Thermal properties of biomaterials on the example of the liver

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Abstract

Lionel Smith Beale, FRS, (1828–1906), a physician and microscopist in an evocative comparison wrote that the liver resembles a magnificent tree with its trunk and branches, with myriad of leaves, synthesizing and detoxifying. The liver in a human is about the size of football, equipped in a circulatory system and is made of about one million primary lobules which are almost identical, like the leaves of the tree. Therefore, the liver from mathematical point of view can be considered as a micro-periodic medium, and the mathematical methods of homogenisation developed for micro-periodic media can be applied to determine some overall properties of the tissue. Pennes equation of heat propagation in a biological tissue is a quasi-nonlinear partial differential equation with coefficients depending on temperature T. It consists of three terms, one of them describes Fourier heat diffusion, with the diffusion coefficient λ depending on T. This term is a subject of the contribution.

Keywords: Pennes equation, micro-periodic structure, effective conductivity

1. Introduction

After the discovery of ultrasound generators it was realized that absorption of high intensity ultrasound waves acts negatively on biological tissues. This observation led to research in tissue heating and healing effects. However, there is still little information on the effect of heating on absorption by tissues, which affects the size and shape of the thermal lesions. The absorption coefficient exhibited by a soft tissue varies widely from tissue to tissue and is a function of temperature, cf. [7].

The liver is composed of four lobes of unequal size and shape, with a rich micro-structure. A normal human liver weighs about 1.5 kg. The liver is a vital organ with a wide range of functions including protein synthesis and storage, transformation of carbohydrates, synthesis of cholesterol, bile salts and phospholipids, detoxification, and production of biochemicals necessary for digestion, [2, 3].

A hepatic lobule (Lat. *lobuli hepatis*) is a small division of the liver defined at the histological scale. It is about 1 million lobules in the human liver, each lobule containing at least 1000 sinusoids 0.5-1.0 mm in length, and 700 nm in breadth. There are over 1 billion sinusoids, with blood sluggishly flowing in parallel through each one.

A hepatocyte is a cell of the main parenchymal tissue of the liver. Hepatocytes make up 70-85% of the liver mass. The typical hepatocyte is similar to a cube with sides of 20-30 μm.

A liver sinusoid is a type of sinusoidal blood vessel (with fenestrated, discontinuous endothelium) that serves as a location for the oxygen-rich blood from the hepatic artery and the nutrient-rich blood from the portal vein. Sinusoidal capillaries are a special type of open-pore capillary also known as a discontinuous capillary, that have larger openings (30-40 μm in diameter) in the endothelium, [10].

2. Acoustic wave

George Döring Ludwig (1922 - 1973), pioneer in medical ultrasound, estimated the velocity of sound transmission in animal soft tissues between 1490 and 1610 m/s, with a mean value of 1540 m/s. He also determined that the optimal scanning frequency of the ultrasound transducer was between 1 and 2.5 MHz, and found that the speed of ultrasound and acoustic impedance values of high water-content tissues do not differ greatly from those of water, [12].

Acoustic wave propagating through a fluid in the direction x with the speed u, amplitude A and angular frequency ω is described by

$$\xi = A \sin \omega \left(t - \frac{x}{u} \right) \tag{1}$$

Here ξ denotes the displacement of the particle in the time t at point x. The velocity of vibrations $v \equiv \partial\xi/\partial t$ has the amplitude $v_0 = A\,\omega$. The amplitude of strain $\varepsilon \equiv \partial\xi/\partial x$ is $\varepsilon_0 = A\,\omega/u = v_0/u$. From Newton's equation we have

$$\rho \frac{\partial^2 \xi}{\partial t^2} = - \frac{\partial p}{\partial x} \tag{2}$$

where ρ is the density of the fluid and $p = p(x,t)$ is the acoustic wave pressure. Hence

$$\frac{\partial p}{\partial x} = A\,\rho\,\omega^2 \sin \omega \left(t - \frac{x}{u} \right) \tag{3}$$

and $p = p_0 + A\,\rho\,\omega\,u \cos\omega\,(t - x/u)$. The integration constant p_0 denotes the pressure in the fluid in absence of wave. The pressure variation is $\widetilde{p} \equiv p - p_0 = \widetilde{p}_0 \cos\omega\,(t - x/u)$, with the amplitude

$$\widetilde{p}_0 \equiv A\,\rho\,\omega\,u = \rho\,u\,v_0 \tag{4}$$

The stream of the energy $\mathcal{J} = wu$, where w denotes the mean value of the wave energy

$$w = \frac{1}{2}\,\rho\,v_0^2 = \frac{1}{2}\,\frac{\widetilde{p}_0}{\rho\,u} \tag{5}$$

