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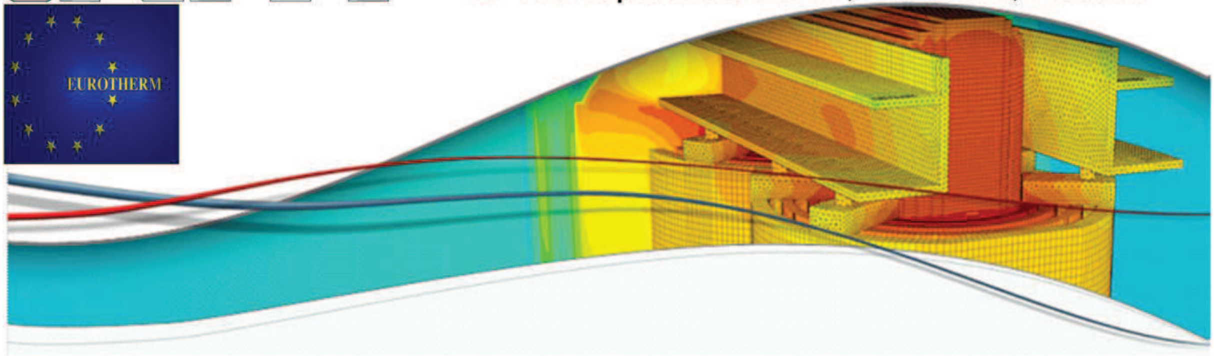

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Macroscopic Thermal Properties of Quasi-linear Cellular Medium on Example of the Liver Tissue

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Key words: liver tissue, Pennes' equation, heat transport, asymptotic homogenization, effective coefficients

Abstract

After discovery of strong sonar systems, it was realized that the high intensity ultrasound waves can be dangerous for biological organisms. This observation led to research in tissue heating effects. The liver tissue from mathematical point of view can be considered as a micro-periodic cellular medium, and in circumstances justified by biological reasons, the mathematical methods of homogenisation developed for micro-periodic media can be applied to determine some overall properties of the tissue. Fourier's heat diffusion term in Pennes' equation is the point of departure in our analysis, [1, 2, 3].

The liver, the largest internal organ in the human body, is connected to two large blood vessels, the hepatic artery and the portal vein. The hepatic artery carries oxygen-rich blood from the aorta, whereas the portal vein carries blood rich in digested nutrients from the entire gastrointestinal tract and also from the spleen and pancreas. These blood vessels subdivide into small capillaries known as liver sinusoids, which then lead to a lobule. A **hepatic lobule** is a small division of the liver defined at the histological scale. The lobules are arranged into an hexagonal lattice.

The ultrasound absorption coefficient increases as a function of protein content, with collagen having a particularly high specific absorption. Collagen accounts for 10% in the liver, and the absorption coefficient is of 0.2 dB/cm. For water and body liquids there is little absorption 0.003 dB/cm, [4].

Pennes' equation describes the thermal response of soft tissue due to hyperthermia

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_k}\left(\lambda \frac{\partial T}{\partial x_k}\right) + w_b\, c_b\,(T - T_a) + q \tag{1}$$

Here, ρ and c are the mass density and specific heat, respectively, w_b is the blood perfusion rate and T_a is the temperature of the arterial supply blood. The heat generation term q encompasses the thermal effects of metabolism and, if necessary, other volumetric heat loads, as microwave irradiation or the heat generated by ultrasound waves.

Nonlinearity of thermal properties. The thermal conductivity properties of water fits to a linear equation over the temperature range 0^0C to 45^0C, $\lambda = 0.5652 + 0.001575\,T$ where T is in ^{0}C, cf. [5]. The thermal diffusivity of tissue matches the thermal diffusivity of water well for both the magnitude and the temperature coefficient.

One-dimensional time-independent problem. Let the section $[x_0, x_0+\ell]$ of x axis consist of two segments $[x_0, x_0+a]$ and $[x_0+a, x_0+\ell]$. Let the heat conductivity be $\lambda_\mathrm{a}(1+\alpha T)$ in $[x_0, x_0+a]$ and $\lambda_\mathrm{b}(1+\beta T)$ in $[x_0+a, x_0+\ell]$. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\lambda_\mathrm{a}(1+\alpha T)\frac{\mathrm{d}T}{\mathrm{d}x}\right\} = 0 \quad \text{for } x_0 < x < x_0 + a \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left\{\lambda_\mathrm{b}(1+\beta T)\frac{\mathrm{d}T}{\mathrm{d}x}\right\} = 0 \quad \text{for } x_0 + a < x < x_0 + \ell \tag{2}$$

with the boundary conditions $T(x_0) = T_0$ and $T(\ell) = T_\ell$. After integration we get, in particular,

$$T_\mathrm{a} = \frac{b\lambda_\mathrm{a} + a\lambda_\mathrm{b}}{b\lambda_\mathrm{a}\alpha + a\lambda_\mathrm{b}\beta} \times \left\{-1 + \sqrt{1 + 2\frac{b\lambda_\mathrm{a}\alpha + a\lambda_\mathrm{b}\beta}{(b\lambda_\mathrm{a} + a\lambda_\mathrm{b})^2}\left(b\lambda_\mathrm{a}(T_0 + \frac{1}{2}\alpha T_0^2) + a\lambda_\mathrm{b}(T_\ell + \frac{1}{2}\beta T_\ell^2)\right)}\right\} \tag{3}$$

