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# REPRESENTATIVE VOLUME ELEMENTS, MICROSTRUCTURAL CALCULATION AND MACROSCOPIC INVERSE IDENTIFICATION OF PARAMETERS FOR MODELLING SOUND PROPAGATION IN RIGID POROUS MATERIALS

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The micro-geometry of porous material is responsible for its sound absorption performance and should be now a design objective. Microstructural calculation of parameters and/or characteristic functions for acoustical models of porous materials with rigid frame requires the so-called Representative Volume Elements, that is, usually cubes which should contain several pores or fibres of typical sizes and distribution. The design of such RVEs, which correctly represent a typical micro-geometry of porous medium, is by no means an easy task since usually the RVE should be also periodic and 'isotropic' (identical with respect to the three mutually-perpendicular directions). The task is simpler in case of two-dimensional microscopic models of some fibrous materials, but such modelling is obviously rather approximative. Designs of periodic RVEs for porous foams and fibrous materials will be presented and used by FE analyses of microstructural problems defined by the application of the Multiscale Asymptotic Method to the problem of sound propagation and absorption in porous media with rigid skeleton. Moreover, a methodology of automatic generation of periodic RVEs with random arrangement of pores based on a simple bubble dynamics will be explained. Among other examples, designs of RVE cubes representative for a corrundum ceramic foam with porosity 90% will be shown and serve for microstructural calculation of some macroscopic parameters used in advanced acoustical modelling of porous media. The curves of acoustic impedance and absorption measured in the frequency range from 500 Hz to 6.4 kHz for two samples of the corrundum foam will be presented. These measurements will be used for inverse identification of relevant macroscopic parameters, namely: the tortuosity, the viscous and thermal permeabilities, and two characteristic lengths. The concurrence of some results obtained by the RVE-based micro-scale calculation and the measurement-based macro-scale identification will be shown.

## 1. Microstructural calculations and macroscopic parameters

Sound propagation in a porous material with rigid frame and open porosity can be very effectively modelled using the Helmholtz equation for time-harmonic acoustics with the porous material treated as an effective fluid resulting from the combined properties of the fluid in pores and the micro-geometry of rigid skeleton. It is, in general, a dispersive medium characterized by the effective speed of sound:

$$c_{\mathrm{eff}}(\omega) = \sqrt{\frac{K_{\mathrm{eff}}(\omega)}{\varrho_{\mathrm{eff}}(\omega)}} \tag{1}$$

which depends on the effective frequency-dependent bulk modulus $K_{\mathrm{eff}}$ and density $\varrho_{\mathrm{eff}}$ ($\omega$ is the angular frequency). Following the advanced modelling developed by many authors, these complex and frequency-dependent functions are expressed as follows[1]

$$K_{\mathrm{eff}}(\omega) = \frac{P_0}{1 - \dfrac{\gamma - 1}{\gamma \alpha'(\omega)}}, \qquad \varrho_{\mathrm{eff}}(\omega) = \varrho_0 \alpha(\omega), \tag{2}$$

where $P_0$ is the ambient mean pressure, $\varrho_0$ is the density of fluid in pores (usually, the air) and $\gamma$ is the ratio of specific heats for pore-fluid, whereas $\alpha = \alpha(\omega)$ and $\alpha' = \alpha'(\omega)$ are the frequency-dependent characteristic functions for the porous medium. The first one is the (dynamic) viscous tortuosity function which accounts for the visco-inertial interactions of the fluid in pores with the skeleton. These interactions make the effective fluid 'heavier' than the pore-fluid, so that the tortuosity function is always greater than 1. The second characteristic function is a thermal analogue of tortuosity. Instead of tortuosity functions the frequency-dependent permeability functions can be used, namely, the dynamic viscous permeability $k = k(\omega)$ and its thermal analogue $k' = k'(\omega)$; the corresponding tortuosity and permeability functions are related as follows:

$$\alpha(\omega) = \frac{\phi\nu}{\mathrm{i}\omega}\big(k(\omega)\big)^{-1}, \qquad \alpha'(\omega) = \frac{\phi\nu}{\mathrm{i}\omega}\big(k'(\omega)\big)^{-1}, \tag{3}$$

where $\nu$ is the kinematic viscosity of pore-fluid and $\phi$ is the porosity of porous medium. Unlike the tortusoity functions, the permeabilities do not depend on the pore-fluid but only on the micro-geometry of porous medium. One should also notice that in case of anisotropic porous materials, the dynamic viscous permeability and tortuosity will be second-order tensors, while their thermal analogues should always remain scalars.

Application of the Multiscale Asymptotic Method (MAM) to the very problem of sound propagation in a rigid porous medium[2,3] eventually leads to two uncoupled micro-scale Boundary-Value Problems (BVPs), namely:

- a harmonic, viscous, incompressible flow with no-slip boundary conditions on the skeleton walls, driven in the specified direction by the pressure gradient of unit amplitude, uniform in the whole fluid domain,
- a harmonic heat transfer (thermal flow) with isothermal boundary conditions on the skeleton walls, and the uniform source of unit amplitude in the whole fluid domain.

Both problems are defined on the fluid domain of a periodic Representative Volume Element (RVE) of the porous medium. Each of the micro-scale problems can be transformed into its re-scaled analogue where the flow velocity field in the first case, and the temperature field in the second case, are replaced by the fields of viscous and thermal permeability, respectively. The problems are uncoupled so they can be separately solved at some computational frequency and the obtained permeability fields are then averaged over the fluid domain, which yields the values of viscous and thermal permeability for this particular frequency.

Instead of calculating the permeability and tortuosity functions by solving the micro-scale BVPs at each of the computational frequencies, they can be determined using the formulas of the Johnson-Allard model.[1] To this end, the following macroscopic parameters (globally characterizing the geometry of porous material) must be known: the global porosity $\phi$, the tortuosity $\alpha_\infty$, the viscous permeability $k_0$, the thermal permeability $k'_0$, and two characteristic lengths – for viscous forces $\Lambda$, and for thermal effects $\Lambda'$. These parameters can be in fact measured experimentally, or they can be calculated from the microstructure. For example, the (static) viscous and thermal permeability parameters, $k_0$ and $k'_0$, are static flow permeabilities obtained from the relevant static micro-scale BVPs, that is, when $\omega = 0$.

## 2. Randomly-generated periodic cells for open-cell foams

In order to calculate the permeability functions or the parameters for the Johnson-Allard model one needs a small periodic cubic cell with a piece of skeleton (solid frame) inside. Such a cell must be representative for the (macroscopically homogeneous) porous medium to constitute its Representative Volume Element (RVE). Porous materials are usually not regular on the micro-scale level though their micro-geometry can be rather well described by some global parameters like porosity, generic characteristics like typical shape of pores or fibres, and some statistical data like, for example, typical sizes of pores and windows linking the pores in case of foams, or typical size of fibres and distances between them in case of fibrous materials. Therefore, it seems reasonable that RVEs may be constructed is a random way basing on some typical statistics of real porous media they tend to represent.

A comparatively simple random method of generation of RVEs for foams with spherical pores is proposed here. The method relies on dynamics of spherical bubbles which move and bounce each other while the cubic cell of RVE decrease its size. The bubbles may freely pass the walls of the cube yet the whole setup is periodic so they at once appear from the other side. (as a matter of fact each bubble is in fact an ensemble of eight identical bubbles set in corners of a cube with edge equal to the current edge-length of RVE cell). The bubbles diameters should be chosen to be consistent with the statistics of the real foam. There is also a penetration parameter which states how much the bubbles may penetrate each other; this parameter together with the assumed bubble sizes must yield typical sizes of windows linking the pores which are known form the foam statistics. Figure 1 depicts the idea in 2D, showing some periodic arrangements of moving bubbles in a few frames of motion (the solid square shows the boundary of periodic cell and the dashed circles inside the bubbles mark the maximal range of penetration). The whole procedure can be stopped when the global porosity is reached or when the size of the RVE cube cannot be decreased.

