Identification of moving loads using the ℓ_1 norm minimization

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Abstract

This contribution deals with the inverse problem of indirect identification of moving loads. The identification is performed based on the recorded response of the loaded structure and its numerical model. A specific feature of such problems is a very large number of the degrees of freedom (DOFs) that can be excited and a limited number of available sensors. As a result, unless the solution space is significantly limited, the identification problem is underdetermined: it has an infinite number of exact, observationally indistinguishable solutions. We propose an approach based on the assumption of sparsity of the excitation, which can be expressed in the form of a requirement of a bounded l_1 norm of the solution. As long as the loads are sparse, the approach allows them to be freely moving, without the usual assumption of a constant velocity. We test the approach in a numerical example with 10% rms measurement noise and describe an experimental setup that is being prepared to perform experimental verification.

Keywords: inverse problems, structural mechanics, moving load identification, sparsity, l1 norm.

1. Introduction

The two main problems in the area of structural health monitoring (SHM) are monitoring for damages and indirect identification of loads. In general terms, these problems correspond respectively to the inverse problems of the first type (system identification) and the second type (input identification).

This contribution is devoted to indirect identification of moving loads based on the recorded responses of the loaded structure. Such a problem has been intensively studied and there is a number of extensive reviews, see, e.g., [1,2]. The problem is important in assessment of pavements and bridges, in traffic monitoring and control, as well as a prerequisite for structural control [3-5]. A specific feature of such problems is a very large number of the degrees of freedom (DOFs) that can be excited by the load and a limited number of sensors available to measure the response. As a result, unless the solution space is significantly limited, the identification problem is underposed: it has an infinite number of exact solutions. In most of the published research the solution space is limited by the assumption of a single vehicle that moves at a constant velocity. Such approaches have led to very good results; however, they significantly restrict the generality of the load being identified and exclude, e.g., freely moving loads and multiple loads.

We propose an approach based on the assumption of sparsity of the excitation, which is well-tailored to practice: even if there are multiple loads, at each time instance only a very limited number of structural DOFs is usually excited. Such an approach fits into the recent research stream on compressed sensing [6], which includes SHM-related application areas such as damage identification [7] or identification of impact load position [8]. To our best knowledge, the concept has not been applied for identification of moving loads. The assumption of sparsity is usually expressed as a requirement of a bounded l_1 norm of the solution [9]. As long as the loads are sparse, the approach allows them to be identified even if they are freely moving or multiple, that is without the usual assumption of a constant velocity.

2. Load identification

2.1. The direct problem

The structure is modelled by means of the finite element (FE) method and is assumed to satisfy the equation of motion in its standard form with zero initial conditions:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f},\ \boldsymbol{x}(0) = \boldsymbol{0},\ \dot{\boldsymbol{x}}(0) = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the structural mass, damping and stiffness matrices, and $\boldsymbol{f}$ represents the excitation of the freely moving load(s). Since the model is linear, responses of linear sensors (such as strain gauges) can be stated in the convolution form,

$$\boldsymbol{\epsilon}(t) = \int_0^t \overline{\boldsymbol{B}}(t-\tau)\boldsymbol{f}(\tau)\mathrm{d}\tau, \tag{2}$$

where the matrix $\boldsymbol{B}$ collects the structural impulse response functions. After time discretization, Eqn (2) takes the form

$$\boldsymbol{\epsilon} = \boldsymbol{B}\boldsymbol{f}, \tag{3}$$

where the vectors $\boldsymbol{\epsilon}$ and $\boldsymbol{f}$ collect all the responses and excitations in all time instances, and $\boldsymbol{B}$ is a block Toeplitz matrix that represents the discretized form of the convolution operator.

2.2. The inverse problem

A discrete set of points that can be visited by the moving load is selected. The set needs to be dense enough to represent the load trajectory. Let vector $\boldsymbol{p}$ collect force excitations in all these points and in all time instances, and let $\boldsymbol{N}$ be the allocation matrix which allocates the points to structural DOFs. Notice that the number of points might be smaller than the number of structural DOFs and that the points need not be aligned with specific DOFs (they are allocated to the DOFs of the involved FE using its shape functions). As a result, Eqn (3) takes the form

$$\boldsymbol{\epsilon} \approx \boldsymbol{B}\boldsymbol{N}\boldsymbol{p}, \tag{4}$$

that is the vector $\boldsymbol{f}$ is approximated by assuming that the load interacts with the structure only in the selected points, $\boldsymbol{f} \approx \boldsymbol{N}\boldsymbol{p}$. In general, load identification is equivalent to solution of Eqn (4).

