8th European Solid Mechanics Conference
July 9-13, 2012
Graz, Austria

Optimal Design of Annular Disks

With Respect to Mixed Creep Rupture Time

Aneta Ustrzycka[†], **Krzysztof Szuwalski**[†]

[†]Faculty of Mechanical Engineering, Cracow University of Technology,
Jana Pawła II 37 Ave., 31- 864 Cracow, Poland
anetaustrzycka@wp.pl, ks@mech.pk.edu.pl

ABSTRACT

The problem of optimal design with respect to mixed creep rupture time is a new one. The first attempt of solution for rotation bar was made in 2010 by Szuwalski and Ustrzycka [5]. Difficulties of the problem result from physical and geometrical nonlinearities, and were observed earlier in solution for ductile creep rupture [4], [6]. Some problems of optimal design for annular rotating disks were discussed by Farshi and Bidabadi [2]. Analytical solution for the elastic-plastic stress distribution in rotating annular disks were obtained by Çğallioğlu et al. [1] and Gun [3].

Because of those difficulties the parametric optimization was adopted. In certain class of function describing the initial shape of the disk we are looking for optimal parameters leading to the longest time to mixed rupture under assumption of constant volume.

The axially symmetric annular disk rotating with constant angular velocity is loaded by centrifugal forces resulting from the own mass of the disk and additional mass uniformly distributed at the outer edge (Figure 1).

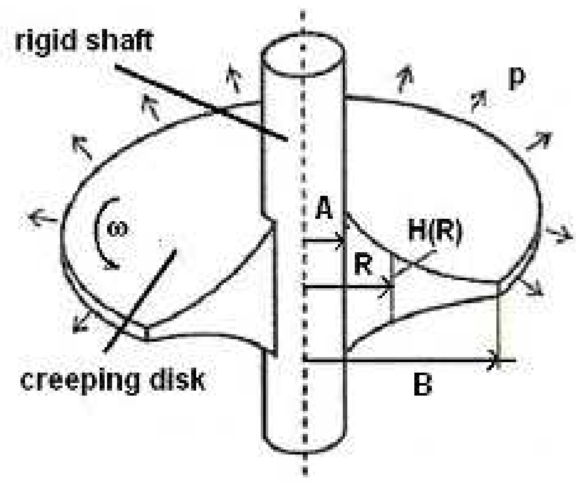

Figure 1: Annular rotating disk.

The mathematical model of mixed creep rupture is described by the system of five partial differential equations in dimensionless form:

$$\bar{\sigma}_r' = \frac{\bar{r}'}{\bar{r}}(\bar{\sigma}_r - \bar{\sigma}_\vartheta) - 2 \cdot \bar{r}\bar{r}'\mu - \frac{\bar{h}'}{\bar{h}}\bar{\sigma}_r$$

$$\bar{\sigma}_\vartheta' = \frac{6\bar{\sigma}_e^2(\bar{\sigma}_r - \bar{\sigma}_\vartheta)\frac{\bar{r}'}{\bar{r}} - \bar{\sigma}_r'\left[(n-1)\left(5\bar{\sigma}_r\bar{\sigma}_\vartheta - 2\bar{\sigma}_r^2 - 2\bar{\sigma}_\vartheta^2\right) - 2\bar{\sigma}_e^2\right]}{(n-1)(2\bar{\sigma}_\vartheta - \bar{\sigma}_r)^2 + 4\bar{\sigma}_e^2}$$

$$\frac{d\bar{r}}{d\bar{t}} = \frac{\bar{r}}{2 \cdot n}\left(\bar{\sigma}_r^2 + \bar{\sigma}_\vartheta^2 - \bar{\sigma}_r\bar{\sigma}_\vartheta\right)^{\frac{n-1}{2}}\left(2\bar{\sigma}_\vartheta - \bar{\sigma}_r\right) \tag{1}$$

$$\bar{h} = \frac{\overline{HR}}{\bar{r}'\bar{r}}$$

$$\frac{\partial \Psi}{\partial \hat{t}} = \frac{-1}{(m+1)\,\Theta}\left[\frac{\hat{\sigma}_e}{\Psi}\right]^m .$$

In the last equation the continuity function Ψ describing damage of material was introduced. The criterion of rupture is fulfilled when Ψ takes values 0. The optimal shape among linear functions (uniparametric optimization) and quadratic functions (biparametric optimization) was found by checking some disks in earlier predicted domain of admissible solutions. The better results were obtained using biparametric optimization. The obtained results are compared with the disks with respect to ductile creep rupture time [6].

References

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