A constitutive model and numerical simulation of sintering processes at macroscopic level

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Abstract

This document presents modelling of single-phase powder sintering processes at the macroscopic level. In particular, its constitutive formulation, numerical implementation and numerical test are described. Numerical tests were carried out for a cylindrical specimen under hydrostatic and uniaxial pressure. Results of macroscopic analysis are compared against the microscopic model results.

Keywords: sintering porcesses, numerical analysis, multiscale modelling

1. Introduction

The paper presents macroscopic model of single-phase powder sintering processes and its numerical implementation developed within the framework of a MUSINT project, realized at the Institute of Fundamental Technological Research, Warsaw, Poland. The overall objective of the MUSINT (Multiscale numerical modelling of sintering processes) is development of numerical models allowing to analyse at various scales manufacturing processes employing sintering as the main technological stage. The paper also shows numerical predictions related to experimental results. The macroscopic constitutive model is based on the assumption that the sintered material is a continuous medium. The parameters of the constitutive model for material under sintering are determined by simulation of sintering at the microscopic level using a micro-scale model. Moreover, numerical simulations are validated by comparison with experimental results. The simulations and preparation of the model are carried out by Abaqus FEA - a software for finite element analysis and computer-aided engineering. A mechanical model is defined by the user procedure "Vumat" which is developed by the first author in Fortran programming language.

2. Problem formulation

2.1. Microscopic model

A microscopic model of sintering based on a new generalized viscoelastic model of sintering proposed by Nosewicz [1] is developed within the framework of the discrete element method. The model is schematically depicted in Figure 1. Thermo-viscoelastic model of sintering is introduced as an author's original extension of viscous model. The rheological model of sintering presented in Figure 1 is enriched by adding elastic and thermal component to the standard viscous model. An addition of elastic part will allow to better redistribute forces in particle assembly with large changes of configuration during sintering. In view of an analogy of a sintered material with a fluid, whose viscoelastic properties are commonly modelled with the viscoelastic Maxwell model, the viscous component is introduced in a series with the elastic element. Furthermore, the rheological model includes the thermal component that allows to include thermal expansion or thermal stresses occurring during temperature evolution.

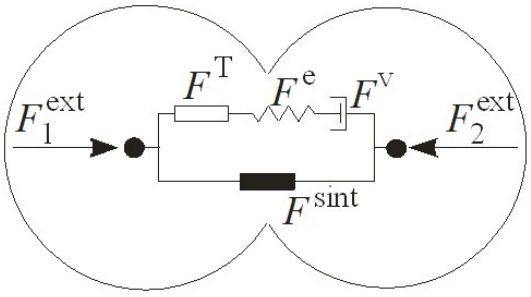

Figure 1: Rheological scheme of thermo-viscoelastic model [1]

The results of the simulations at the microscopic level will be used to determine parameters of the macroscopic model in multiscale simulations.

2.2. Macroscopic model

At the macro scale sintered material is treated as continuous medium. Its constitutive model is tightly related to obser-

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vations made at the micro scale (single grain level) and fully consistent with the model describing in the microscopic mechanical phenomena of the sintering process. The macroscopic model is derived with the use of methodology known from works of e.g. Zhang [2]. The total strain rate is a sum of three parts: thermal, elastic and viscous, as follows:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{th} + \dot{\boldsymbol{\varepsilon}}^{e} + \dot{\boldsymbol{\varepsilon}}^{visc} \tag{1}$$

Thus, the constitutive equation has the form:

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{th} - \dot{\boldsymbol{\varepsilon}}^{visc}\right) + \dot{\boldsymbol{\sigma}}^{s} \tag{2}$$

where:

- thermal strain

$$\boldsymbol{\varepsilon}^{th} = \alpha T \mathbf{I} \tag{3}$$

- viscous strain rate

$$\dot{\boldsymbol{\varepsilon}}^{visc} = \frac{\boldsymbol{\sigma}'}{2\eta_s} + \frac{tr(\boldsymbol{\sigma}) - 3\sigma_s}{9\eta_b}\mathbf{I}, \tag{4}$$

- sintering stress

$$\boldsymbol{\sigma}^{s} = \sigma_s \mathbf{I} \tag{5}$$

$$\sigma_s = \frac{\beta\gamma_s\hat{\rho}^2}{r_0}\frac{1}{\left[(1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)\right]^2} \tag{6}$$

All parameters in the above equations — viscosity η_s, η_b, relative density $\hat{\rho}$, surface energy γ_s etc. — are related to microscopic material properties. They may also be functions of temperature. These relations are subject to investigation in our research.

