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Mutually Orthogonal Golay Complementary Sequences in Synthetic Aperture Imaging Systems

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The main objective of this study is to improve the ultrasound image by employing a new algorithm based on transducer array element beam pattern correction implemented in the synthetic transmit aperture (STA) method combined with emission of mutually orthogonal complementary Golay sequences. Orthogonal Golay sequences can be transmitted and received by different transducer elements simultaneously, thereby decreasing the time of image reconstruction, which plays an important role in medical diagnostic imaging.

The paper presents the preliminary results of computer simulation of the synthetic aperture method combined with the orthogonal Golay sequences in a linear transducer array. The transmission of long waveforms characterized by a particular autocorrelation function allows to increase the total energy of the transmitted signal without increasing the peak pressure. It can also improve the signal-to-noise ratio and increase the visualization depth maintaining the ultrasound image resolution.

In the work, the 128-element linear transducer array with a 0.3 mm pitch excited by 8-bits Golay coded sequences as well as one cycle at nominal frequencies of 4 MHz were used. The comparison of 2D ultrasound images of the phantoms is presented to demonstrate the benefits of a coded transmission. The image reconstruction was performed using the synthetic STA algorithm with transmit and receive signals correction based on a single element directivity function.

Keywords: coded excitation, mutually orthogonal Golay codes, synthetic aperture, ultrasound imaging.

1. Introduction

In the past decade ultrasound imaging has become one of the preferred diagnostic techniques primarily because of its accessibility, the use of nonionizing radiation, and real-time display (Chan, Perlas, 2014; Maeda, 2014). High resolution ultrasound images are routinely obtained by employing phased array transducers and delay-and-sum beamforming techniques (Matrone *et al.*, 2015; Rabinovich *et al.*, 2013; Tanter, Fink, 2014). In this approach, however, the adequate focusing of the examined volume is achieved at the expense of the limited frame rate. Synthetic aperture (SA) methods offer a number of advantages over the conventional beamforming methods based on phased arrays: the higher spatial resolution due to the full dynamic focusing on the transmit and receive, lower power consumption due to the small number of elements used for ultrasound wave field generation at each transmission, and so on.

Another crucial factor for image quality in ultrasound imaging is decreasing of the signal-to-noise ratio (SNR) with depth. The severe attenuation of the ultrasonic signals in the tissue results in echoes from large depths literally buried in noise. To overcome this problem, the long wide band transmitting sequences and compression techniques on the receiver side can be applied. The average transmitted power increases proportionally to the length of the code. There are several papers in literature concerning a similar boundary-condition problem of signal compression in medical diagnostic imaging (Kim *et al.*, 2007; Klimonda *et al.*, 2005; Liu, Insana, 2005; Nowicki *et al.*, 2003).

Among the different excitation sequences proposed in ultrasonography, Golay codes evoke more and more interest in comparison with other signals. The reason for that lies in the fact that Golay codes, like no other signals, suppress to zero the amplitude of side-lobes. This type of complementary sequences has been introduced by Golay in the sixties (Golay, 1961). For the convenience of the reader a step-by-step principle of construction and properties of these sequences as well as correlation principle are described in (Trots *et al.*, 2004).

However, they are disadvantageous in that the frame rate is reduced to one half of a conventional pulse based B-mode imaging system due to the requirement that the pair of constituent codes need to be fired one after another. This stems from the fact that the two member codes are not orthogonal, so that separate transmission is inevitable to avoid crosscorrelation. In order to overcome this problem, mutually orthogonal Golay complementary codes were used to transmit coded sequences at the same time so as not to decrease the frame rate while increasing the SNR (Bae *et al.*, 2002; Kim, Song, 2003; Yang, Chakrabarti, 2012).

2. Synthetic transmit aperture method

As an alternatative to the conventional phased array imaging technique the synthetic transmit aperture (STA) method can be used instead (Trots *et al.*, 2009; 2010). It provides the full dynamic focusing, both in transmit and receive modes, yielding the highest imaging quality. In the STA method at each time one array element transmits a pulse and all elements receive the echo signals, where data are acquired simultaneously from all directions over a number of emissions, and the full image can be reconstructed from these data. The advantage of this approach is that a full dynamic focusing can be applied to the transmission and the receiving, giving the highest quality of image.