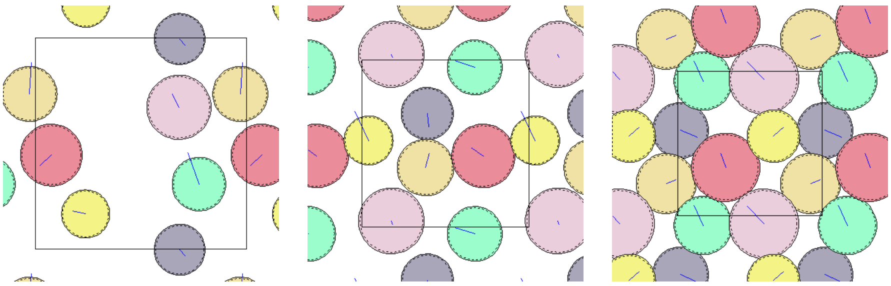

**Figure 1.** Periodic arrangements of moving bubbles

Figure 2 presents a periodic RVE (with five pores of various size) randomly-generated for an open-cell foam with global porosity equal to 70%. It shows the final arrangement of bubbles (pores), the solid skeleton, and a finite-element mesh of fluid domain; some geometric data is given in Table 1.

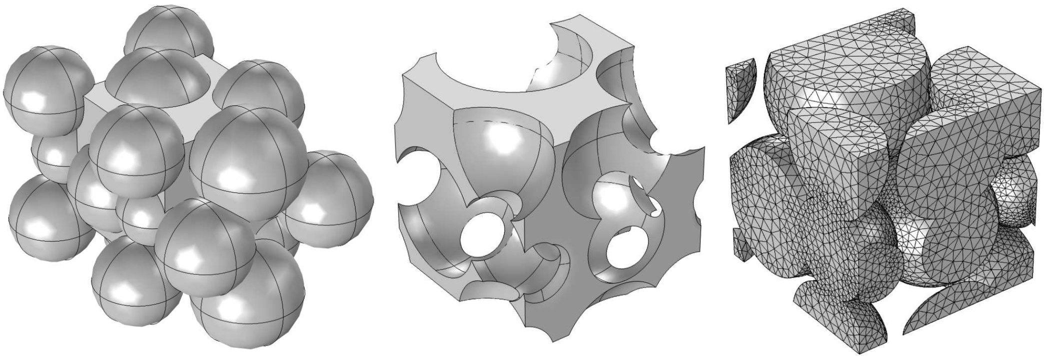

**Figure 2.** Randomly-generated periodic RVE of an open-cell porous foam with porosity 70%: bubbles (pores) and the RVE cube, the skeleton (frame), and a FE mesh of fluid domain

**Table 1.** Geometric data of the randomly-generated periodic RVE for open-cell foam of porosity 70%

| | | | | | |
|---|---|---|---|---|---|
| Porosity: **70%** | Number of pores in RVE: **5** | | | | |
| Ratio of the pore-diameters to the RVE edge-length: | 0.4948 | 0.6263 | 0.6301 | 0.6618 | 0.7761 |
| Pore-diameters [mm] for the assumed RVE edge-length of **0.5159 mm**: | 0.2553 | 0.3231 | 0.3251 | 0.3414 | 0.4004 |
| Volume [$mm^3$] of | RVE: 0.1373 | frame: 0.0412 | pores: 0.0961 | | |

