Improvement of penalty approach in contact modeling*

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Abstract

An improved approach to penalty modeling in contact mechanics is proposed. The presented algorithm enables evaluation of the optimum values of the penalty factor for each constrained degree of freedom in the finite element model. The values are chosen so as to ensure desired accuracy in fulfillment of the geometric constraints while keeping the condition number of the modified stiffness matrix at a moderate level. Thus, the main weakness for which the penalty approach is often criticised — excessive worsening of the system conditioning — is fairly limited. Numerical examples confirm advantages of the method.

Keywords: contact modeling, penalty approach, matrix condition number

1. Introduction

Penalty approach is one of the commonly applied method of dealing with contact constraints in FE modeling in structural mechanics. Its advantage is simplicity of formulation; in particular, no additional unknown variables appear in the system of equilibrium equations. Among its drawback one may mention only approximate fulfillment of the contact constraints and worsening of the system conditioning, as the additional terms that modify the system coefficient matrix have much higher order of magnitude that its intact values. Manipulating the assumed penalty factor value, one may decrease one of the mentioned drawbacks, but usually at the cost of increasing the other. The main difficulty is, however, that there are no good universal rules to tell what this value should be in the particular problem to solve.

We propose the numerical method to compute this value, or rather values (as the penalty factors may be different for each contact condition), based on current values of the structural stiffness matrix, loads, displacements, and the desired accuracy of contact modeling. The penalty modifications of the system coefficient (stiffness) matrix and the r.h.s. vector allow to fulfill the contact constraints with the assumed accuracy while increasing the matrix condition number as little as it is necessary in the particular configuration.

2. Methods

Let $\mathbf{q}$, $\mathbf{r}$ be the vectors of nodal displacements and loads and $\mathbf{K}$ the stiffness matrix of a system without contact costraints. In the linear case, the system potential energy is expressed as $\Pi(\mathbf{q}) = \frac{1}{2}\mathbf{q}^{\mathrm{T}}\mathbf{K}\mathbf{q} - \mathbf{q}^{\mathrm{T}}\mathbf{r}$ and its minimization leads to the standard system of N algebraic equations $\mathbf{K}\mathbf{q} = \mathbf{r}$.

Let contact constraints at each node have the linear form of node-to-rigid-plane contact,

$$u_x n_x + u_y n_y + u_z n_z \leq \hat{u} \tag{1}$$

where $\{n_x, n_y, n_z\}$ is the unit normal to the plane (directed inwards) and $\hat{u}$ is the current distance to the plane. If the solution $\mathbf{q}$ (containing components u_x, u_y, u_z for all nodes) violates some of the constraints (let their number be M), additional M equations appear in the problem

$$\mathbf{C}^{\mathrm{T}}\mathbf{q} = \hat{\mathbf{u}} \tag{2}$$

where $\mathbf{C}_{N\times M}$ is the constraint matrix whose columns contain unit normal components for the involved d.o.f. and zeros elsewhere. The equations generate additional contact forces, not included in $\mathbf{r}$.

In the penalty method, the potential energy of the system is modified as [3]

$$\Pi(\mathbf{q}) = \frac{1}{2}\mathbf{q}^{\mathrm{T}}\mathbf{K}\mathbf{q} - \mathbf{q}^{\mathrm{T}}\mathbf{r} + \frac{1}{2}(\mathbf{C}^{\mathrm{T}}\mathbf{q} - \hat{\mathbf{u}})^{\mathrm{T}}\boldsymbol{\epsilon}(\mathbf{C}^{\mathrm{T}}\mathbf{q} - \hat{\mathbf{u}}) \tag{3}$$

where $\boldsymbol{\epsilon}_{M\times M}$ is a diagonal matrix of penalty coefficients. In the traditional approach, the coefficients are arbitrary "large" positive numbers (usually the same for all constraints), typically estimated as several orders of magnitude higher than stiffness matrix elements. Minimization of the energy leads to the system of equations with the modified stiffness matrix and the r.h.s. vector,

$$(\mathbf{K} + \mathbf{C}\boldsymbol{\epsilon}\mathbf{C}^{\mathrm{T}})\mathbf{q} = \mathbf{r} + \mathbf{C}\boldsymbol{\epsilon}\hat{\mathbf{u}}. \tag{4}$$

in which the contact costraints are enforced with the better accuracy, the higher the penalty coefficients are. On the other hand, their too high values worsen conditioning of the modified stiffness matrix which affects accuracy of the overall solution.

