A model of stiffness of normal interaction of spherical particles embedded in matrix

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Abstract

A model of a normal interaction of deformable spherical particles embedded in matrix is examined analytically. Normal interaction was modelled by linearly elastic one dimensional springs transferring axial forces only. Two, upper and lower, limits of an axial stiffness of the spring for the normal interaction are proposed and validated using a 3D finite element analysis with different ratios of elastic moduli of particles and matrix and distances between particles. The validation showed that the results of FEM are between the proposed limits of stiffness of the normal interaction for a connecting element with particles stiffer than interface member.

Keywords: stiffness of normal interaction, spherical particles, particulate composite, heterogeneous materials

1. Introduction

An approximation of a medium by one dimensional bars is widely used in the modelling of various heterogeneous and composite materials: concrete, rocks, geomaterials, biomaterials, etc. [1,2,3,4]. Since a unified approach of bonded particles still does not exist various Lattice and/or Spring Network Models can be applied to model particulate composites.

This work is aimed at the assessment of the axial stiffness of a normal elastic interaction of spherical particles embedded in a weaker or stiffer matrix. Two upper and lower limits of the stiffness were achieved and verified by 3D FEM. An analysis showed that the results obtained by FEM are between the developed limits when matrix is weaker than particles and the FEM results are slightly bit greater than the obtained upper limit.

2. Modelling concept and governing equations

A particulate composite consisting of particles embedded in a matrix is approximated by one-dimensional springs connecting centres of particles (Fig. 1). The springs are characterized by their length and axial stiffness K_s only (Fig. 1b).

It is assumed that the particle and interface materials obey linearly elastic law, the connecting springs undertake only axial forces being normal interaction forces, particles of the composite do not rotate, particles interact with an interface member by an entire surface of hemispheres.

All parameters related to particles are denoted by subscripts p while quantities related to interface members are denoted by a subscript b. The particle is characterised by radius R_p, elasticity modulus E_p and Poisson's ratio ν_p. The interface member is characterised by the cylinder radius R_b and the elasticity constants E_b and ν_b, respectively.

Two limits $K_{s,I}$ and $K_{s,II}$ of the axial stiffness K_s can be obtained by considering different operations of division of a connecting element: in the form of parallel and series connected springs (Fig. 2).

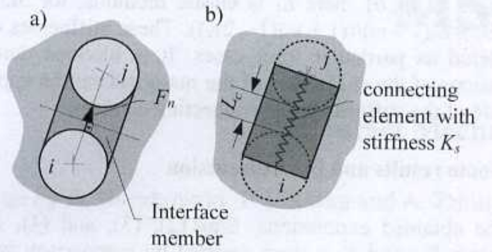

Figure 1: A normal interaction of two spheres through a conditional interface member (a) and a spring representing an interaction of the spheres (b)

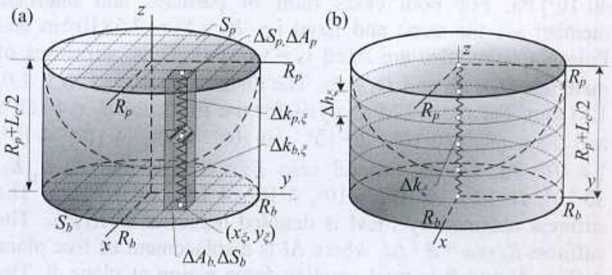

Figure 2: Concepts of the discretisation of a half of the conditional connecting members: by parallel connected prisms (*a*) and by sequentially connected rings (*b*)

For the two approaches, the total stiffnesses $K_{s,I}$ and $K_{s,II}$ of the connecting elements are as follows:

$$K_{s,I} = \lim_{\substack{\Delta A_{p,\xi} \to 0 \\ \Delta A_{b,\xi} \to 0}} \sum_{\xi} \frac{\Delta k_{p,\xi} \Delta k_{b,\xi}}{\Delta k_{p,\xi} + \Delta k_{b,\xi}}, \; K_{s,II} = \lim_{\Delta h_\xi \to 0} \frac{1}{\sum_\xi 1/\Delta k_\xi} \tag{1}$$

where $\Delta k_{p,\xi}$ and $\Delta k_{b,\xi}$ are stiffnesses of series connected infinitesimal prisms of areas $\Delta A_{p,\xi}$ and $\Delta A_{b,\xi}$ for the particle and interface member respectively (Fig. 2a), Δk_ξ is stiffness of a ring of infinitesimal thickness Δh_ξ (Fig. 2b).

Limits $K_{s,I}$ and $K_{s,II}$ given in Eqn (1) can be expressed as integrals

$$K_{s,I} = \pi \int_0^{R_b} \frac{\dfrac{D_p D_b}{\sqrt{R_p^2 - r^2}\left(L_c/2 + R_p - \sqrt{R_p^2 - r^2}\right)}}{\dfrac{D_p}{\sqrt{R_p^2 - r^2}} + \dfrac{D_b}{L_c/2 + R_p - \sqrt{R_p^2 - r^2}}} r dr \qquad (2)$$

$$K_{s,II} = \frac{1}{L_c/\left(\pi D_p R_p^2\right) + I} \qquad (3)$$

where I

$$I = 2\int_0^{R_p} \frac{dz}{\pi\left(R_p^2 - z^2\right)D_p + \pi D_b z^2} \qquad (4)$$

The integrals given in Eqns (2) and (4) can be computed analytically or evaluated numerically. It is easy to see that when $D_b = D_p$ Eqns (2) and (3) are equal, i.e. $K_{s,I} = K_{s,II}$.

