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load transferring mechanism between the matrix and the filler. Single layer graphene sheets (SLGS) can be modeled using atomistic simulation techniques that are computationally expensive and cannot be used effectively for assessing the properties of graphene reinforced nanocomposites. In the present work a continuum mechanics approach is followed for the modeling of graphene. A methodology, proposed in our previous work, is used for the calculation of the effective properties of an equivalent shell element (ESE) that can accurately represent both the membrane and the plate behavior of the SLGS, regardless its size. The interfacial load transferring mechanism between the SLGS and the polymer matrix is simulated by use of a cohesive zone model (CZM) that successfully captures delamination and debonding phenomena. The parameters' values of the traction separation law implemented in the CZM cannot be uniquely chosen due to the dispersion of the available experimental data and their effect on the mechanical properties of the nanocomposite is investigated through an extensive parametric study. Representative volume elements subjected to cyclic loading are used for the estimation of expected nanocomposite properties.

**T.4.6** - *Aerodynamic Heating of Ballistic Missile Fin Configuration During Supersonic Flight Conditions* - Stevan Maksimović, Ognjen Ognjanović, M. Maksimović, I. Vasović

This work considers aerodynamic heating and aero-thermo-mechanical analysis of fin type structures on the missile at supersonic flight. At high Mach number the heat due to friction between body and flow, i.e. viscous heating must be taken into account because the velocity field is coupled with the temperature field. The flow field around the fins of the missile and especially the temperature distribution on its surface, as well as aerodynamic-thermal/structural analyses are numerically modeled in ANSYS Workbench environment. Detail investigation was carried out for two Mach numbers at supersonic flight. Available structural experimental results have been used for computational structural mechanics validation and verification, in order to assure credibility of numerical fluid-thermal-structure interaction. In this work a multidisciplinary framework for numerical aerodynamic-thermal/structural analyses, based on only one multi-module software, was used to analyse thermal effects on fin structure during supersonic flights conditions.

**T.4.7** - *Thermo-Mechanical Numerical Analysis of Transformation-Induced Stress Relaxation During Pseudoelastic Behavior of SMA* - Vladimir Dunić, Radovan Slavković, Elżbieta Pieczyska

A stress relaxation phenomenon is observed by coupled thermo-mechanical numerical analysis of SMA subjected to uniaxial test. The thermo-mechanical coupling is realized in the partitioned approach. The software components for the structural analysis (PAKS) and the heat transfer (PAKT) based on the Finite Element Method (FEM) have been used. The latent heat production is correlated with the amount of the martensitic volume fraction. The thermo-mechanical numerical analysis of a belt type specimen has been investigated for the strain controlled loading with the break during the martensitic transformation. The thermally induced martensitic transformation induced the significant stress change during the loading break what was expected according to the experimental results from literature.

**T.4.8** - *Solving Contact Problems Using One-dimensional Finite Elements as Elastic Supports and Application in Angioplasty and Stent Deployment Modeling* - Velibor Isailović, Nenad Filipović, Miloš Kojić

Solving contact problem of two or more solid bodies is a very challenging issue. Contact problems occur very often in engineering, for instance: collision of two cars in crash tests, tire rolling, metal forming processes, coupled engineering parts with clearance, transition or interference fit, etc. There are several approaches that can be implemented in order to solve this problem (Wriggers 2008). This paper shows procedure how simple one-dimensional finite elements can be used as an elastic supports with a goal to provide specific boundary condition between two or more solid bodies. The basic idea is to analyze positions of nodes placed on outer body surface according to the other body surfaces and to add additional stiffness in nodes which are in contact with certain faces of some other solid body. This methodology is general and can be applied to modeling of angioplasty endovascular procedure or modeling of medical stent deployment problem. Initial results of angioplasty numerical model are given in the results section.

**T.4.9** - *Bursting Oscillations on Multiple Time Scales: Quantitative Techniques for Two Types of Systems with Cubic Nonlinearity* - Ivana Kovačić

This work is concerned with bursting oscillations – a kind of mixed-mode oscillations in which fast flows occur along periodically changeable slow flows. The corresponding governing equation stems from a non-autonomous oscillator, with a harmonic external excitation that has a low-valued angular frequency. Two cases regarding the existence and influence of cubic geometric nonlinearity are considered: a bistable one and a pure cubic case. Their distinctive characteristics in terms of the corresponding S-curve and the related slow and fast flows are discussed first. Then, two different quantitative techniques are presented to obtain their slow and fast flows as well as the overall response.

