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NODAL POSITIONS DISPLACEMENT SENSITIVITY OF AN ELEMENTARY ICOSAHEDRON TENSEGRITY STRUCTURE

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1 INTRODUCTION

The tensegrity structures form the cells of living organisms[1]. We will deal with the displacement sensitivity of elementary icosahedrons. They are important from the point of view of computational biology since they are the simplest models of a cell.

The structures consisting of struts and tendons undergo finite deformations. We are using an Updated Lagrangian formulation to describe it. The same refers to the design sensitivity formulation[2]. The constitutive model is visco-elastic (standard solid). Our design parameters are the positions of the nodes what gives the shape sensitivity. The design constraints are set on the chosen displacements. The elementary icosahedron is an element of a monolayered matrix of the cells.

2 FORMULATION

We adopt the incremental formulation in the Updated Lagrangian frame. The equation of equilibrium reads

$$\left(\left(\int_{\Omega^t} \mathbf{B}_{\mathrm{L}}'^{\mathrm{T}}\, {}_t^t\bar{\boldsymbol{\tau}} \mathbf{B}_{\mathrm{L}}' \, d\Omega^t \right) + \int_{\Omega^t} \mathbf{B}_{\mathrm{L}}^{\mathrm{T}} \Delta \mathbf{S} \, d\Omega^t \right) \Delta \mathbf{q} = \int_{\Omega^t} \mathbf{N}^{\mathrm{T}} \Delta \mathbf{f} \, d\Omega^t + \int_{\partial\Omega_\sigma^t} \mathbf{N}^{\mathrm{T}} \Delta \mathbf{t} \, d\left(\partial \Omega_\sigma^t\right) \tag{1}$$

where $\mathbf{B}_{\mathrm{L}}'^{\mathrm{T}}$ and $\mathbf{B}_{\mathrm{L}}^{\mathrm{T}}$ are the nonlinear and linear operators, $\mathbf{N}$ is the shape functions matrix, $\Delta\mathbf{S}$ is the stress increment, $\bar{\boldsymbol{\tau}}$ is the Cauchy stress matrix, $\Delta\mathbf{q}$ is the displacement increment, $\Delta\mathbf{f}$ and

$\Delta\mathbf{t}$ are the body forces and the boundary tractions increments. The integration is done over the domain $\mathbf{\Omega}$ and its boundary $\mathbf{\Omega}_{\sigma}$.

The constitutive model is visco-elastic such as the stress increment depends on total stress $\mathbf{S}$, the shear modulus (G), the bulk modulus (K) and the strain increment $\Delta\mathbf{E}$ as follows

$$\Delta\mathbf{S} = \mathbf{D}^{const}(\mathbf{S}, G, K)\Delta\mathbf{E} \qquad G(t) = G_o + \sum_{i=1}^{n} G_i \exp\left(\frac{-t}{\lambda_i}\right) \tag{2}$$

where t is the time and λ_i are the relaxation times of the particular parallel dampers. The equations (2) describe the generalized Maxwell model.

The design sensitivity equation obtained by differentiating of the equation of equilibrium (1) with respect to the design variable h is of the form

$$\left(\left(\int_{\Omega^t} \mathbf{B}_{\mathrm{L}}'^{\mathrm{T}}\, {}^t_t\boldsymbol{\tau} \mathbf{B}_{\mathrm{L}}' \, d\Omega^t \right) + \int_{\Omega^t} \mathbf{B}_{\mathrm{L}}^{\mathrm{T}} \mathbf{S} \, d\Omega^t \right) \frac{d\Delta\mathbf{q}}{dh} = \frac{d\Delta\mathbf{Q}}{dh} - \frac{d\Delta\mathbf{F}}{dh}\Big|_{\Delta\varepsilon\ fixed} \tag{3}$$

where $\Delta\mathbf{Q}$ is the load increment, r.h.s. of (1), $\Delta\mathbf{F}$ is the internal forces increment and $\mathbf{S}$ is the total stress at time t.

$$\frac{d\Delta\mathbf{F}}{dh} = \frac{d}{dh}\left(\left(\int_{\Omega^t} \mathbf{B}_{\mathrm{L}}'^{\mathrm{T}}\, {}^t_t\boldsymbol{\tau} \mathbf{B}_{\mathrm{L}}' \, d\Omega^t \right) + \int_{\Omega^t} \mathbf{B}_{\mathrm{L}}^{\mathrm{T}} \Delta\mathbf{S} \, d\Omega^t \right)\Big|_{\Delta\varepsilon\ fixed} \tag{4}$$

The equations (3) and (4) define the direct differentiation method, DDM.