The simple model for the STA ultrasound imaging is given in Fig. 1. In transmission only a single element is used. It creates a cylindrical wave (in the elevation plane the shape of the wavefront is determined by the height of the transducer) which covers the whole region of interest. The received echo comes from all imaging directions, and the received signals can be used to create a whole image – in other words all of the scan lines can be beamformed in parallel. The created image has a low resolution because there is no focusing in the transmit. After all of the transducer elements have transmitted, the low-resolution images are summed and a high-resolution image is created.

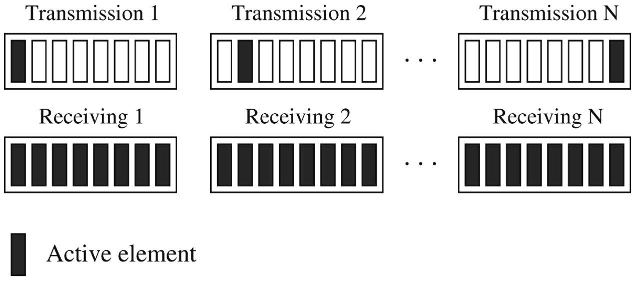

Fig. 1. Principle of the STA imaging method.

In the SA ultrasound imaging methods, for each point in the resulting image every combination of transmit-receive pairs contributes according to the round-trip propagation time only. The angular dependence is not taken into account in the applied point-like source model. But when the width of the array element is comparable to the wavelength corresponding to the nominal frequency of the emitted signal, the point-like source model becomes inaccurate. Here, a STA imaging algorithm, which accounts for the element directivity function and its influence is applied (Tasinkevych *et al.*, 2012; 2013).

3. Golay complementary sequences

Among the different excitation sequences proposed in ultrasonography, Golay codes evoke more and more interest in comparison with other signals. The reason of that lies in the fact that Golay complementary sequences, like no other signals, suppress to zero the amplitude of side-lobes. The pairs of Golay codes belong to a bigger family of signals, which consist of two binary sequences of the same length n, whose auto-correlation functions have the side-lobes equal in magnitude but opposite in sign. The sum of these auto-correlation functions gives a single auto-correlation function with the peak of $2n$ and zero elsewhere.

Figure 2 shows a pair of complementary Golay sequences, their autocorrelations, and the zero side-lobes sum of their autocorrelations.

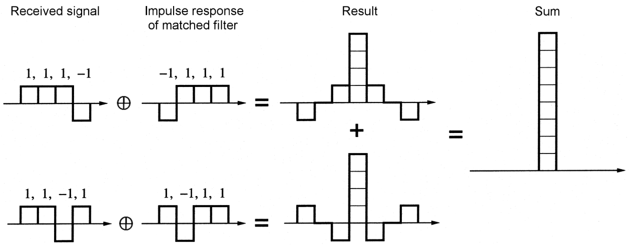

Fig. 2. Principle of side-lobes cancellation using a pair of Golay complementary sequences of the 4 bit length, where ⊕ denotes correlation.

As it can be seen from Fig. 2, the key to the side-lobes cancelling property of Golay code pairs is that the range side-lobes of one are equal in amplitude and opposite in sign to the side-lobes of the other.

4. Orthogonal Golay complementary sequences

The main advantage of Golay coding sequences is that they can be mutually complementary orthogonal codes that allow the transmission of coded strings at the same time. This makes it possible to maintain the number of frames unchanged while increasing the SNR (Chiao, Thomas, 2000). In the case where the orthogonal codes are transmitted simultaneously the echo signal for each of them should be separated by compression, i.e., during the correlation process.

A set of codes A_i and B_i, with the sequence length L for each $i = 1, 2, ..., M$, each of which forms a complementary set with a given impulse autocorrelation function is presented by

$$\begin{aligned} R_A(n) &= \sum_{i=1}^{M} \sum_{j=0}^{L-n-1} A_i(j)A_i(j-n) = ML\delta(n), \\ R_B(n) &= \sum_{i=1}^{M} \sum_{j=0}^{L-n-1} B_i(j)B_i(j-n) = ML\delta(n). \end{aligned} \tag{1}$$

Moreover, both A_i and B_i are mutually orthogonal if the sum of the correlation between them is zero, as given by

$$R_{AB}(n) = \sum_{i=1}^{M} \sum_{j=0}^{L-n-1} A_i(j)B_i(j-n) = 0, \tag{2}$$

where $\delta(n)$ is the Kronecker delta function (Tseng, Liu, 1972).

Hence, both A_i and B_i are complementary pairs of orthogonal codes when they meet (2).