**T.4.10** - *Software Solution of Cupula's Membrane Deformation Shew for Use in Clinical Praxis* - Radun

# Thermo-mechanical numerical analysis of transformation-induced stress relaxation during pseudoelastic behavior of SMA

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**Abstract**

A stress relaxation phenomenon is observed by coupled thermo-mechanical numerical analysis of SMA subjected to uniaxial test. The thermo-mechanical coupling is realized in the partitioned approach. The software components for the structural analysis (PAKS) and the heat transfer (PAKT) based on the Finite Element Method (FEM) have been used. The latent heat production is correlated with the amount of the martensitic volume fraction. The thermo-mechanical numerical analysis of a belt type specimen has been investigated for the strain controlled loading with the break during the martensitic transformation. The thermally induced martensitic transformation induced the significant stress change during the loading break what was expected according to the experimental results from literature.

**Keywords:** shape memory alloys, stress relaxation, thermo-mechanical coupling, phase transformation, partitioned coupling

## 1. Introduction

The thermo-mechanical coupling of Shape Memory Alloys (SMA) constitutive model has been investigated in previously published papers (Dunić et al. 2014, Dunić et al. 2016). The previous numerical tests verified that the influence of the temperature change during the martensitic phase transformation is significant on the stress magnitude (Dunić et al. 2014). Also, it was noticed that for the low strain rates, the temperature change is induced by the martensitic transformation but also by the convection and the thermoelastic effect. The idea of this paper is to describe numerically what happens if the loading process has a loading break in the middle of martensitic transformation for the pseudoelastic effect. The expected behavior based on experimental investigation is the monotonic stress and temperature drop (Pieczyska et al. 2006).

The paper will be organized as follows. In the section 2, the fundamental details about the SMA constitutive model implemented into the structure analysis program PAKS are given. In Section 3 the information about the heat transfer and the structure analysis FEM programs are given with details about the partitioned coupling algorithm. In Section 4, the thermo-mechanical numerical analysis of the SMA belt type specimen is conducted to show stress relaxation phenomenon. At the end in Section 5, conclusions about the achieved results are presented.

## 2. A brief review of SMA constitutive model

The phenomenological constitutive model for SMA is derived from Gibbs free energy $g(\boldsymbol{\sigma}, T, \xi, \mathbf{e}_{tr})$ which depends on the stress $\boldsymbol{\sigma}$, the temperature $T$ and the internal state variables $\xi$, $\mathbf{e}_{tr}$ (Lagoudas, 2010). The main relation which simplifies the model is the assumption that: "any change in the current microstructural state of the material is strictly a result of a change in the martensitic volume fraction" (Lagoudas, 2010), (Boyd, J., Lagoudas, D., 1996), (Qidwai, M., Lagoudas, D., 2000):

$$\dot{\mathbf{e}}_{tr} = H\mathbf{n}_{tr}\dot{\xi} \tag{1}$$

where H is the maximal transformation strain and $\xi$ is the martensitic volume fraction. The direction of the strain tensor $\mathbf{n}_{tr}$ depends on the transformation direction (forward or reverse):

$$\mathbf{n}_{tr} = \begin{cases} \dfrac{3\mathbf{S}}{2\bar{S}}; & \dot{\xi} > 0 \\ \dfrac{\mathbf{e}_{tr}}{\bar{e}_{tr}} & \dot{\xi} < 0 \end{cases}. \tag{2}$$

Using the second law of thermodynamics it was shown by Lagoudas, (2010) that:

$$\Pi\dot{\xi} \geq 0\,, \tag{3}$$

what leads to the transformation function in the following form (Lagoudas, 2010):

$$\Phi = \begin{cases} \Pi - Y; & \dot{\xi} > 0 \\ -\Pi - Y; & \dot{\xi} < 0 \end{cases} \tag{4}$$

where $\Pi(\boldsymbol{\sigma}, T, \xi)$ is the thermodynamic force and $Y$ is the value which depends on transformation hardening functions (Lagoudas, 2010). In the further evolution of the transformation function, the assumption of constant integration direction (deviatoric stress or transformation strain) of the stress integration procedure is used together with the separation of the total stress on deviatoric and mean part. This gives the possibility to solve only one equation on the integration point level (Kojić, Bathe, 2005).

## 3. Thermo-mechanical coupling

### *3.1 Heat transfer program - PAK-T*

The heat transfer program PAK-T (Kojić et al. 1999) computes heat transfer through the solids with the boundary conditions: convection on the part of the surface, prescribed surface flux, prescribed temperature, radiation, etc. From the Fourier's Law of heat conduction (Kojić, 1998):

$$\mathbf{q} = -\mathbf{k}\nabla T \tag{5}$$

the energy balance equation is derived as (Lagoudas, 2010, Dunić et al. 2016):

$$-\rho c\frac{\partial T}{\partial t} + \nabla^{T}(\mathbf{k}\nabla T) + q + \left(q_{dis} - T_0 \alpha c_m \dot{e}_m\right) = 0 \tag{6}$$