In the proposed approach, we define values of allowed inaccuracy in fulfillment of each contact constraint (allowed penetrations) and gather their values in a vector $\boldsymbol{\delta}_{M\times 1}$. Thus, in the worst case we require

$$\mathbf{C}^{\mathrm{T}}\mathbf{q} - \hat{\mathbf{u}} = \boldsymbol{\delta} \tag{5}$$

Substituting $\mathbf{q}$ from Eqn (4) we get a system of M equations

$$\mathbf{C}^{\mathrm{T}}\left(\mathbf{K} + \mathbf{C}\boldsymbol{\epsilon}\mathbf{C}^{\mathrm{T}}\right)^{-1}(\mathbf{r} + \mathbf{C}\boldsymbol{\epsilon}\hat{\mathbf{u}}) - \hat{\mathbf{u}} = \boldsymbol{\delta} \tag{6}$$

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in which the only unknowns are M diagonal elements of the penalty matrix $\boldsymbol{\epsilon}$. After several transformations, skipped here, we come at the following equation,

$$\boldsymbol{\epsilon}\boldsymbol{\delta} = \mathbf{C}^{\mathrm{T}}\mathbf{r} - \mathbf{C}^{\mathrm{T}}\mathbf{K}\mathbf{C}(\hat{\mathbf{u}} + \boldsymbol{\delta}) \tag{7}$$

from which the diagonal elements of $\boldsymbol{\epsilon}$ can be determined in a trivial way.

3. Computational example

Deep drawing of a thin circular isotropic sheet with a hemispherical punch is considered as an example [2]. The geometrical configuration of the problem is shown in Figure 1. The initial radius of the blank is 2.22 in and its initial thickness 0.035 in. The Coulomb friction coefficient was assumed 0.2. The strain hardening law was assumed in the form (stress unit is [ton/in^2])

$$\bar{\sigma} = \begin{cases} 5.4 + 27.8\bar{\varepsilon}^{0.504}, & \text{for} \quad \bar{\varepsilon} \leq 0.36 \\ 5.4 + 24.4\bar{\varepsilon}^{0.504}, & \text{for} \quad \bar{\varepsilon} > 0.36 \end{cases} \tag{8}$$

The analysis was performed with the use of MFP2D program [1]. Fifty axisymmetric linear bending elements have been used. A contact algorithm with classical penalty approach (penalty factor $\epsilon = 10^7$) and next with the modified penalty approach was applied. In the second case, two different accuracy levels of contact modeling were considered: $\delta = 10^{-7}$in and $\delta = 10^{-3}$in. The depth of penetration of blank in tool is presented in Figure 2. Summary matrix condition numbers in all time steps of numerical simulations are shown in Figure 3. It should be noted that almost in all time increments the matrix condition number is improved with the use of modified approach compared to classical penalty method approach even when very high accuracy (10^{-7}in) is assumed.

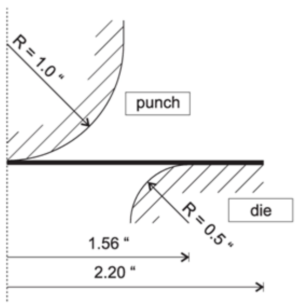

Figure 1: Hemispherical punch deep drawing problem, dimensions are given in inches

4. Discussion

The proposed method allows to adapt the order of magnitude of penalty coefficients to the assumed error tolerance level. One may thus avoid using too high values of the coefficients which leads to excessive increase of the matrix condition number in the considered system of equations. Moreover, the penalty coefficients are different for different contact conditions which allows to accommodate possible large differences in structural stiffness with respect to different deformation modes (e.g. tension and bending).

There is no difficulty in extending the approach to nonlinear problems. In this case, the penalty coefficients $\boldsymbol{\epsilon}$ are computed from Eqn. (7) in each Newton–Raphson iteration, for the current geometric configuration and current state of activity of all potential contact conditions. Modification of the presented approach towards contact of two deformable bodies is possible, too. The only drawback of the method is that it does not include friction constraints at the moment — in this area, the penalty coefficients have still to be chosen arbitrarily.

References

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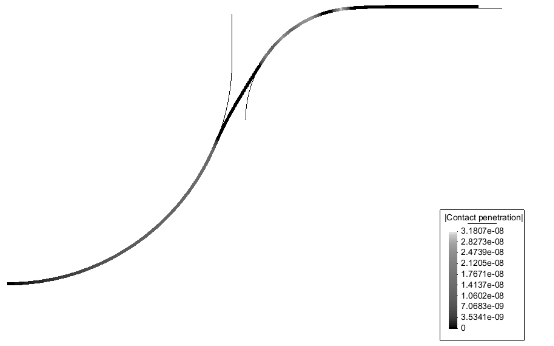

Figure 2: Penetration of the blank in tools distribution, 0 – without contact. Contact modeling accuracy equal 10^{-7}.

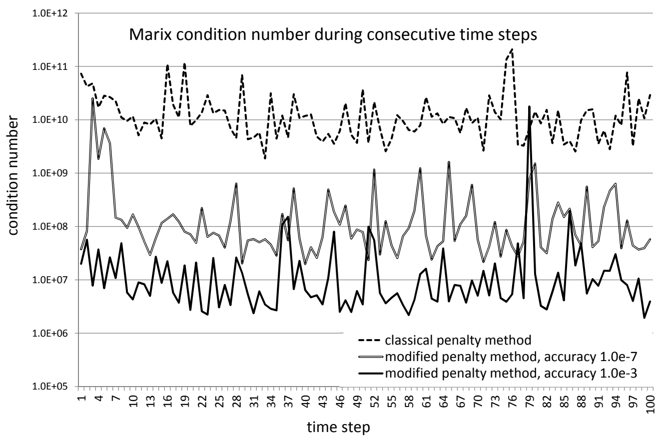

Figure 3: Matrix condition number during consecutive steps