, c_n, and d_n is, respectively, only three, four, and two.

Concrete/foam/concrete sandwich walls. Resistance for the wall with plastic ties is significantly higher than for the similar wall with metal ties; however, dynamic properties are similar.

Heavy concrete blocks. Dynamic thermal properties of empty-core masonry blocks are essentially different as compared with those for blocks filled with insulation.

5. Test simulations

The test simulations were performed using, as the outside surface temperature excitation, the sol–air temperature calculated for a vertical surface facing west for a sunny day of February in Warsaw (see Fig. 3). The sol–air temperature (see ASHRAE Handbook) represents combined effect of the air temperature and solar radiation in problems with linear boundary conditions of the third kind. We used it here just as an example of rapidly varying thermal excitation. Temperature at the inside surface of a wall assembly represented periodic variations with amplitude of 1 °C [1.8 °F], around mean value of 20 °C [68 °F]. The same daily temperature courses were repeated several times, to eliminate the effect of initial conditions.

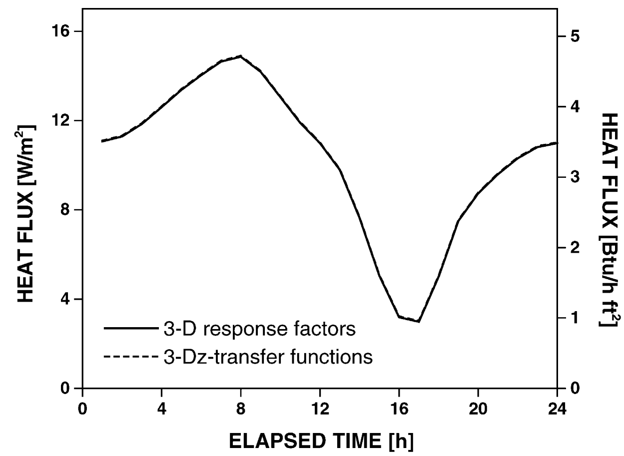

Fig. 4. Comparison of the heat-flux simulation results for the 2 × 4 wood-framed system clear wall.

The heat flux across the inside surface of a wall assembly was simulated in two ways, using the response factors for a 3-D model and the z-transfer function coefficients, derived from the response factors. The results of simulations for the 2 × 4 wood-framed clear wall are presented in Fig. 4. Differences between the heat flux values, calculated using the 3-D response factors and the 3-D z-transfer function coefficients, are almost invisible. Fig. 5 presents the results of simulations for the corner; differences here are also very small.

Figs. 6–8 present the heat flux courses for the 2 × 4 steel-stud system clear wall, the corner, and the wall/floor intersection. Differences are significant only for the last case.

Fig. 9 presents the results of simulations for the 2 × 6 steel-stud wall covered with 1-in. EPS foam and brick, Fig. 10 for the massive concrete blocks, filled with

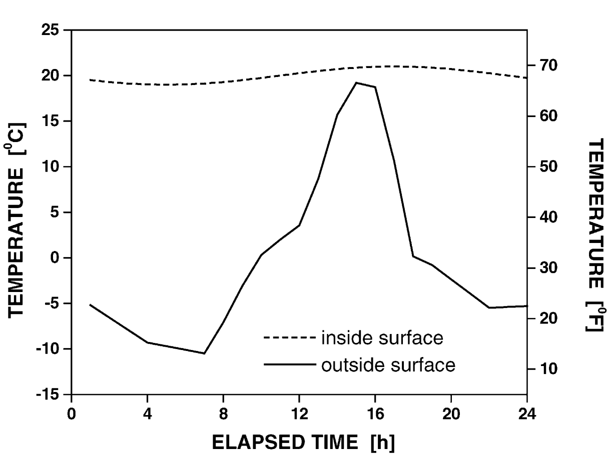

Fig. 3. Inside and outside surface temperature courses used for simulations.