2 NUMERICAL EXAMPLE

The icosahedral tensegrity structure is presented in Fig. 1. The cytoskeleton can slide on the surface. The right side of cellural matrix is fixed. We adopt the following data, namely, height of the cell 19 μm, cross-sectional areas of the tendons (filaments) 10nm^2, cross-sectional areas of the struts (microtubules) 190 nm^2, Young's modulus of the tendons 2.6GPa and the struts 1.2GPa, initial prestressing forces 20 nN, maximum loading 0.1N, relaxation time 1.0 sec, G_i/G_o ratio 0.91. The honeycomb pattern of the cells is shown in Fig 1a and the elementary icosahedral tensegrity structure which models the individual cells is shown in Fig 1b. The 6 pairs of the parallel struts are marked in blue color and the remaining bars are the tendons.

We observe the behaviour of matrix during the growth of the single cell which is placed in the middle of the matrix. Except for the displacement and stress field we follow the design sensitivity fields. We wish to observe the effects of elongation of all of the elements of particular cell. The local design derivatives of the internal forces with respect to the initial lengths of the bar are calculated separately for each bar and then they are assembled into the global r.h.s. in the design sensitivity equation.

The observed points are at the middle of loaded edge (3069) and close to the growing cell (1609). We set three cases for the design sensitivity, namely, case A – the design cell is the growing cell, case B – the design cell is close to loaded edge in the upper quarter of the matrix and case C – the design cell is in the lower quarter close to the fixed edge.

The displacement field is shown in Figure 2 (left). The field is not symmetric since the growing cell is not symmetric. The equilibrium paths are shown in Figure 2 (right). Observing Figure 3 (left and right) and Figure 4 (left) we may notice that the highest x-displacement sensitivity values at the observed points are in the situation when the design cell is the growing one (case A). Considering the case B we note that the design cell is more important for the x-displacement at node at the edge then for the x-displacement at node in the middle of the matrix. Analysing the case C we see that the design cell is almost equally important for both observed x-displacements, the sensitivities are almost equal there. The x-displacement sensitivities fields are shown in the Figure 4 (right) and in the Figure 5.

This type of the deducing can be important when analyzing the mechanotransduction and division of the cells[3].

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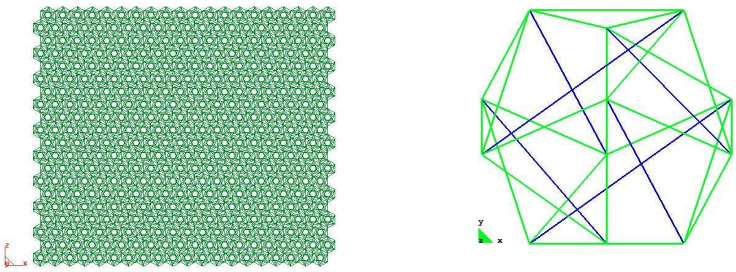

Figure 1. Honeycomb pattern of the cells (left), elementary cell (right)

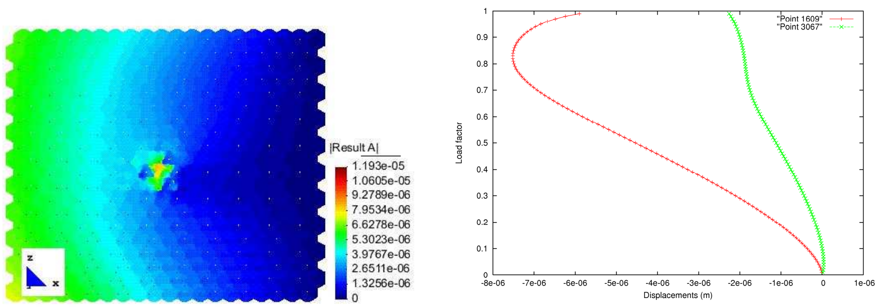

Figure 2: Displacement field at the end of the process (left), load factor versus x-displacement at observed points (right)