In practice, M mutually orthogonal sets can be transmitted simultaneously by M transducers from the linear array without interfering on the reception side. To illustrate the proposed method, consider the following example of transmission by two elements ($M = 2$) using the orthogonal Golay sequences of the length $L = 4$. The two orthogonal sets of $\{A, B\}$ are then as follows:

$\mathrm{A}_1 = [\ 1\ -1\ -1\ -1\ ]$; $\mathrm{A}_2 = [\ -1\ -1\ 1\ -1\ ]$;

$\mathrm{B}_1 = [\ -1\ 1\ -1\ -1\ ]$; $\mathrm{B}_2 = [\ 1\ 1\ 1\ -1\ ]$.

The two code sequences $\{A_i, B_i\}$ are transmitted by two transducers {1 and 2} and repeated for two transmissions $i = 1, 2$, and the echo, recorded by each receiving transducer for each transmission. Then, the signal from the given transducer can be compressed and isolated from the echoes received by the neighboring transducers after summing correlations between each of the received sequence with the corresponding sequence transmitted by the same transducer. For example, in order to recover the echo for transducer 1 we obtain

$$P_1 = \sum_{j=1}^{2} S_j \oplus A_j, \tag{3}$$

where $\{\mathrm{S}_j, j\text{=}1,2\}$ are the two echo sequences and "$\oplus$" means the correlation operator.

The echo from the single point reflector for transmission from the transducer 1 will be compressed to $[0, 0, 0, 8, 0, 0, 0]$, while the echoes from the other transducers will be compressed to $[0, 0, 0, 0, 0, 0, 0]$.

A Golay pairs matrix may be constructed using the Hadamard matrix (Gan *et. al.*, 2010). A Hadamard matrix of order N (an even integer) is a square $N \times N$ matrix of binary elements with the property that any row (or column) differs from any other row (or column) in exactly $N/2$ positions. If the elements are denoted by '+1' and '−1', then the rows (or columns) of a Hadamard matrix are mutually orthogonal. For $N = 2^n$ (n is a positive integer), the well-known Walsh-Hadamard matrix is generated by the recursion (Huang, 2006):

$$H_N = \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix} \tag{4}$$

with the initial matrix $H_1 = [+1]$. By permuting all the rows in the upper half of H_N with the respective rows in the lower half of H_N, a permuted version of H_N can be defined as

$$\widetilde{H}_N = \begin{bmatrix} H_{N/2} & -H_{N/2} \\ H_{N/2} & H_{N/2} \end{bmatrix}. \tag{5}$$

As an example, the Golay-paired Hadamard 8×8 matrix is constructed using recursion (4) as

$$H_8 = \begin{bmatrix} +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 \\ +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 \\ +1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 \\ +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 \end{bmatrix}. \tag{6}$$

One can easily see that any given sequence in a row in the upper half of the matrix H_8 and the sequence given in the corresponding line in the lower part are the pairs of Golay sequences so that all sequences in the H_8 are complementary sets of sequences. These sequences are also orthogonal to each other.

Figures 3 and 4 show the mutually orthogonal Golay sets for $M = 2$ and $L = 8$.

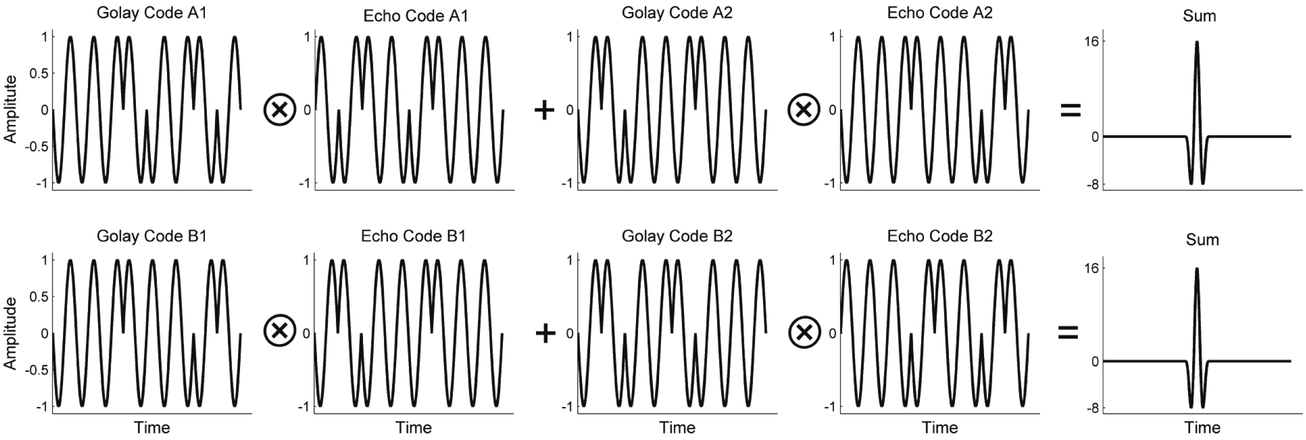

Fig. 3. Mutually orthogonal pairs of Golay complementary sequences of length 8 bits.