2.3. Numerical simulations

In order to verify the model and its numerical implementation, numerical analyses of sintering were performed. Example of the Finite Element Method (FEM) model is shown in the Figure 2. In this model following displacement boundary conditions were assumed: radial displacements on the lateral wall and all displacements at the bottom are zero.

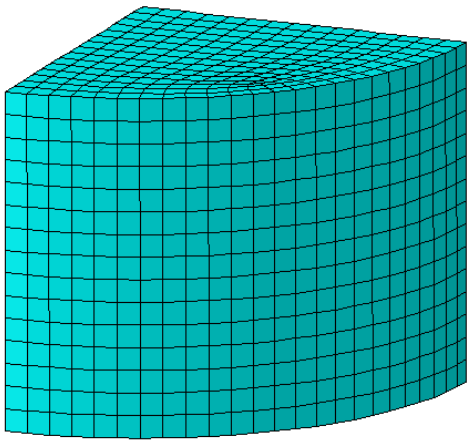

Figure 2: FEM model of sintering.

At very first stage, processes under hydrostatic pressure and under uniaxial compression of a cylindrical specimen were simulated, both excluding thermal effects. As a result shrinkage of the element and so on change of relative density were obtained. Evolution of relative density is presented in the Figure 3 and Figure 4. Figure 4 presents comparison of the FEM simulation against results of the Discrete Element Method analysis with the microscopic model implemented. In this figure one can see a test for various pressure load: 5, 15 and 30 MPa and for example "micro 5MPa" or "macro 5MPa" refers to the micro and macro model under load 5 MPa respectively.

Further numerical results will be presented during the Conference.

3. Conclusion

The preliminary tests show that the presented model is qualitatively valid and the numerical implementation is correct and

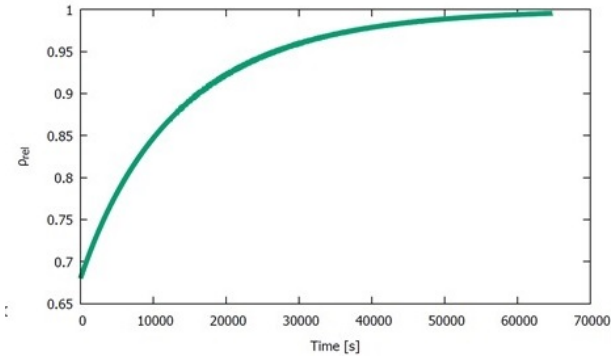

Figure 3: Relative density evolution in time - hydrostatic experiment

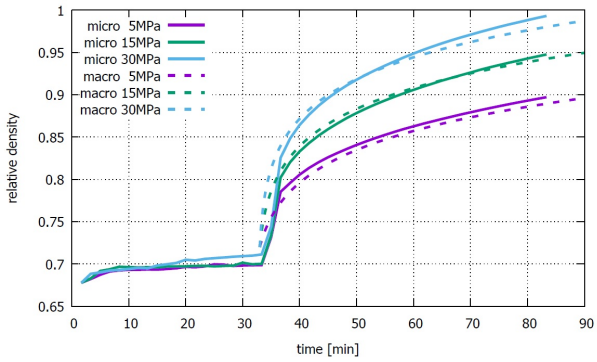

Figure 4: Relative density evolution in time at the uniaxial compression - comparison with numerical results of micro model analysis

therefore calibration of the model is required in order to achieve quantitative agreement. Futhermore the model has to be extended to simulate all other stages of powder metallurgy, in particular to take into account heating and cooling stage.

References

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