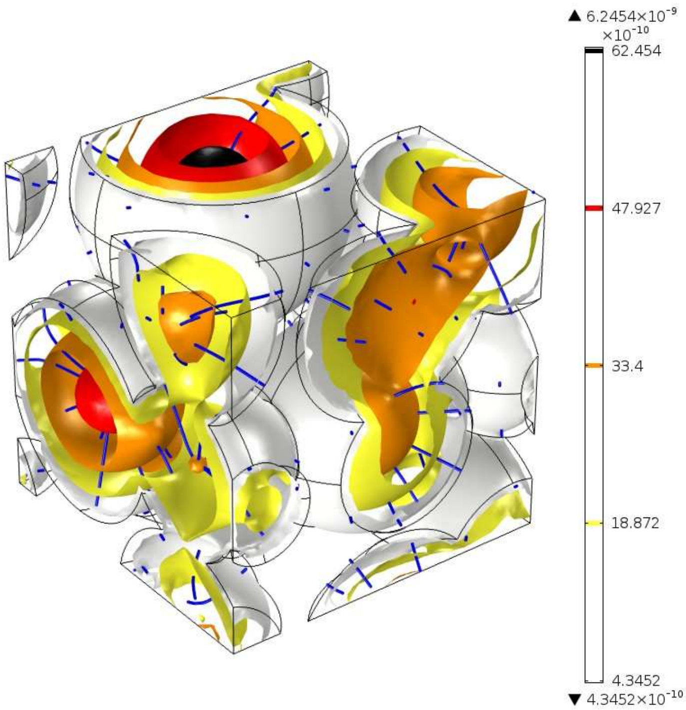


**Figure 3.** Thermal permeability field [$m^2$] for the randomly-generated RVE of open-cell porosity 70%

The finite-element mesh was used to solve the scaled heat transfer problem at the micro-scale for the unit source in the whole fluid domain – the solution (a thermal permeability field) is shown in Fig. 3; it was averaged over the fluid domain to yield the parameter of thermal permeability: $k'_0 = 1.65 \times 10^{-9}$ $m^2$. The thermal characteristic length was calculated as the ratio of the fluid domain volume to the doubled surface of pores and equals $\Lambda' = 0.138$ mm.

## 3. Regular periodic cells for fibrous and open-cell porous materials

Figure 4 presents a periodic RVE constructed for a fibrous material with porosity of 97%. The edge-length of RVE is 0.2 mm and it contains four fibres: one with diameter 0.024 mm, two with diameter 0.020 mm, and one with diameter 0.018 mm. The finite-element mesh shown also in Fig. 4 was used to solve the static viscous incompressible flow problem (or rather its re-scaled analogue) driven by the unit vector of pressure gradient in the $x_1$-direction. The solution, that is, the field of viscous permeability in that direction, is shown Fig. 5; the field was averaged over the fluid domain in order to determine the static permeability tensor component: $k_{0,11} = 1.43 \times 10^{-9}$ m$^2$.