After time averaging $\langle(...)\rangle \equiv (1/t)\int_0^t (...)\mathrm{d}\tau$ we get, so called, the acoustic pressure

$$p^* \equiv \langle \widetilde{p}_0 \rangle = w = \frac{\mathcal{J}}{u} = \frac{1}{2}\,\frac{(\widetilde{p}_0)^2}{\rho u^2} \tag{6}$$

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In a soft biologica tissue an acoustical wave with a speed 1500 m/s and frequency $\nu \equiv \omega/(2\pi) = 1$ Mhz has the length 1.5 mm. Let the acoustic pressure p^* be 1 N/m^2 = 1 J/m^3. Then $v_0 = (2/3) \cdot 10^{-6}$ m/s, and the amplitude of vibrations $A = (2/3) \cdot 10^{-3}$ nm, what is a sub-atomic length.

3. Attenuation of sound

Due to a sound propagating, there is always thermal loss of energy caused by viscosity. For inhomogeneous media, besides viscosity, acoustic scattering is another reason for the removal of acoustic energy, [5].

Absorption of ultrasound in biological tissues strongly depends on the molecular composition of the tissue. The absorption coefficient increases in function of a protein content, with collagen of a particularly high specific absorption. Collagen accounts for 10% in the liver, and the absorption coefficient is of 0.2 dB/cm. In water and body liquids there is little absorption 0.003 dB/cm, [1].

4. Thermal properties and Pennes equation

The heat in a living body is transferred by three different mechanisms: conduction, convection (natural or forced) and radiation. Average thermal conductivity of the liver in (W/(m ·K) is 0.52 with a standard deviation 0.03. The same numbers are found for the blood, [6]. The Pennes equation reads, [9],

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_k}\left(\lambda \frac{\partial T}{\partial x_k}\right) + w_b\, c_b\,(T - T_a) + q \tag{7}$$

Here, ρ and c are the mass density [kg/m^3] and specific heat [J/(kg · K)], respectively, w_b is the blood perfusion rate [m^3 blood / (s kg tissue)], T_a is the temperature of the arterial supply blood. The heat generation term q encompasses the thermal effects of metabolism and, if necessary, other volumetric heat loads, as microwave irradiation or the heat generated by ultrasound waves.

5. Nonlinearity of thermal properties

There is a considerable variation in thermal properties from tissue to tissue, from species to species, and even within tissues from the same donor. Thermal properties of water taken from [11] were fit to a linear equation over the range 0^0 to 45^0, it is $\lambda = 0.5652 + 0.001575T$ where λ is in (W/(m ·K). Thermal conductivity of a tissue is lower than that of water, while the temperature dependence approximates that of water, it is $\lambda = 0.4882 + 0.001265T$. Thermal diffusivity of a tissue matches the thermal diffusivity of water well for both the magnitude and the temperature coefficient.

6. One-dimensional time-independent problem

Let the section $[x_0, x_0 + \ell]$ of x axis consist of two subsections (segments) $[x_0, x_0 + a]$ and $[x_0 + a, x_0 + \ell]$. Let the heat conductivity be $\lambda_\mathrm{a}(1+\alpha T)$ in $[x_0, x_0+a]$ and $\lambda_\mathrm{b}(1+\beta T)$ in $[x_0 + a, x_0 + \ell]$.

For small α and β the temperature $T_\mathrm{a} = T(x_0 + a)$ is given by

$$T_\mathrm{a} = \\ = \frac{1}{b\lambda_\mathrm{a} + a\lambda_\mathrm{b}}\left(b\lambda_\mathrm{a}(T_0 + \frac{1}{2}\alpha T_0^2) + a\lambda_\mathrm{b}(T_\ell + \frac{1}{2}\beta T_\ell^2)\right) \tag{8}$$

In the linear case, for $\alpha = 0$ and $\beta = 0$ we have

$$T_\mathrm{a} = \frac{1}{b\lambda_\mathrm{a} + a\lambda_\mathrm{b}}\,(b\lambda_\mathrm{a} T_0 + +a\lambda_\mathrm{b} T_\ell) \tag{9}$$

For $\alpha > 0$ and $\beta > 0$ the expression (8) is always greater than (9).