where $q$ is the local heat source, $\mathbf{k}$ is the material's conductivity, $\mathbf{T} = \nabla T$ is the temperature gradient and $q_{dis}$ is the elementary dissipative energy. The term $-T_0 \alpha c_m \dot{e}_m$ describes the Gough-Joule effect (Schweizer, Wauer, 2001). The elementary dissipative energy $q_{dis}$ of the martensitic transformation is given as (Lagoudas, 2010, Dunić et al. 2016):

$$q_{dis} = \eta \left( \Pi - \rho \Delta s_0 T \right) \dot{\xi} \tag{7}$$

where $\eta$ is the dissipative factor, $\Pi$ is the thermodynamic force of the martensitic transformation, $\xi$ denotes the martensite volume fraction rate, the product $\rho \Delta s_0$ is the stress influence coefficient and $\Delta s_0$ is the difference of effective entropy at zero stress for the martensitic and the austentic phase (Lagoudas, 2010).

### *3.2 Structure analysis program - PAK-S*

The structure analysis program PAKS (Kojić et al. 1999) solves linear and nonlinear structural problems. For the elastic and thermoelastic material, the stress can be determined by the constitutive relation:

$$\boldsymbol{\sigma} = \mathbf{C}_{el} \left( \mathbf{e} - e_{th} \mathbf{I} \right) \tag{8}$$

where $\mathbf{C}_{el}$ is the elastic constitutive matrix and $e_{th}$ is the thermal strain. The equilibrium equations can be derived in the following form (Matthies et al. 2006), (Matthies, Steindorf, 2002):

$$-\mathbf{div}\left(\tau\right) = \mathbf{r}_s, \qquad \boldsymbol{\tau} = 2G\mathbf{h} + c_m \left( tr\mathbf{h} \right) \mathbf{I}, \qquad \mathbf{h} = \ln \mathbf{v}, \qquad \mathbf{b} = \mathbf{F}\mathbf{F}^T = \mathbf{v}^2 \tag{9}$$

in which "tr" indicates the trace function, $\boldsymbol{\tau}$ is the Kirchhoff stress, $\mathbf{h}$ is the logarithmic strain, $\mathbf{F}$ is the gradient of deformation, $\mathbf{v}$ is the left stretch tensor, $\mathbf{b}$ is the left Cauchy-Green deformation tensor and $\mathbf{r}_s$ is the body load. More details and possible formulations are given in (Bathe, 2006).

### *3.3 The partitioned coupling approach for thermo-mechanical coupling*

The thermo-mechanical coupling has very strong influence on SMA behavior (Pieczyska et al., 2013), (Pieczyska et al., 2014), (Dunić et al. 2014). There are two important advantages of partitioned coupling approach: easier implementation and reusing of existing software solutions (Matthies et al. 2006), (Matthies, Steindorf, 2002). This coupling approach can provide strong coupling, if the nonlinear Block-Gauss-Seidel numerical algorithm is used (Matthies et al. 2006). For the coupling realization (Dunić et al. 2016), the FEM programs for structural (PAKS) and the heat transfer analysis (PAKT) are connected using the CTL – Component Template Library developed by Niekamp (2005). The CTL is used as middleware interface necessary for communication between the components.

## 4. Thermomechanical modeling of stress relaxation in the SMA

A belt type specimen (160 mm × 10 mm × 0.38 mm) is modeled by the FEM 3D elements (Dunić et al. 2014) (Pieczyska et al. 2014). The FEM mesh consists of 400 elements (80×5x1) with 972 nodes (Fig. 1). The specimen ends in the FEM model has a constant temperature because in the experimental setup presented in (Piecyzska et al 2013) (Dunić et al 2014), the grips of the testing machine are very large in comparison to the specimen thickness. The rest of the model specimen has a possibility of free convection. The finite element mesh is the same for both PAKS and PAKT. The boundary and loading conditions are shown in Fig. 1.


**Fig. 1.** Finite element mesh with loading and boundary conditions (Dunić et al. 2014)

| Material parameter | Value | Material parameter | Value |
|---|---|---|---|
| $E_A$ [GPa] | 59.2 | ν[-] | 0.41 |
| $E_M$ [GPa] | 45 | $h$[W m$^{-2}$ K$^{-1}$] | 6.5 |
| $\alpha_{A,M}$[K$^{-1}$] | $1.1\times10^{-5}$ | $c_p$ $c_p$[J kg$^{-1}$ K$^{-1}$] | 460 |
| $H$ [-] | 0.06 | $\lambda_c$ [W m$^{-1}$K$^{-1}$] | 18 |
| $\rho\Delta s_0^{(A,M)}$ $\rho\Delta s_0^{(A,M)}$[MPa K$^{-1}$] | -0.378 | ρ[g cm$^{-3}$] | 6.29 |
| $M_{0s}$ [K] | 213 | $M_{0f}$[K] | 209 |
| $A_{0s}$[K] | 270 | $A_{0f}$[K] | 276 |

**Table 1.** Material parameters of the SMA (Dunić et al. 2014)

## 5. Results and conclusions

The numerical stress relaxation test with a relaxation break in the branch of loading was introduced according to the experimental tests given in Pieczyska et al. (2006) in a following way - loading until a given strain value 0.36 and advanced martensitic transformation is obtained, maintaining the strain value at the same level for 3 minutes, reloading and next unloading until the stress achieves zero.