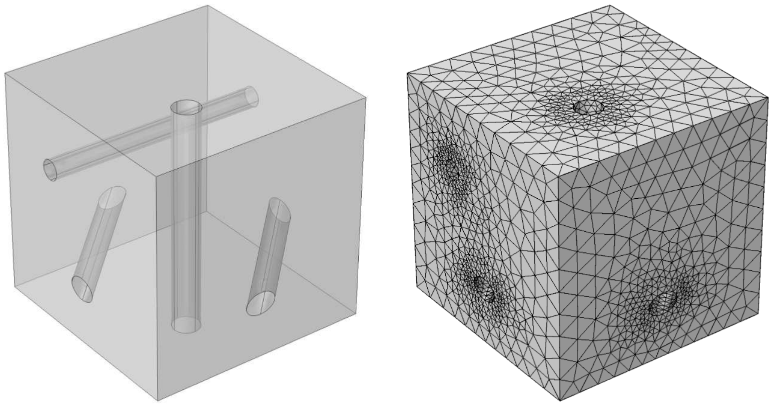

**Figure 4.** Regular periodic RVE containing four fibres and its FE mesh for a fibrous material with porosity 97%

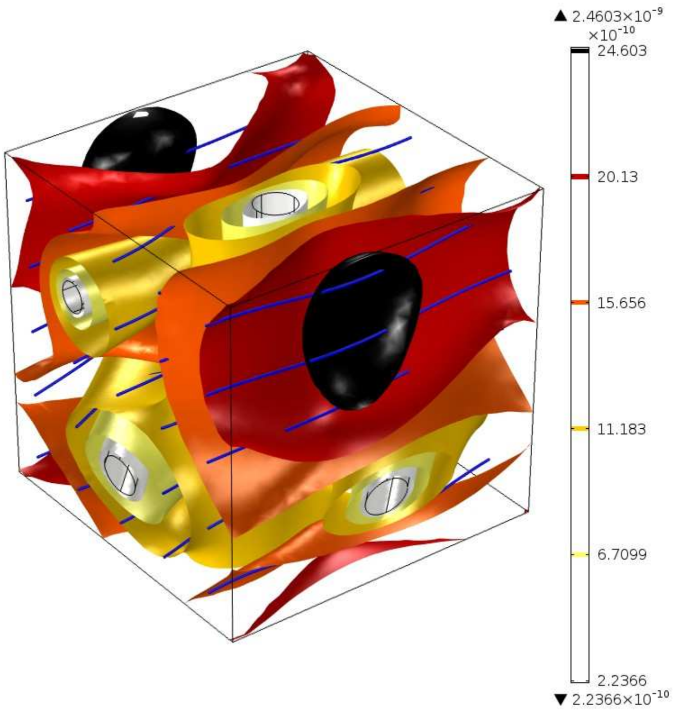


**Figure 5.** Viscous permeability field [m$^2$] for the regular RVE of fibrous material with porosity 97%

Figure 6 presents a periodic RVE constructed for an open-cell foam with porosity of 90%. This regular RVE is not only periodic but also identical with respect to the three mutually perpendicular directions; therefore, the viscous flow should be identical for each of these directions which renders the viscous permeability isotropic. The RVE was scaled so that its edge length is 0.8 mm and it contains eight pores: one internal pore (diameter 0.592 mm), three face pores (diameter 0.368 mm),

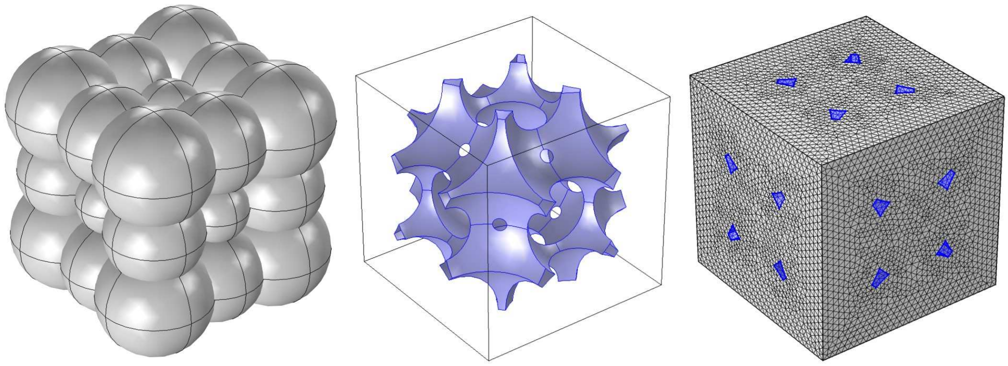

**Figure 6.** Regular RVE for an open-cell porous foam with porosity 90%: bubbles (pores), skeleton (frame), and a FE mesh of fluid domain

