ESTIMATION OF LAYER THICKNESS BY THE COST FUNCTION OPTIMIZATION: PHANTOM STUDY

JURIJ TASINKIEWICZ, JERZY PODHAJECKI, JANUSZ WOJCIK, KATARZYNA FALIŃSKA, JERZY LITNIEWSKI

Ultrasound Department
Institute of Fundamental Technological Research Polish Academy of Sciences,
Pawińskiego 5b, 02-106 Warszawa, Poland
yurijtas@ippt.pan.pl

The aim of this work is to present preliminary results of the layer thickness assessment method based on optimization approach. The developed method is based on a multilayer model structure. The measured acoustic signal reflected from the layer is compared with a simulated signal on the basis of a multilayer model. The cost function is defined as the difference between the reflected signal measured using pulse echo approach and the simulated signal. The thickness of the solid layer is the parameter which minimizes the cost function yielding desired solution. Minimization of the cost function is performed with the simulated annealing algorithm. The results obtained with the developed method using measurement data of a custom design model are compared with the reference value and the accuracy of the method is checked. The relative error of the thickness estimation is 1.44%.

INTRODUCTION

The main objective of this work is to present a noninvasive method of the layer thickness assessment. Several methods have been reported which allow the thickness of the layer to be measured using reflected acoustic waves. The envelope method [1] or autocorrelation method [2] can be applied in the case when the reflections from layer interfaces are separated in time. In the case when the internal reflections overlap the reflected signal can be subjected to the cepstral analysis [3]. There are also other methods like the parametric method [4] or the maximum entropy analysis method [5], which allow the thickness of the layer to be determined when the reflected signal consists of interfering components. In this paper we present a new approach of the layer thickness estimation based on the cost function optimization scheme. The reported method is based on the analysis of the acoustic wave-field reflected from the multilayered system. Specifically, the liquid – solid layer– porous medium structure is considered. The method was tested using measurement data of a custom design model consists solid layer and porous material. The proposed method can be applied for the cortical bone thickness assessment using reflected ultrasound what could be helpful information in diagnosis of osteoporosis.

1. THEORETICAL BACKGROUND

The analyzed multi-layer structure is shown in Fig. 1. In this case a straightforward analysis of acoustic wave propagation may be conducted. The incident wave propagates in first layer media and impinges the second layer at the right angle. In this case only a longitudinal wave exists. It was observed earlier that the temporal spectrum of the ultrasonic pulse reflected from the layered media depends strongly on the thickness of the corresponding layers [3]. This property is used in the further part of the analysis.

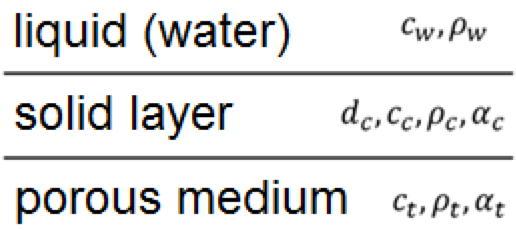

Fig. 1. Multi-layer structure.

In Fig. 1 ρ denotes a mass density, c is the acoustic wave velocity (longitudinal wave is assumed), α is the attenuation coefficient; the subscripts w, s, p denote the first, second and third regions (water, solid and porous media), respectively; d_s is the thickness of the solid layer. The acoustic signal of the reflected wave can be modeled using the following formula resulting from the linear system approach:

$$\hat{S}^R(\omega,\theta,d_s) = R(\omega,\theta,d_s)S^I(\omega), \tag{1}$$

which gives the dependence between the temporal spectra of the interrogating pulse $S^I(\omega)$ (the superscript I denotes an incident wave) and the reflected signal $\hat{S}^R(\omega,\theta,d_c)$ in the layered media, shown in Fig. 1, which is characterized by the reflection coefficient $R(\omega,\theta,d_s)$. In the Eq. (1) ω is the temporal angular frequency and θ denotes the parameters of the model (attenuation, acoustic wave velocity, mass density). For the analyzed structure an analytical solution for the reflection coefficient can be derived as shown in [6]:

$$R = \frac{B_{21}+B_{22}Z_p - (B_{11}+B_{12}Z_p)Z_w}{B_{21}+B_{22}Z_p + (B_{11}+B_{12}Z_p)Z_w}, \tag{2}$$

where the elements of the transmission matrix are given below:

$$B_{11} = B_{22} = \cos k_s d_s\,;\; B_{12} = \frac{j}{Z_s}\sin k_s d_s\,;\; B_{21} = \,j\,Z_s \sin k_s d_s. \tag{3}$$

In the Eq. (3) k_i and $Z_i = \rho_i c_i,\; i = w,s,p$ denote the wave number and acoustic impedance of liquid (water), solid and porous media regions, respectively. The corresponding velocities c_s and c_p in the second and third regions are complex valued functions of ω due to the attenuation taken into account in the model above. In this case the corresponding wave numbers are defined as follows:

$$k_i \equiv \frac{\omega}{\overline{c_i}} = \frac{\omega}{c_i} - j\alpha_i, \quad i = s,p, \tag{4}$$