7. Effective thermal conductivity of liver

Effective medium approximations describe a medium (composite material) based on properties and relative fractions of its components. These approximations include a Clausius-Mossotti's formula (CMF) for the effective conductivity of the medium consisting of the matrix substance of conductivity λ_M, in which small spherical inclusions of conductivity λ_I, are disseminated the ratio of the volume of all small speheres to that of whole being f,

$$\lambda^{\mathrm{eff}} = \frac{\lambda_I + 2\,\lambda_M + 2\,(\lambda_I - \lambda_M)\,f}{\lambda_I + 2\,\lambda_M - (\lambda_I - \lambda_M)\,f}\,\lambda_M \tag{10}$$

Vladimir Mityushev defined the effective thermal conductivity when the conductivity coefficient is a function of the temperature T, and found a generalization of CMF for a family of strongly non-linear and weakly inhomogeneous composites, [8]. Unfortunately, in the liver tissue the inhomogeneity of conductivities (collagen vs fluid) is strong.

A. Gałka, J. J. Telega and S. Tokarzewski noticed that soft tissues are usually anisotropic, and using asymptotic methods derived the formula for the effective heat conductivity in this general case, [4]. In one-dimensional case their formula reads

$$\lambda^{\mathrm{eff}} = A(\xi) + B(\xi) + \frac{C(\xi)}{T - D(\xi)} \tag{11}$$

where $\xi = a/\ell$, cf. Sect. 6, and A, B, C, D are given function of ξ. Their effective conductivity λ^{eff} is no more linear in the temperature, however. This formula is applied to describe the λ^{eff} of the liver.

References

[1] Barnett, S.B., Rott, H.-D., ter Haar, G.R., Ziskin, M.C., Maeda, K., The sensitivity of biological tissue to ultrasound, *Ultrasound in Medicine & Biology*, 23, pp. 805-812, 1997.

[2] Beale, L.S., *How to work with the microscope*, 4th edition, Harrison, London, 1868.

[3] Cotran, R.S., Kumar, V., Fausto, N., Nelso, F., Robbins, S.L., Abbas, A.K., *Pathologic basis of disease*, 7th edition, Elsevier Saunders, St. Louis, MO, 2005.

[4] Gałka, A., Telega, J.J., Tokarzewski, S., Nonlinear transport equation and macroscopic properties of microheterogeneous media, *Arch. Mech.*, 49, pp. 293-310, 1997.

[5] Girault, J., Ossant, F., Ouahabi, A., Kouame, D., Patat, F., Time-varying autoregressive spectral estimation for ultrasound attenuation in tissue characterization *Ultrasonics, Ferroelectrics, and Frequency Control, IEEE Transactions on*, 45, pp. 650-659, 1998.

[6] http://www.itis.ethz.ch/itis-for-health/tissue-properties/database/thermal-conductivity/

[7] Kruglenko, E., Gambin, B., Cieślik, L., Soft tissue-mimicking materials with various number of scatterers and their acoustical characteristics, *Hydroacoustics*, 16, pp. 121-128, 2013.

[8] Mityushev, V., First order approximation of effective thermal conductivity for a non-linear composite, *J. Tech. Phys.*, 36, pp. 429-432, 1995.

[9] Pennes, H. H., Analysis of tissue and arterial blood temperatures in the resting human forearm, *J. Appl. Physiol.*, 1, pp. 93 -122, 1948.

[10] Teutsch, H.F., Schuerfeld, D., Groezinger, E., Three-dimensional reconstruction of parenchymal units in the liver of the rat, *Hepatology*, 29, pp. 494-505, 1999.

[11] Touloukian, Y.S., Liley, P.E., Saxena, S.C., *Thermophysical Properties of Matter*, The TPRC Data Series (IFI/Plenum, New York, 1970), 3, pp. 120, 209, 10, pp. 290, 589.

[12] Woo, J., *A short History of the development of ultrasound in obstetrics and gynecology, esource Discovery Network*, University of Oxford, retrieved March 12, 2012.