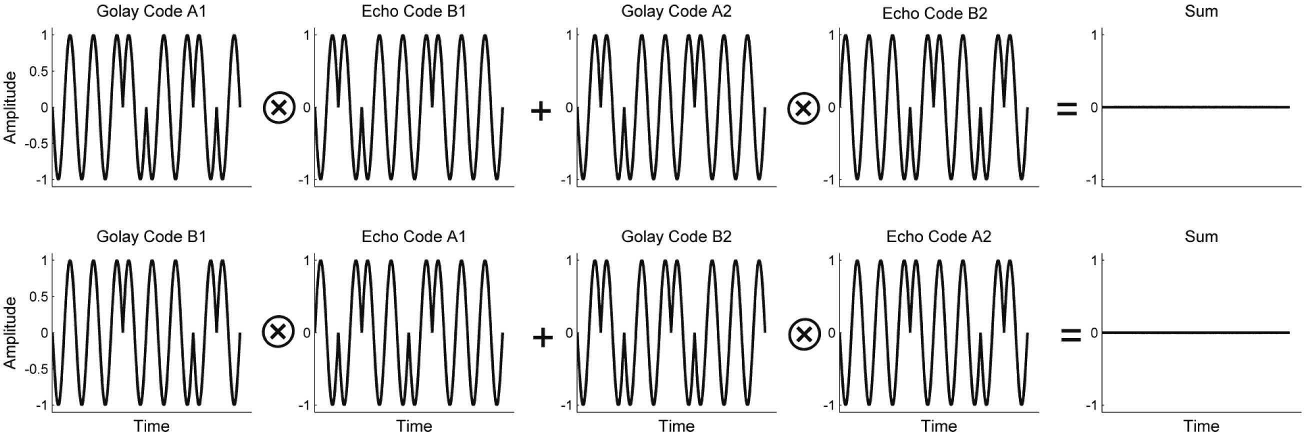

Fig. 4. Principle of mutually orthogonal Golay complementary sets for $M = 2$, $L = 8$.

5. Imaging methods

The two mutually orthogonal Golay sets are used: the first set consists of A_1 and B_1, and the second consists of A_2 and B_2, so that the sequences of each can be transmitted simultaneously without causing any harmful interference to each other.

In the used method the first transducer array element is excited by the sequence A_1 and the second transducer array element is excited by the sequence B_1 during the first transmission using orthogonal Golay sets. Next the first transducer array element is excited by the sequence A_2 and the second transducer array element is excited by the sequence B_2 during the second transmission. The above firing process is repeated again with another pair of the transducer array elements. The echo data is acquired simultaneously from all directions over a number of emissions, and the full image is reconstructed. As a result the two image lines can be constructed after two transmissions, which allows to increase the image frame rate twice in comparison to the conventional B-mode imaging method using one pair of Golay complementary codes only. Figure 5 shows the transmission scheme for orthogonal Golay sets in the STA method.

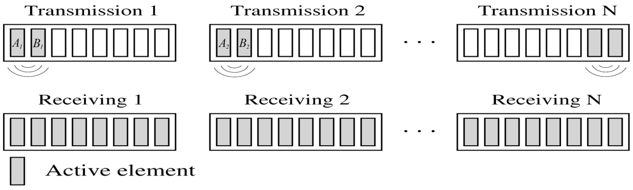

Fig. 5. Principle of the STA imaging method using mutually orthogonal Golay sequences for $M = 2$.

The application of mutually orthogonal sets allows to improve the image resolution in different depths. Figure 6 illustrates the ultrasound pulse transmission process using a mutually orthogonal Golay sequence set [A_1, A_2; B_1, B_2; C_1, C_2; D_1, D_2] with the length of L and $M = 4$ and the transmission to four focal points F_1, F_2, F_3, and F_4, which are on the same scan line.