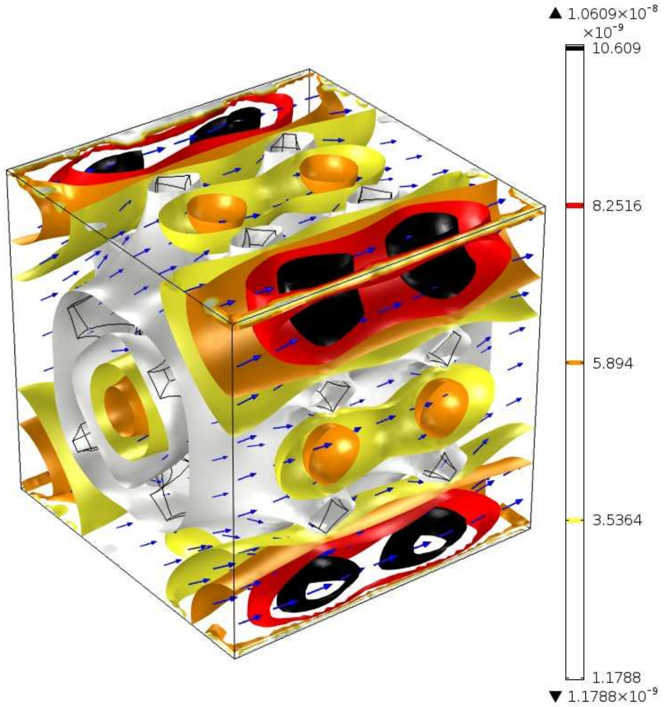


**Figure 7.** Viscous permeability field [$m^2$] for the regular RVE of open-cell foam with porosity 90%

three edge pores (diameter 0.544 mm), and one corner pore (diameter 0.640 mm). The solid skeleton as well as a finite-element mesh of the fluid domain are also depicted in Fig. 6. The FE mesh was used to find the viscous permeability field – see Fig. 7, and eventually, the parameter of static permeability $k_0 = 7.50 \times 10^{-10}$ m$^2$.

## 4. Acoustic absorption and inverse identification of macroscopic parameters of a porous ceramic material

Figure 8 shows the microstructure of corrundum ($Al_2O_3$) ceramics with porosity 90% and two cylindrical samples made of it. Statistical data for this material can be found in work by Potoczek.[4] Both samples have the same diameter of 29 mm and they only differ in height, namely, for the first sample it is $h_1 = 18$ mm and for the second one $h_2 = 24$ mm. Both samples were measured in the so-called impedance tube in order to determine their macroscopic acoustical characteristics, namely, the acoustic impedance and absorption coefficient (see Fig. 9), in the frequency range from 500 Hz to 6.4 kHz. Then, two curves (i.e., the real and imaginary part) of acoustic impedance for the first sample

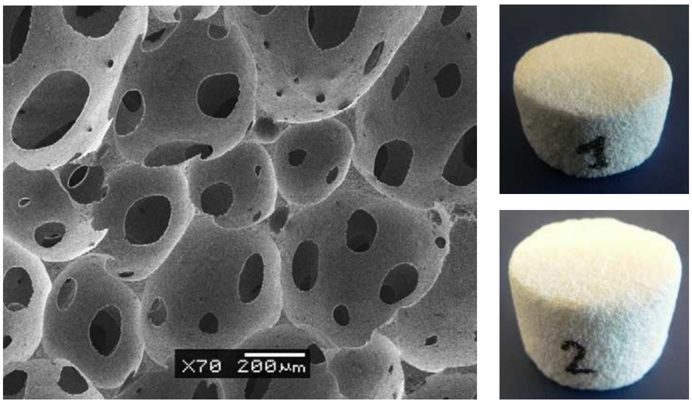


**Figure 8.** Microstructure[4] and two samples of porous corrundum ceramics with porosity 90%

**Table 2.** The initial and identified values of model parameters (and their ratio) for corrundum ceramics of porosity 90%

| Parameter | Symbol | Unit | Initial value | Identified value | Ratio |
|---|---|---|---|---|---|
| tortuosity | $\alpha_\infty$ | – | 2.0000e+000 | 1.5149e+000 | 0.76 |
| viscous permeability | $k_0$ | $m^2$ | 4.3456e-010 | 7.1304e-010 | 1.64 |
| thermal permeability | $k'_0$ | $m^2$ | 3.0603e-009 | 8.3739e-009 | 2.74 |
| viscous char. length | $\Lambda$ | m | 8.7895e-005 | 6.2392e-005 | 0.71 |
| thermal char. length | $\Lambda'$ | m | 1.1662e-004 | 2.7323e-004 | 2.34 |

