Searching for an optimal sensor location in the identification of laminates' parameters

Witold Beluch[1], **Tadeusz Burczyński**[1,2] **and Mirosław Habarta**[1]

[1]*Department of Strength of Materials and Computational Mechanics, Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland*
e-mail: witold.beluch,tadeusz.burczynski,wacław.kus@polsl.pl

[2]*Institute of Computer Science, Cracow University of Technology, ul. Warszawska 24, 31-155 Cracow, Poland*

Abstract

The aim of the paper is to find an optimal sensor location in order to perform an identification of laminates' elastic constants. As a result the identification procedure can be more accurate and less time-consuming. Simple and hybrid (with laminas composed of different materials) laminates are considered. To collect data necessary for the identification procedure modal analysis methods are used. This attitude allows reducing the number of sensor points to one. Sensitivity analysis of the measurements to the variation of identified parameters is performed. Global optimization methods in form of Evolutionary Algorithm and Artificial Immune System are employed to perform the identification task. Commercial finite element method software is employed to solve the direct problem for laminates. Numerical examples showing the influence of the sensor point location on the identification results are attached.

Keywords: laminates, inverse problems, sensitivity, identification, evolutionary methods

1. Introduction

Laminate elements are often produced in short series. To determine the elastic constants of the product non-destructive methods of the identification should be employed. The identification is typically performed on the basis of measurements of state fields, like displacements or stresses. Application of the modal analysis methods allows reducing the number of sensors to even one, as one can obtain a diagram instead of a single value at sensor point.

Sensor (or sensors) location strongly influences the results of the identification procedure. The aim of the paper is to find the best location of the sensor for laminates' identification and show the influence of the location of the sensor on the identification results. Commercial FEM software package MSC.Patran/Nastran is used to solve the direct boundary-value problem for laminate structures.

2. Identification of laminates' elastic constants

2.1. Simple and hybrid laminates

Laminates state a group of composites which consist of many layers. Each layer is composed of continuous phase (matrix) and long fibres, usually placed unidirectionally in each ply. Laminates have especially high strength/weight ratio comparing with other types of composites. Laminates can be usually treated as thin two-dimensional structures with four independent elastic constants [4]: two Young module E_1, E_2, one shear modulus G_{12} and one Poisson ratio ν_{12}.

In hybrid laminates particular plies are composed of different materials. The main reason of designing hybrid laminates is to find a balance between the cost and the required properties of the laminate. Interply hybrid laminates [1] have the internal layers made of a low-strength but cheaper material while the external layers are composed of a high-stiffness, but more expensive material. In the case of interply hybrids it is necessary to identify 4 elastic constants for each material and additionally the material densities, which increases the number of identified parameters to 10.

2.2. Laminates' identification task

The identification can be treated as the minimization of the functional $J(\mathbf{x})$ with respect to a design variables vector $\mathbf{x}$:

$$\min : \left[J(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^{N} \left(v_i - \hat{v}_i \right)^2 \right] \tag{1}$$

where: $\mathbf{x}$ – the vector of the design variables, $\hat{v}_i$ – measured values of the state fields, v_i – the same state fields values calculated from a numerical model, N – the number of measurements.

Gradient-based optimization methods are fast and precise, but in many engineering problems the calculation of the objective function gradient is complicated or even impossible. If the objective function has many optima, the gradient methods can direct to the local ones. To avoid the mentioned problems, the global optimization methods, like Evolutionary Algorithms or Artificial Immune Systems can be employed [2,3].

3. Optimal sensor location

3.1. Formulation of the problem

Identification belongs to inverse problems, which are mathematically ill-posed. In order to solve the identification task it is necessary to collect measurements in form of state fields' values from the considered structure. Identification procedure compares them with the values of the state fields calculated from the numerical model of the structure. An important problem is the choice of sensors location, which can significantly influence the effectiveness and precision of the identification process.

Typically, the location of sensors is determined by physical conditions and/or by intuition. The other approach is an application of efficient numerical algorithms of optimum experimental design. These algorithms usually base on the Fisher Information Matrix and A-optimality or D-optimality criterion [5] .

The research is financed from the Polish science budget resources in the years 2009-2012 as the research project

In the present paper modal analysis methods are employed to collect data necessary for the identification of laminates' elastic constants. The structure is excited by the sinusoidal signal of changing frequency and the accelerations in one sensor point are measured to obtain the frequency response diagram [6].

3.2. Sensitivity analysis

The aim is to find the location $\mathbf{x}_0$ of the sensor point where the measurements are the most sensitive to the variation of the identified parameters – material constants $\mathbf{C}=(E_1, E_2, G_{12}, \nu_{12})$. For a given sensor location $\mathbf{x}$ and vector of material constants $\mathbf{C}$, $F(\mathbf{x},\mathbf{C})$ denotes the corresponding frequency response function. It should be emphasized that $F(\mathbf{x},\mathbf{C})$ is not a single value, but a function belonging to the normed space $\mathbf{X}$ of all functions which can represent the frequency-amplitude response.