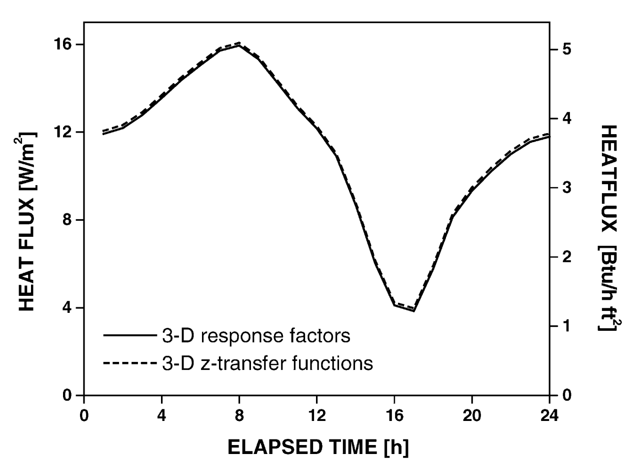

Fig. 5. Comparison of the heat-flux simulation results for the corner; 2 × 4 wood-framed wall system.

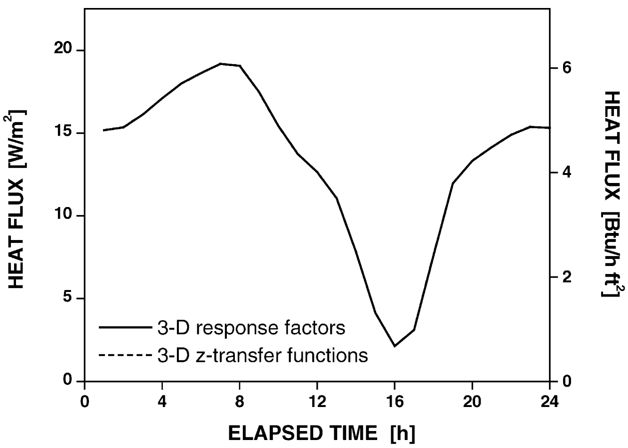

Fig. 6. Comparison of the heat-flux simulation results for the 2 × 4 steel-stud system clear wall.

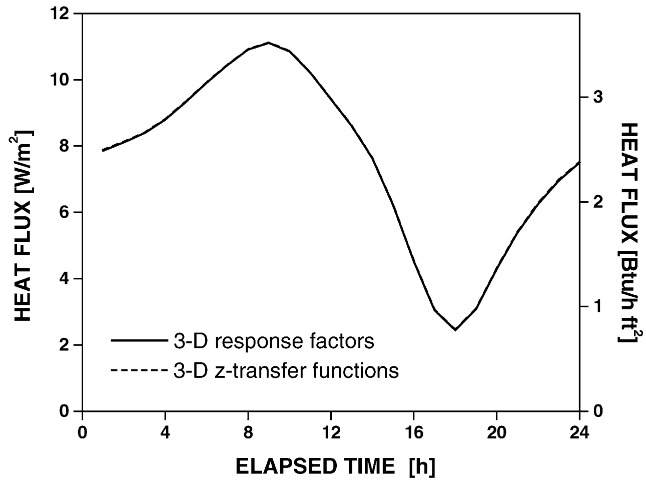

Fig. 9. Comparison of the heat-flux simulation results for the 2 × 6 steel-stud wall with 2.5 cm [1 in.] EPS foam and brick.

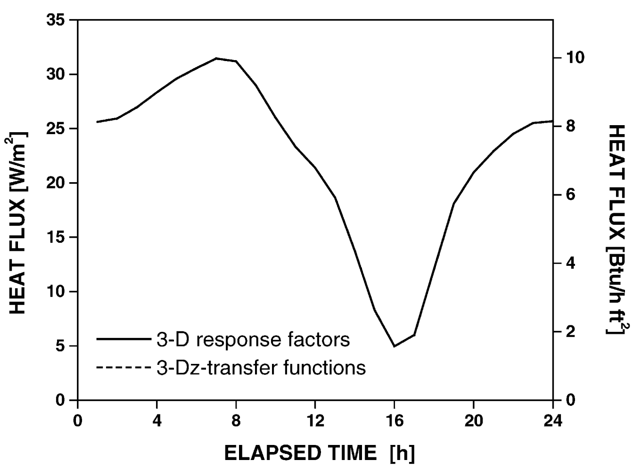

Fig. 7. Comparison of the heat-flux simulation results for the corner; 2 × 4 steel-stud wall system.