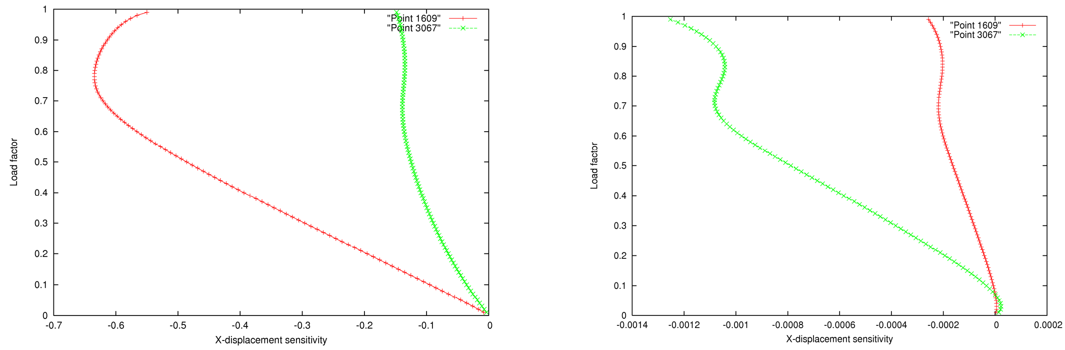

Figure 3: X-displacement sensitivities at observed points, case A (left), case B (right)

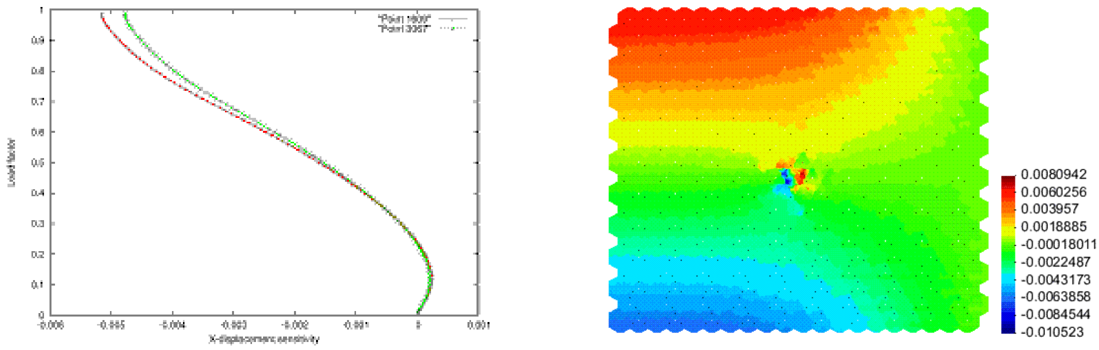

Figure 4: X-displacement sensitivity, case C (left), x-displacements sensitivity field, case A (right)

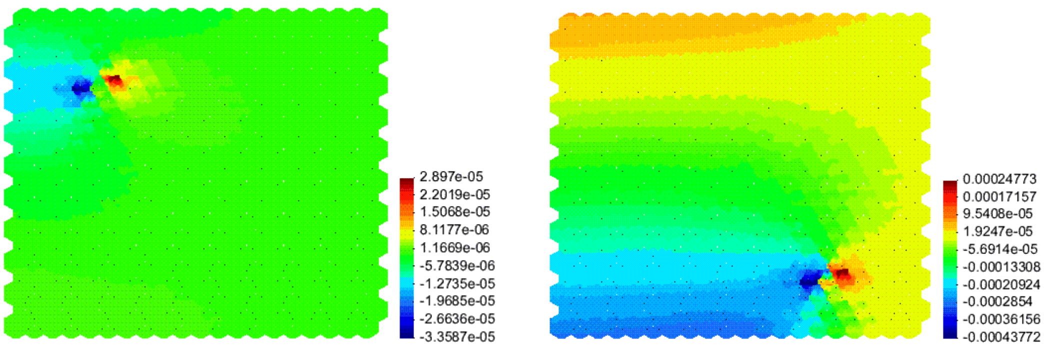

Figure 5: X-displacements sensitivity fields, case B (left), case C (right)