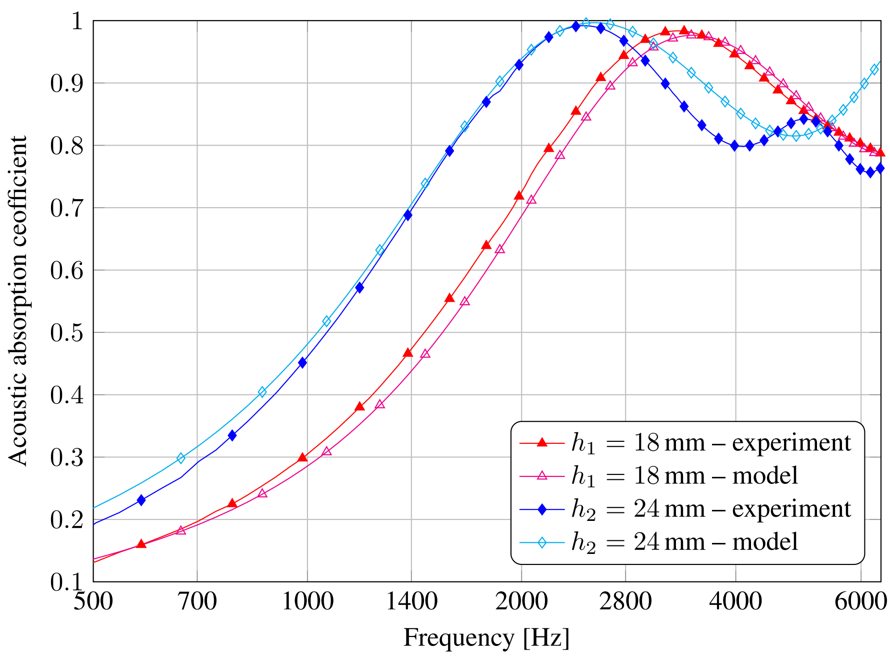


**Figure 9.** Acoustic absorption of both samples of corrundum ceramics with porosity 90%: the results of measurements and calculations using the Johnson-Allard model with the identified parameters given in Table 2

served for the inverse identification procedure – proposed by Zielinski[5] – which allowed to determine five missing macroscopic parameters for Johnson-Allard model; the directly and precisely specified (measured) porosity parameter of 90% was assumed to be known. The experimental curves obtained for the second sample served only for verification purposes. The initial and identified values of these parameters are given in Table 2. One should notice that the identified viscous permeability parameter, $k_0 = 7.13 \times 10^{-10}$ m$^2$, is similar with the value $7.50 \times 10^{-10}$ m$^2$ found from the microstructural analysis described in the previous section. This FE analysis used the RVE (see Fig. 6) scaled so that the main window linking the pores was similar with the average value found by Potoczek for this ceramic material.[4]

The parameter of porosity together with the five identified parameters were used to calculate the curves of acoustic impedance and absorption coefficient from the Johnson-Allard model. These curves proved to be in a very good agreement with the corresponding experimental curves obtained for both samples – as can be seen in Fig. 9 which compares the results of modelling and measurements for the absorption coefficient.

## 5. Conclusions

A procedure of automatic generation of periodic cells with random arrangement of pores (dedicated for foams with spherical pores) was demonstrated, together with some regular designs of periodic cells for fibrous and open-cell porous materials. The proposed examples of Representative Volume Elements served for microstructural, finite-element calculations of viscous and thermal permeabilities – two important parameters used in modelling of sound propagation in porous media. The FE analyses gave very reasonable results. Moreover, the viscous permeability parameter calculated for the specifically designed and scaled RVE dedicated for an open-cell corrundum ceramic foam with porosity 90% appeared to be similar with the value found from the inverse identification procedure using some of the curves of acoustic impedance and absorption experimentally determined for two samples manufactured from the very ceramics.

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