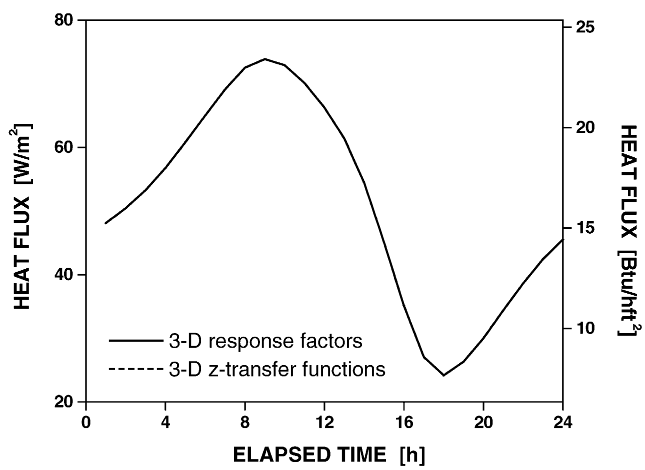

Fig. 10. Comparison of the heat-flux simulation results for the insulated concrete blocks.

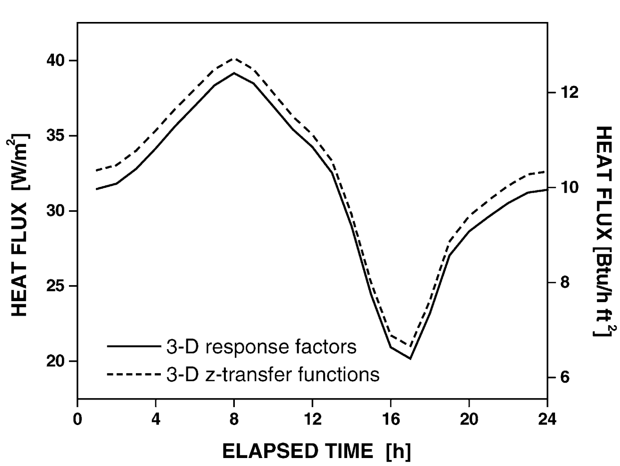

Fig. 8. Comparison of the heat-flux simulation results for the 2 × 4 steel-stud system wall/floor intersection.

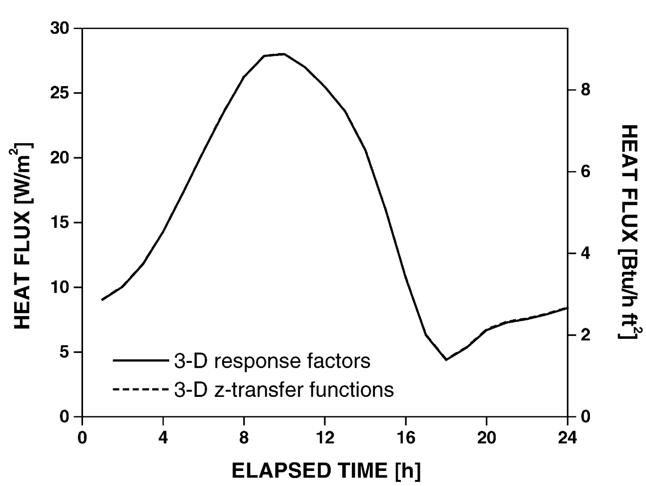

Fig. 11. Comparison of the heat-flux simulation results for the sandwich wall with metal ties.

insulation, and Fig. 11 for the sandwich wall with metal ties. Compatibility of the heat-flux courses here also is excellent.

6. Conclusions

The method of derivation of the conduction z-transfer function coefficients from response factors for three-dimensional wall assemblies gives satisfactory results.