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2.3. *Assumption of sparsity and the l_1 norm minimization*

In practical cases the length of the excitation vector $\boldsymbol{p}$ is much larger than the length of the measurement vector $\boldsymbol{\epsilon}$, so that there are infinitely many solutions. To obtain a unique solution, an additional knowledge about the load has to be used to constrain the solution space. Usually, a single load is assumed with known trajectory and velocity. Here, we propose an assumption of sparsity, which can be expressed through the l_1 norm as the task of minimization of the following weighted objective function [9]:

$$\boldsymbol{F}(\boldsymbol{p}) = \|\boldsymbol{\epsilon} - \boldsymbol{BNp}\|^2 + \alpha\|\boldsymbol{p}\|_1, \tag{4}$$

where the coefficient α weights the importance of sparsity.

3. Numerical example

The measuring section of the experimental stand, see Figs. 2 and 3, is modelled as a 2D FE beam. The material, geometric and excitation parameters are tuned to represent the experimental setup. The beam is divided into 21 finite elements. The moving load is assumed to excite the beam vertically in 15 equally spaced points. The strain measurements in the four points of the beam (Fig. 3) are simulated numerically and contaminated with a 10% rms uncorrelated Gaussian noise. The number of unknowns in Eqn (4) is thus equal to $15n_t$, and they have to be identified based on only $4n_t$ measurements (n_t is the number of the time steps).

Figure 1 presents an exemplary result obtained by means of the L1packv2 [10] and Wolfram Mathematica. Two moving mass loads are simulated. Their actual trajectories are marked by the orange and green curves, while the identification result is shown in the form of the density plots. A good qualitative agreement is evident. More results will be shown during the conference, including those based on the experimental measurements.

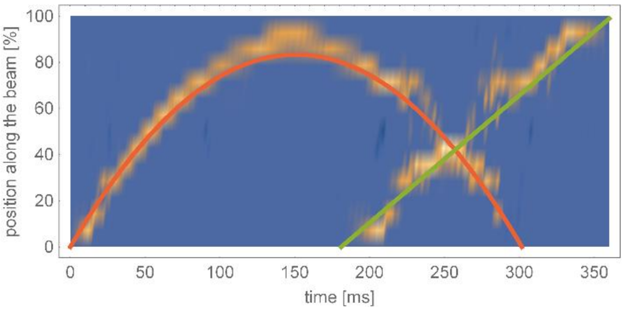

Figure 1: Numerical example, identification of two moving loads

4. Experimental stand

An experimental stand is built to measure strains in chosen points of a simply supported beam loaded by a moving mass. The beam has the form of a steel plate (0.55m x 0.03m x .0015m) with a fixed support on the beginning and a rolling support on the end. The beam with supports is installed on thick wall steel profiles, see Fig. 2. The moving mass is simulated by a steel roller. The roller has a groove on its diameter to provide a better guidance when travelling along the beam. The moving mass is accelerated on a inclined plane with an adjustable angle, which allows to get various speeds of the moving mass.

The structural response to the moving mass is measured in four points across the beam by strain gauges, see Fig. 3. A half-bridge is used and the gauges are installed on the top and bottom surface of the beam. Additionally, two optoelectronics sensors are installed in order to record the speed of the traveling mass.

Figure 2: Front view of the experimental stand

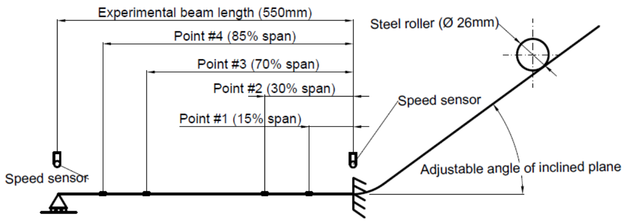

Figure 3: Schematic draft of the experimental stand. Four points with strain gauges are marked

5. Conclusions

This contribution proposes a method for identification of freely moving multiple loads based on the assumption of sparsity. Experimental results will be shown during the conference.

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