For a given sensor location $\mathbf{x}$, the derivative of $F(\mathbf{x},\mathbf{C})$ with respect to the each material parameter C_i is calculated (via finite difference approximation). The overall measure of the sensor location point quality may be defined as:

$$J(\mathbf{x},\mathbf{C}) = \left\| \frac{\partial F(\mathbf{x},\mathbf{C})}{\partial \mathbf{C}} \right\| = \sqrt{\sum_{i=1}^{4} \left\| \frac{\partial F(\mathbf{x},\mathbf{C})}{\partial \mathbf{C}} \right\|^2} \tag{2}$$

where the $\|\cdot\|$ sign denotes the norm in the X space - in practice calculated using discretised values of frequency-amplitude response.

The determination of the point where $J(\mathbf{x},\mathbf{C})$ attains maximum for wide range of material constants can make the identification procedure faster and less ambiguous.

It should be noted that the concepts and expressions given above are somehow simplified. In real problems, material parameters scaling should be performed to guarantee the equal influence of all material constants – it must be remembered that ν_{12} and E_{ij} (or G_{12}) are quantities which orders of magnitude are extremely different, which is also true for the corresponding sensitivities. Hence, all presented sensitivities are called "normalised sensitivities".

4. A numerical example – sensitivity analysis

A square laminate plate 0.2x02m made of 12 plies is considered (Figure 1). The stacking sequence of the laminate is: (10/15/45/60/0)s, where "s" denotes symmetry. Each ply is made of the same glass-epoxy material having elastic constants values: $E_1 = 38.6$GPa, $E_2 = 8.27$GPa, $G_{12} = 4.14$GPa, $\nu = 0.26$.

To solve the direct problem the plate is divided into 25 4-node finite elements. The plate is subjected to the sinusoidal load P in node 6 with maximum value P_{max} = 100kN. It is assumed that the excitation location does not change and the possible sensor locations are limited to chosen nodes of the mesh (points 1-16).

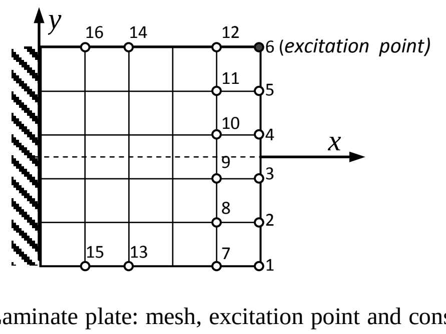

Figure 1: Laminate plate: mesh, excitation point and considered sensor locations (points 1-16)

The acceleration amplitudes in direction perpendicular to the plate plane are measured (numerical experiment) and the sensitivities are calculated for each location of the sensor point.

The results of the sensitivity analysis for selected points (the highest and the lowest normalised sensitivity values) are collected in Table 1.

Table 1: Sensitivity analysis – results

Pt.	$\left\|\frac{\partial F}{\partial E_1}\right\|$	$\left\|\frac{\partial F}{\partial E_2}\right\|$	$\left\|\frac{\partial F}{\partial G_{12}}\right\|$	$\left\|\frac{\partial F}{\partial \nu_{12}}\right\|$	$J(\mathbf{x},\mathbf{C})$	Sensor's rank
1	0.643	0.638	0.637	0.637	1.277	2
2	0.585	0.528	0.581	0.581	0.116	14
3	0.467	0.464	0.463	0.464	0.093	16
6	1.000	1.000	1.000	1.000	2.000	1
7	0.109	0.372	0.311	0.311	0.586	4
11	0.079	0.049	0.049	0.049	0.116	15
12	0.396	0.434	0.434	0.434	0.849	3

The proposed rank of sensors ordered accordingly to values of J is presented in the last column of Table 1. It can be seen that the best selection for sensor location are points: 6, 1, 12 and 7. To find the optimal location of the sensor point more precisely the adaptive mesh techniques can be employed. This attitude can be especially useful if the geometry of the laminate structure is more complicated.

5. Final conclusions

The paper is devoted to the searching of the optimal sensor location for identification problems. The identification of laminate material constants problem is considered. Influence of the sensor point location on the sensitivity of identified parameters is presented. As the result the location of the sensor (or sensors) significantly influences identification results. In the next step the global optimization methods supported by finite element method are employed to solve the identification task.

References

[1] Adali, S. et al., Optimal design of symmetric hybrid laminates with discrete ply angles for maximum buckling load and minimum cost, *Composite Structures*, 32, pp. 409-415, 1995.

[2] Beluch, W., Burczyński, T. and Kuś, W., Parallel Artificial Immune System in Optimization and Identification of Composite Structures, *Proc. of the XI Conference Parallel Problem Solving from Nature*, LCNS 6239, Springer-Verlag, Berlin Heidelberg, Vol. 1, p p. 171-180, 2010.

[3] Beluch, W., Evolutionary Identification and Optimization of Composite Structures, *Proceedings of the III European Conference on Computational Mechanics, ECCM 2006,* LNEC, Lisbon, CD-ROM, 2006.

[4] German, J., *The basics of the fibre-reinforced composites' mechanics*, Publ. of the Cracow University of Technology, Cracow, Poland, 2001 (in Polish).

[5] Patan, M. and Uciński, D., Optimal location of sensors for parameter estimation of static distributed systems, *Proceeding of PPAM 2001*, Wyrzykowski R. et al. Eds., LNCS 2328, pp.729-737, 2002.

[6] Uhl, T., *Computer-aided Identification of Constructional Models,* WNT, 1997 (in Polish).