In Equations (2), (3) and (4) R_p and R_b are radii of a particle and interface member respectively, L_c is distance between surfaces of particles (Fig. 1b), D_p and D_b are elasticity constants of the particles and the interface member respectively. These stiffnesses a generally depend on a stress–strain state of the particles and a bond member. For an uniaxial stress state $D_z = E_z$, $z \in \{p, b\}$, here E_z is elastic modulus, for 3D stress state $D_z = E_z(1 - \nu_z)/((1 + \nu_z)(1 - 2\nu_z))$. These stiffnesses can be considered as particular limit cases. It is obvious that other expressions of the stiffnesses of the materials can be applied to estimate of the stiffness of the connecting element.

3. Some results and brief discussion

The obtained expressions, Eqns (2), (3), and (4), for the stiffnesses $K_{s,I}$ and $K_{s,II}$, were verified by comparison with the results of the 3D FEM analysis. Two cases were considered: the first assuming fixed properties of a particle $E_p = 40$ GPa, and the enveloping values of matrix $E_b \in [1.4 \cdot 10^6, 40 \cdot 10^9]$ Pa; while for the second one vice versa: $E_b = 40$ GPa, $E_p \in [1.4 \cdot 10^6, 40 \cdot 10^9]$ Pa. For both cases radii of particles and interface member are the same and fixed i.e. $R_p = R_b = 7.5 \times 10^{-3}$ m and Poison's ratios also are fixed $\nu_p = \nu_b = 0.0$. Thus, stifnesses of materials $D_p = E_p$ and $D_b = E_b$. The distance $L_c \in \{0.1, 0.5, 1.0, 1.5\}$ mm. For the FEM analysis for the first case $E_p = 40$ GPa and $E_b \in \{E_p, 30 \cdot 10^9, 20 \cdot 10^9, 10 \cdot 10^9, 4 \cdot 10^9, 4 \cdot 10^8, 4 \cdot 10^7, 1.4 \cdot 10^6\}$ Pa, for the second case $E_b = 40$ GPa and $E_p \in \{E_p, 30 \cdot 10^9, 20 \cdot 10^9, 10 \cdot 10^9, 4 \cdot 10^9, 4 \cdot 10^8, 4 \cdot 10^7, 1.4 \cdot 10^6\}$ Pa. The stiffness obtained by FEM is denoted hereafter as $K_{s,FEM}$. The stiffness $K_{s,FEM} = R / \Delta l$, where Δl is displacement of free plane A (Fig. 3) and R is total reaction force acting at plane B. The FEM analysis was performed by ANSYS 12.

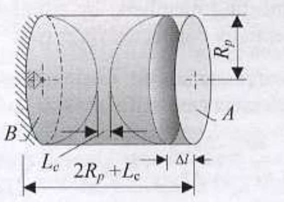

Figure 3: Geometry of the model under investigation

Dependence of the stiffnesses $K_{s,I}$ and $K_{s,II}$ on the modulus of elasticity of the interface member E_b and the particles E_p calculated by Eqns (2) and (3) (solid and dotted lines) is depicted in Fig. 4 and Fig. 5.

We can see it from Fig. 4 and Fig. 5 when $E_p \geq E_b$ then $K_{s,FEM} \in [K_{s,I}, K_{s,II}]$ and $K_{s,FEM}$ is closer to $K_{s,I}$ than $K_{s,II}$. When $E_b \geq E_p$ then for some values of E_p $K_{s,FEM} \geq K_{s,II} \geq K_{s,I}$.

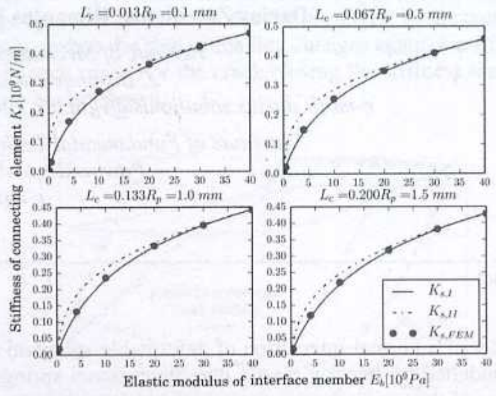

Figure 4: The dependence of the stiffnesses on the modulus of elasticity of interface member

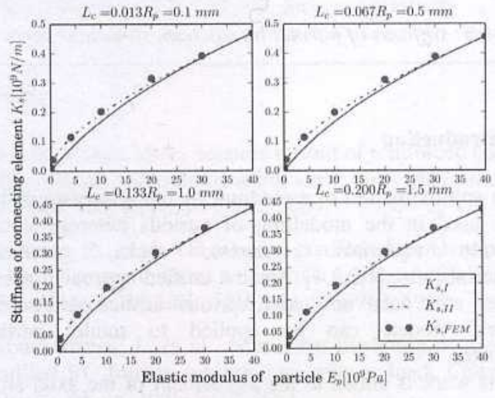

Figure 5: The dependence of the stiffnesses on the modulus of elasticity of particles

4. Concluding remarks

Evaluation of upper and lower limits of the stiffness of a normal interaction of spherical particles embedded in a matrix is suggested. Validity of the model was verified and confirmed by the results of the 3D FEM analysis. It was shown that the results obtained by FEM are between the established limits when the matrix is weaker than the particles.

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