The difference between the temporal spectrum of the measured reflected signal and the one obtained from the model presented by Eq. (1) can be defined as follows:

$$e(\omega,\theta,d_s) = S^R(\omega,\theta,d_s) - \hat{S}^R(\omega,\theta,d_s) = S^R(\omega,\theta,d_s) - R(\omega,\theta,d_s)S^I(\omega), \quad (5)$$

where $S^R(\omega,\theta,d_s)$ is temporal spectrum of the measured reflected signal; $e(\omega,\theta,d_s)$ is the difference between the measured and modeled spectra. The reflected signal in the Eq. (6) is modeled using the reflection coefficient defined by Eq. (2). The main aim of the discussed method is to fit the temporal spectrum of the reflected wave calculated on the basis of the model $\hat{S}^R(\omega,\theta,d_s)$ to the spectrum of the reflected wave measured experimentally $S^R(\omega,\theta,d_s)$. To this end the so-called cost function is defined as the least square error between the measured and simulated temporal spectra:

$$E(\omega,\theta,d_s) \equiv e(\omega,\theta,d_s)e(\omega,\theta,d_s)^T = \sum_k |e(\omega_k,\theta,d_s)|^2, \quad (6)$$

where summation is over all frequency samples (discrete temporal spectrum is assumed). In the Eq. (6, 7) $\theta = \{\alpha_s, \alpha_p, \rho_s, \rho_p, \rho_w, c_s, c_p, c_w\}$ denotes the vector of system parameters which assumed to be known. Minimizing the cost function allows one to determine the value of the layer thickness d_s which best fit the measurements and provide the desired solution.

2. MATERIALS AND METHODS

The developed method was tested using experimental data obtained from the custom design model of solid thin layer glued to the porous material shown in Fig. 2. The layer material (Sawbones, Pacific Research Laboratory Inc, Vashon, WA) was made of oriented glass fibers mixed with epoxy. The material is transverse isotropic, with elastic properties close to those of real bones . The attenuation coefficient of the layer material was α_s = 2.6 dB/cm as reported in [7]. The longitudinal wave velocity c_s measured with the pulse echo method at a center frequency of 6 MHz was equal 2900 m/s. The frequency of 6 MHz was chosen to obtain separation of the pulses reflected from the bottom surfaces of the plate. The solid plate was bonded to a porous material being a rigid polyurethane foam core and having a thickness of about 30 mm, which models the cancellous bone [8], as illustrated in Fig. 2. Both plate and porous material was immersed in water. To simplify the applied model (Eq. 1-4), the attenuation of the water was neglected. To simplify the analysis the acoustic wave velocities c_s and c_p were assumed constant, taking values measured at certain reference frequency. For the attenuation coefficients in solid layer and porous regions the following frequency dependence was assumed [9]:

$$\alpha_i(\omega) = \alpha_i(\omega_0)\left(\frac{\omega}{\omega_0}\right)^{n_i}, i = s, p, \quad (7)$$

where $\alpha_i(\omega_0)$ is the attenuation coefficient at the reference frequency ω_0; n_i – certain empirical constant, which was chosen from the range (1 – 2).

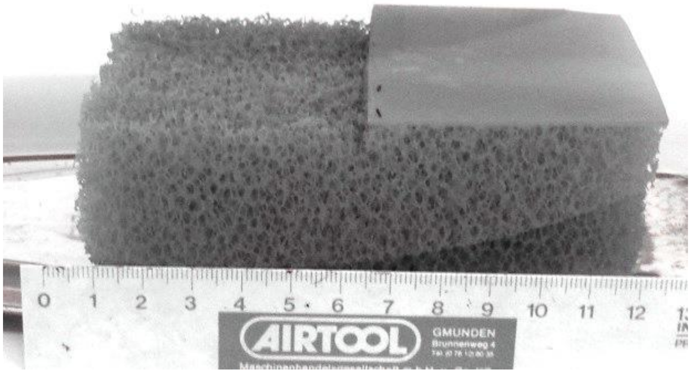

Fig. 2. The sample used in experiments.

The mass density of the foam ρ_f was equal 1200 kg/m^3 were considered [8]. The longitudinal wave velocity c_p in the sample of the porous material and corresponding attenuation coefficient α_p were measured with the pulse echo method at a center frequency of 0.6 MHz. The frequency of 0.6 MHz was chosen because of the high attenuation (see Table 1) of the porous material and the thickness of the sample (approximately 30 mm). The main parameters of the custom design model used in experiments are referenced in Table 1.