The stress and the average temperature change vs. strain curves of the TiNi SMA, subjected to the loading-unloading tension tests at strain rate of $10^{-2}$ s$^{-1}$ is presented in Fig. 2. As is seen, the proposed approach predicts decrease in stress if strain is kept constant during loading. Changes in the stress and in the average specimen temperature vs. time for the aforementioned relaxation tests, are presented in Fig 3. A monotonic stress drop 100 MPa is observed while the temperature of the specimen also drops monotonically towards its initial temperature.

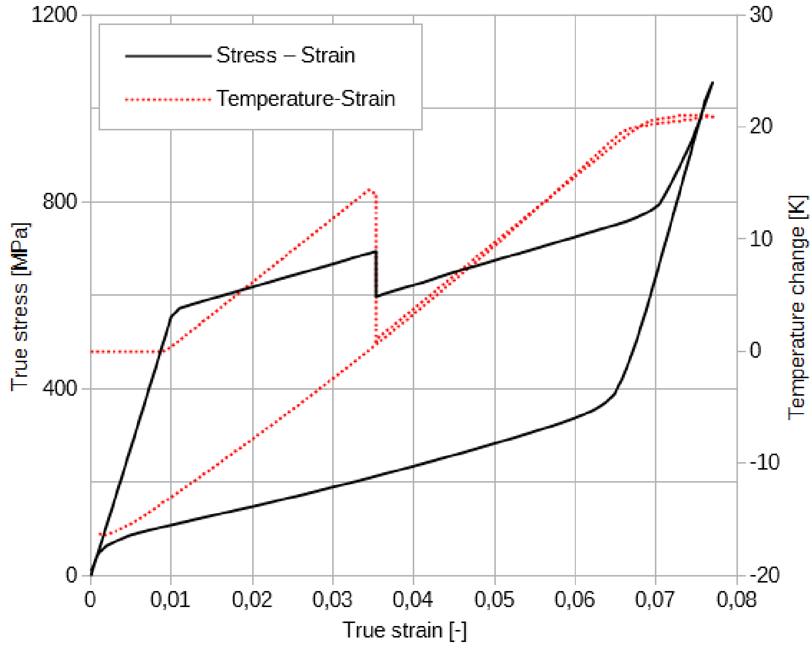


**Fig. 2.** Stress and average temperature change as a function of strain for TiNi SMA tension test with 3min stress relaxation brake in the branch of loading

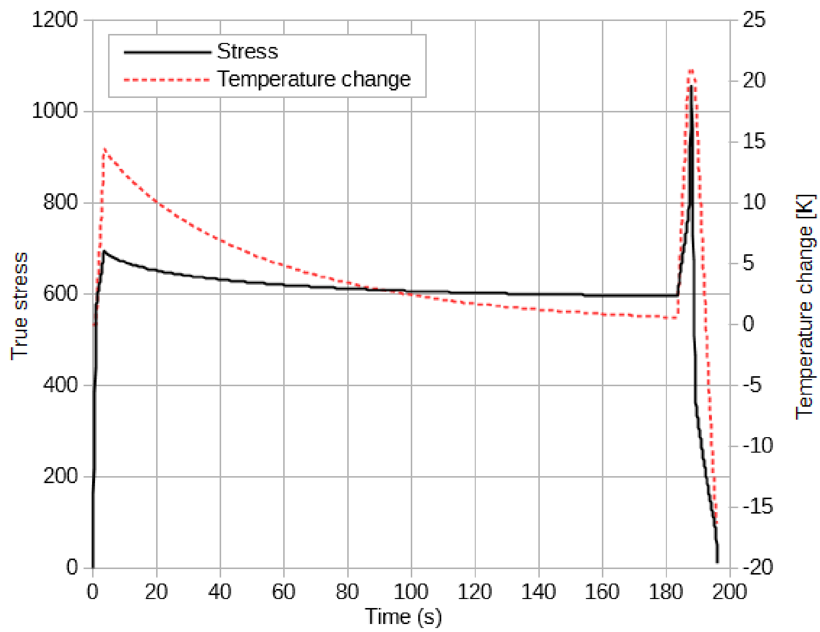


**Fig. 3.** Stress and average temperature changes vs. time for TiNi SMA tension test with 3min stress relaxation brake in the branch of loading

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