Homogenisation of quasi-linear heat equation with periodic coefficients applying an asymptotic method for λ^{eff} was developed in the paper [6]. Its idea is the following one. Let Ω be a bounded regular domain, and $\Gamma = \partial\Omega$ be its boundary. We introduce a parameter $\varepsilon \equiv \ell/L$ where ℓ and L are typical lengths associated with micro-inhomogeneities and the region Ω, respectively. Let $q = q(x,t)$ be a heat source. We study the quasi-linear heat transport equation

$$\frac{\partial}{\partial x_i}\left(\lambda^{\varepsilon}_{ij}\,\frac{\partial T^{\varepsilon}}{\partial x_j}\right) = -q \quad \text{in} \quad \Omega \quad \text{with} \quad T^{\varepsilon}|_{\Gamma} = 0 \qquad \text{on} \quad \Gamma \tag{4}$$

where the heat conductivity coefficient $\lambda^{\varepsilon}_{ij}(x,T^{\varepsilon}) = \lambda_{ij}\,(x/\varepsilon, T^{\varepsilon}))$, $i,j = 1,2,3$ and $x \in \Omega$. We introduce the basic cell $Y = (0,Y_1)\times(0,Y_2)\times(0,Y_3)$. The material coeffcients are assumed to be Y-periodic. The following two-scale asymptotic assumption is made $T^{\varepsilon}(x) = T^{(0)}(x,y) + \varepsilon T^{(1)}(x,y) + \varepsilon^2 T^{(2)}(x,y) + \cdots$, where $y \equiv x/\varepsilon$ is known as the local variable, and the functions $T^{(0)}(x,y)$, $T^{(1)}(x,y)$, $T^{(2)}(x,y)$, and so on, are Y-periodic.

The homogenised (effective) heat coefficient is given by

$$\lambda^{\mathrm{eff}}_{ij}(T^{(0)}(x)) = \frac{1}{|Y|}\int_Y \left(\lambda_{ij}(y,T^{(0)}(x)) + \lambda_{kj}(y,T^{(0)}(x))\,\frac{\partial \chi_i(y,T^{(0)}(x))}{\partial y_k}\right) dY \tag{5}$$

The functions $\chi_k = \chi_k(y,T^{(0)}(x))$, called the local functions, are solutions to the equation

$$\frac{\partial}{\partial y_j}\left\{\lambda_{ij}(y,T^{(0)})\left(\delta_{ik} + \frac{\partial \chi_k(y,T^{(0)})}{\partial y_i}\right)\right\} = 0 \tag{6}$$

Ritz's method offers a possibility of determination of local functions in an approximate way. We take $\chi_i(y,T) = \sum_a \chi_{ia}(T)\,\Phi_a(y)$, where $\Phi_a(y)$, $a = 1,2,\cdots$ are prescribed functions, and $\chi_{ia}(T)$ are unknown constants for given T, which satisfy the algebraic equation $\chi_{ia}A_{ab} = B_{ib}$ with A_{ab} and B_{ab} expressed by suitable integrals.

To illustrate the procedure we have evaluated the dependence of λ^{eff} on T for the composite consisting of the basic cells arranged in a two-dimensional periodic system and built of the collagen capillaries filled with the water.

Validation. Analytical and numerical results are going to be verified by measurement of temperature using magnetic resonance imaging (MRI) and through measurement of backscattered ultrasound waves, [3, 7, 8].

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References

[1] Woo, J., A Short History of the Development of Ultrasound in Obstetrics and Gynecology, esource Discovery Network, University of Oxford, retrieved March 12, 2012

[2] Teutsch, H.F., Schuerfeld, D., and Groezinger, E., Three-Dimensional Reconstruction of Parenchymal Units in the Liver of the Rat, Hepatology 29: 494–505 (1999)

[3] Kruglenko, E., The Influence of the Physical Parameters of Tissue on the Temperature Distribution during Ultrasound Interaction, Acta Bio-Optica et Informatica 18: 250–254 (2012)

[4] Barnett, S.B., Rott, H.-D., ter Haar, G.R., Ziskin, M.C., Maeda, K., The Sensitivity of Biological Tissue to Ultrasound, Ultrasound in Medicine & Biology 23: 805–812 (1997)

[5] Touloukian, Y.S., Liley, P.E., and Saxena, S.C., in *Thermophysical Properties of Matter*, The TPRC Data Series (IFI/Plenum, New York, 1970), Vol. 3, pp. 120, 209; Vol. 10, pp. 290, 589

[6] Gałka, A., Telega, J.J. and Tokarzewski, S., Nonlinear Transport Equation and Macroscopic Properties of Microheterogeneous Media, Arch Mech 49: 293–310 (1997)

[7] Kruglenko, E., Gambin, B., RF Signal Amplitude Statistics during Temperature Changes in Tissue Phantoms , HYDROACOUSTICS 17: 115–122 (2014)

[8] Doubrovina, O., Gambin, B., Kruglenko, E., Temperature Level and Properties of Wavelet Approximations of Back Scattered Ultrasound, HYDROACOUSTICS 17: 37–46 (2014)