Table. 1. Material properties of the custom design model.

d_s, mm	c_s, m/s	α_s, dB/cm at 1 MHz	ρ_s, kg/m^3	c_p, m/s	α_p, dB/cm at 0.5 MHz	ρ_f, kg/m^3	ρ_p, kg/m^3
1.05	2900	2.6	1640	1641	10.8	1200	1457

The thickness of the solid layer sample was measured with a digital caliper. The corresponding density of the foam ρ_p was computed using the following equation:

$$\rho_p = P\rho_w + (1 - P)\rho_f, \tag{8}$$

where the mass density of the water $\rho_w = 1000$ kg/m^3 and P is a porosity of the foam which was measured and equals to 0.88 (88%). The pulse-echo method was used in experimental verification of the developed method. For this purpose a flat transducer of 25 mm in diameter excited by a sine cycle of 600 kHz was used. The transmit pulse was obtained using pulse generator (Tektronix AFG 3252 dual channel arbitrary function generator). The reflected signal was detected using the same transducers. The radio frequency (RF) data collected on each measurement point were first amplified using high power pulse amplifier and then recorded by the digital storage oscilloscope (model DSO9104A, Keysight Technologies Inc.) at 400 MHz temporal sampling rate and transferred to a PC for further off-line processing in Matlab® in order to test the method developed. To solve the optimization problem, that is to minimize the cost function Eq. (6), the simulated annealing algorithm implemented in *simulannealbnd* function from the Matlab® package 'Optimization Toolbox' was used. The method is based on comparison of the temporal spectra of measured and simulated reflected signals, Eq. (6). In the numerical simulations the corresponding temporal spectra were truncated at the upper frequency 1.25 MHz. Five measurements were taken at spatial positions spaced 5 mm apart. At each measurement point the optimization with simulated annealing was repeated N=50 and the mean values of $\bar{d}_s$ were evaluated. Then the obtained values were averaged yielding the final estimate of the thickness d_s. This value was further compared with the reference values shown in Table 1.

3. RESULTS AND DISCUSSION

The results of the simulations performed in Matlab® are presented in Table 2.

Table. 2. Measured values and relative errors of the thickness of the solid layer fixed to the porous material.

measurement	$\bar{d}_s$, mm	d_s, mm	$\hat{d}_s$, mm	δ_d, %
#1	1.0418	1.0651	1.05	1.44
#2	1.0659			
#3	1.0783			
#4	1.0753			
#5	1.0641			

In Table 2 $\hat{d}_s$ denotes the reference value of the layer thickness from Table 1 and δ_d is corresponding relative error:

$$\delta_d = \frac{d_s - \hat{d}_s}{\hat{d}_s} \cdot 100\%, \tag{9}$$

In Fig. 3 the results summarized in Tables 2 are illustrated in a graphic form, presenting the estimated mean values of the layer thickness.

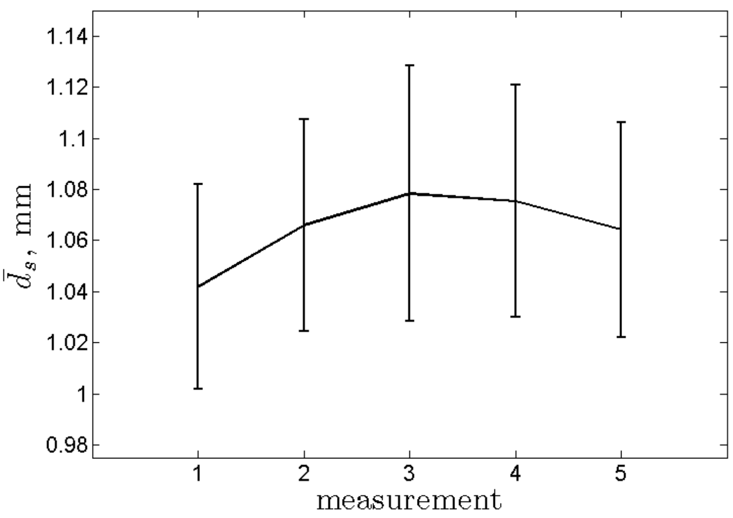

Fig. 3. Mean values $\bar{d}_s$ and standard deviations σ_d of the layer thickness for *N*=50 estimations at each measurement.

The error bars show standard deviations σ_d estimated from 50 measurements at each spatial point spaced 5 mm apart along the custom design model. As can be seen from Table 2 the relative error of the layer thickness evaluation was 1.44%. This proves that the developed method is promising for further study and development. Specifically, it is important to point out that in the presented method the acoustic wave speed in the solid layer was known. In real situations c_s is usually unknown. Therefore, the further research will be focused on generalization of the discussed method for the case of simultaneous estimation of the layer thickness and acoustic (longitudinal) wave velocity in the layer.

This method may be potentially used in estimation of the thickness of cortical bone layer. The influence of cortical bone thickness can substantially reduce the accuracy of Broadband

Ultrasound Attenuation (BUA) measurements from cancellous bone area, which is main indicator of bone disease, such as osteoporosis [10-13].Therefore, for reliable assessment of the trabecular bone quality it is necessary to take the cortical bone thickness into account. This can be achieved only when the thickness of the cortical bone layer is known.

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