The list of 20 wall assemblies analyzed includes: clear walls and details of the wood- and steel-framed wall systems, insulated concrete forms (ICF wall), sandwich walls with metal and plastic ties, and two-core block masonry walls, with or without insulation inserts.

The response factors for three-dimensional models were calculated with the help of the finite-difference computer code HEATING 7.2, for boundary conditions of the first kind. They were used as ‘‘input data’’ to determine the z-transfer function coefficients from the primarily infinite set of linear equations, which includes relationships with response factors and compatibility conditions. For each case, different kinds of cut off were considered, and minimum-error procedure was applied while seeking for the solution to satisfy, as best as possible, the compatibility conditions.

With accuracy of five decimal digits, the maximum index of a z-transfer function coefficient does not exceeds five. It was necessary to admit negative values of the coefficients b_n with higher indices for the lightweight wood- and steel-framed wall assemblies, and for the empty concrete blocks, to satisfy, with sufficient accuracy, compatibility equations. For the coefficients c_n and d_n, the sign sequence is always + and −, alternately. The ‘‘reversibility test’’ shows very high accuracy in reproducing response factors from the z-transfer function coefficients.

Test simulations, performed for a continuously varying temperature excitation of high amplitude, show perfect compatibility of the heat flux calculated using the 3-D response factors and the 3-D z-transfer function coefficients, derived from the response factors.

Appendix A

Comparison of the original dimensionless 3D response factors with those recalculated from the z-transfer function coefficients, for the 2 × 4 wood-stud wall

n	Dimensionless response factors $R{\cdot}X_n$			Dimensionless response factors $R{\cdot}Y_n$		
	Recalculated from c_n, d_n	$R{\cdot}X_n$ original values	$R{\cdot}\Delta X_n$	Recalculated from b_n, d_n	$R{\cdot}Y_n$ original values	$R{\cdot}\Delta Y_n$
0	7.6488000	7.6488030	−0.0000027	0.1933700	0.1933737	−0.0000037
1	−5.3440320	−5.3440460	0.0000146	0.4215911	0.4215949	−0.0000039
2	−0.8861932	−0.8878742	0.0016811	0.1847053	0.1847107	¯0.0000054
3	−0.2442194	−0.2442498	0.0000304	0.0922046	0.0922092	−0.0000046
4	−0.0897458	−0.0897582	0.0000124	0.0494752	0.0494763	−0.0000011
5	−0.0406983	−0.0406981	−0.0000002	0.0268640	0.0268632	0.0000008
6	−0.0205236	−0.0205199	−0.0000036	0.0145756	0.0145795	−0.0000038
7	−0.0108082	−0.0108069	−0.0000013	0.0078935	0.0078996	−0.0000061
8	−0.0057850	−0.0057861	0.0000010	0.0042696	0.0042756	−0.0000060
9	−0.0031149	−0.0031169	0.0000020	0.0023080	0.0023128	−0.0000048
10	−0.0016808	−0.0016829	0.0000020	0.0012473	0.0012508	−0.0000035
11	−0.0009077	−0.0009094	0.0000017	0.0006740	0.0006764	−0.0000024
12	−0.0004904	−0.0004916	0.0000012	0.0003642	0.0003657	−0.0000016
13	−0.0002649	−0.0002658	0.0000009	0.0001968	0.0001978	−0.0000010
14	−0.0001431	−0.0001437	0.0000006	0.0001063	0.0001069	−0.0000006
15	−0.0000773	−0.0000777	0.0000004	0.0000574	0.0000578	−0.0000004
16	−0.0000418	−0.0000420	0.0000002	0.0000310	0.0000313	−0.0000002
17	−0.0000226	−0.0000227	0.0000001	0.0000168	0.0000169	−0.0000001
18	−0.0000122	−0.0000123	0.0000001	0.0000091	0.0000091	−0.0000001
19	−0.0000066	−0.0000066	0.0000001	0.0000049	0.0000049	−0.0000001
20	−0.0000036	−0.0000036	0.0000000	0.0000026	0.0000027	−0.0000000
	Mean standard deviation σ_X = 0.000367			Mean standard deviation σ_Y = 0.0000032		

References

[1] D.G. Stephenson, G.P. Mitalas, Calculation of heat conduction transfer functions for multi-layer slabs, ASHRAE Transactions 77 (2) (1971) 117–126.

