Micromechanical modelling of elasto-plastic composites: efficient and robust finite-element implementation of Mori–Tanaka model

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The Mori–Tanaka (MT) model [1] belongs to the wide class of mean-field homogenization methods that have been developed for predicting overall properties of heterogeneous materials. While more elaborate multiscale approaches, such as the FE^2 method [2], are readily available, the mean-field models are still attractive in view of their simplicity, good predictive capabilities and low computational cost [3].

The MT model is a mean-field model originally dedicated to the estimation of effective properties of linearly elastic two-phase composites. It originates from Eshelby's solution to the problem of an ellipsoidal inclusion embedded into an infinite linearly elastic matrix [4]. The MT scheme, originally formulated for elastic composites, can be adapted to elasto-plastic composites by applying incremental linearization, as proposed by Hill [5]. In order to use the incremental MT method in practice, e.g., as a two-phase material model in the framework of the finite element method, the rate form of: constitutive equations of phases, so-called interaction equation and averaging rule, must be integrated over time. This leads to an incremental finite-step formulation and several practical realisations, dating back to [6], [7], are known. However, computational efficiency, consistent linearization, and related issues have not attracted sufficient attention yet. The goal of this work is to develop an efficient and robust incremental MT scheme, suitable for large-scale finite-element computations.

Consistent linearization of the macroscopic stress predicted by the MT scheme is considered a necessary condition to achieve the goal of this work. The automatic differentiation (AD) technique [8], [9] has been employed for that purpose. The corresponding compact AD-based formulation of the incremental MT scheme has been introduced. The AD-based formulation constitutes the basis for finite-element implementation and for automation of the related tasks. The incremental MT scheme involves a nested Newton-type algorithm with inner iterations corresponding to the equations of incremental plasticity formulated for the individual phases and with outer iterations corresponding to the micro-macro transition scheme of the MT method. Considering additionally the global equilibrium iterations, the complete computational scheme can be classified as a doubly-nested iteration-subiteration scheme. Exact linearization, which is necessary for achieving quadratic convergence rate, has been performed at each level so that the Newton method can be used to efficiently solve the nonlinear equations at each level.

However, consistent linearization was not sufficient to obtain a fully robust computational scheme. This is because convergence problems were encountered in some situations, especially for higher content of inclusions and in large-scale boundary value problems. It has been found that the problems were caused by discontinuities in the incremental finite-step response [10]. The discontinuities may occur at the instant of the elastic-to-plastic transition in the matrix phase and result from the related abrupt change of the reference stiffness for which the MT interaction equation is formulated. As a remedy, the incremental MT scheme has been enhanced with a *substepping strategy*.

A highly robust and efficient computational scheme has been finally obtained, as illustrated in Fig. 1. It presents results of a plate with a hole made of a metal-matrix composite stretched in the longitudal direction. The material is composed of spherical ceramic inclusions uniformly dispersed in an aluminum-alloy matrix. The inclusions are assumed elastic, and the matrix is elasto-plastic, governed by J2-plasticity with isotropic hardening. The finite-element mesh comprises 368,640 trilinear hexahedral elements which corresponds to approximately 1.2 million displacement unknowns. It can be seen that consistent linearization of the incremental Mori–Tanaka scheme was not enough to achieve desired robustness of the computational scheme, Fig. 1(a). Upon employing the substepping strategy, the developed computational scheme [11] can serve as an extremely efficient micromechanical model in large-scale FE problems dedicated to elasto-plastic composite materials, Fig. 1(b).

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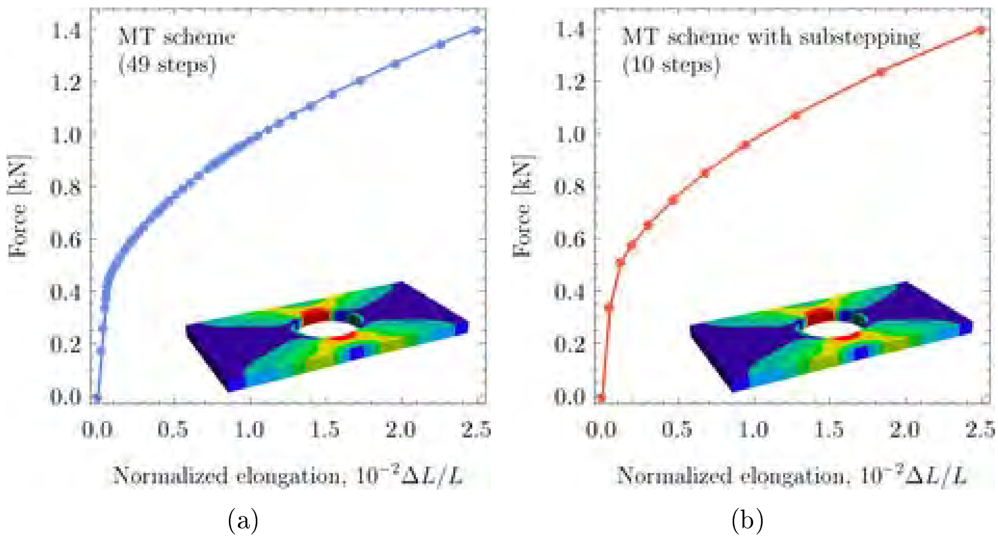

Fig. 1: The impact of response discontinuities on the convergence behaviour and robustness of the incremental MT scheme employed in finite-element computations for: (a) basic MT scheme exhibiting severe convergence problems that are caused by the discontinuities and (b) MT scheme enhanced with the substepping strategy [10]

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