[2] J.E. Seem, S.A. Klein, W.A. Beckman, J.W. Mitchell, Transfer functions for efficient calculation of multidimensional heat transfer, Journal of Heat Transfer 111 (1989) 5–12.

[3] J.E. Seem, S.A. Klein, W.A. Beckman, J.W. Mitchell, Comprehensive room transfer functions for efficient calculation of the transient heat transfer processes in buildings, Journal of Heat Transfer 111 (1989) 264–273.

[4] D.M. Burch, R.R. Zarr, B.A Licitra., 1990, A Dynamic Test Method for Determining Transfer Function Coefficients for a Wall Specimen Using a Calibrated Hot Box, Insulation Materials, Testing and Applications, ASTM STP 1030, Philadelphia.

[5] D.M. Burch, J.E. Seem, G.N. Walton, B.A. Licitra, Dynamic evaluation of thermal bridges in a typical office building, ASHRAE Transactions 98 (1992) 1.

[6] W.C. Brown, D.G. Stephenson, A guarded hot-box procedure for determining the dynamic response of full-scale wall specimens, ASHRAE Transactions 99 (1993), Part I 632–642, Part II 643–660.

[7] T. Kusuda, Thermal response factors for multi-layer structures of various heat conduction systems, ASHRAE Transactions 75 (1) (1969) 241–271.

[8] J.A. Clarke, Energy Simulation in Building Design, Adam Hilger Ltd., 1985.

[9] E.I. Jury, Theory and Application of the *Z*-transform Method, John Wiley & Sons, Inc., New York, 1964.

[10] ASHRAE Handbook: Fundamentals, ASHRAE Inc., Atlanta, Georgia, USA, 1989, 1997.

[11] E. Kossecka, Relationships between structure factors, response factors and *z*-transfer function coefficients for multi-layer walls, ASHRAE Transactions 104 (1A) (1998) 68–77.

[12] B.R. Anderson, The measurement of *U*-values on site, in: Proceedings of the ASHRAE-DOE-BTECC Conference on Thermal Performance of the Exterior Envelopes of Buildings III, Clearwater Beach, Florida, 2–5 December 1985.

[13] ISO/DIS 9869.2. 1991, International Standard, thermal insulation – building elements – in situ measurement of thermal resistance and thermal transmittance, International Organization for Standardization.

[14] Enermodal Engineering Limited, 2001, Modeling two- and three-dimensional heat transfer through composite wall and roof assemblies in hourly energy simulation programs, ASHRAE 1145-TRP Final Report, Part I, Part II, Library of Wall Assemblies, prepared for ASHRAE Inc., Atlanta, Georgia.

[15] E. Kossecka, J. Kosny, Conduction *z*-transfer function coefficients for common composite wall assemblies, in: Proceedings of the Conference on Thermal Performance of the Exterior Envelopes of Buildings VIII, Clearwater Beach, Florida, 2–6 December 2001.

[16] Childs K.W., 1993, HEATING 7.2 User's Manual, ORNL/TM-12262, Oak Ridge National Laboratory, Oak Ridge, TN.

[17] S. Carpenter, J. Kosny, E. Kossecka, Modeling transient performance of 2- and 3-d building assemblies: ASHRAE 1145-RP, ASHRAE Transactions 109 (1) (2003) 566–571.

[18] E. Kossecka, J. Kosny, Relations between structural and dynamic thermal characteristics of building walls, in: Proceedings of the 1996 International Symposium of CIB W67 on Energy and Mass Flows in the Life Cycle of Buildings, Vienna, 4–10 August 1996, pp. 627–632.

[19] E. Kossecka, J. Kosny, Equivalent wall as a dynamic model of the complex thermal structure, Journal of Thermal Insulation and Building Envelopes 20 (1997) 249–268.