In the first transmission, the transducer array elements focus ultrasound signals corresponding to the first coded sequence A_1 of the first Golay code at the

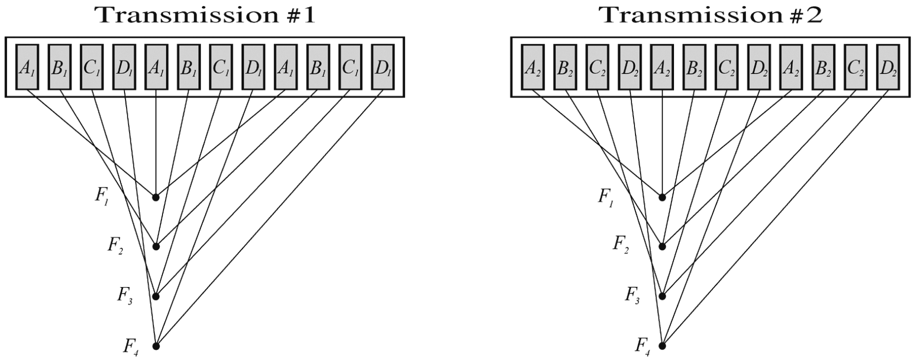

Fig. 6. Transmission schemes for an orthogonal Golay set for $M = 4$ and focal points at different depths.

focal point F_1. The transducer array elements corresponding to the first coded sequence B_1 focus ultrasound signals at the focal point F_2. The other elements which transmit coded sequences C_1 and D_1 focus signals at the focal points F_3 and F_4, respectively. The reception of the reflected signals occurs simultaneously by all transducer elements.

In the second transmission, the transducer array elements focus ultrasound signals corresponding to the second coded sequence A_2 of the first Golay code at the focal point F_1. The transducer array elements corresponding to the second coded sequence B_2 focus ultrasound signals at the focal point F_2. The other elements which transmit coded sequences C_2 and D_2 focus signals at the focal points F_3 and F_4, respectively. The reception of the reflected signals occurs simultaneously by all transducer elements.

Another novel imaging method that can be applied using mutually orthogonal Golay sets is sector mode imaging (Peng *et al.*, 2006). Each emission simultaneously transmits two plane waves with different transmission angles. The received echo signals related to different angles are isolated by the orthogonality property of the excited signal and used to construct two images of different areas (Fig. 7). Finally the two images are synthesized to one frame of a sector mode image.

One of the differences between such an imaging system and conventional sector B-mode imaging is that the imaging area of the former is rectangular while the one in the latter is a sector. In order to widen the view of the imaging area, two or more transmit waves with different angles are needed to illuminate the object to be imaged. As shown in Fig. 7, a plane wave with the transmission angle θ is transmitted from the transducer to illuminate the imaging area *AREA 1*, and another plane wave with the transmission angle $-\theta$ is then transmitted to illuminate the imaging area *AREA 2*. The above two imaging areas are synthesized to obtain an imaging area approximate to a sector format. The fact that only two transmissions are required to construct 2D or 3D images is a characteristic feature of this system. However, it must be pointed out, because each imaging area depends only on one transmission, the quality of such a system is badly influenced by noise.

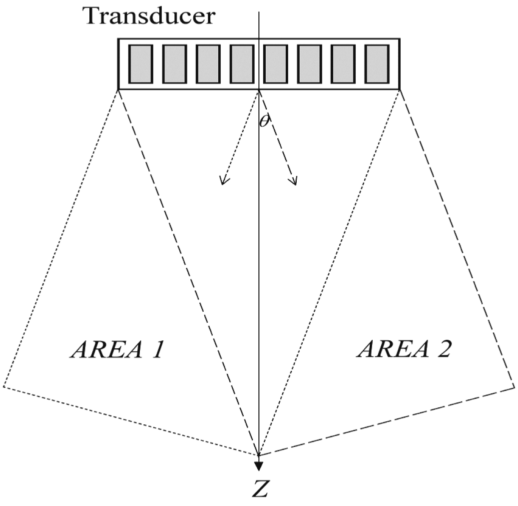

Fig. 7. Sector mode imaging using mutually orthogonal Golay sequences.

6. Results and discussion

All simulations in this work are carried out with a powerful software, *Field II* (Jensen, 1996) run under Matlab®. In the computer simulation the 128-element linear transducer array with the 0.3 mm inter-element spacing was applied. The mutually orthogonal Golay coded sets of the 8-bit length and one cycle pulse at a nominal frequency of 4 MHz were used. The phantom attenuation is equal to 0.5 dB/(MHz·cm). In the applied STA algorithm the element directivity correction scheme, discussed in (Tasinkevych *et al.*, 2012), was implemented to improve the image quality near the transducer aperture. The comparison of the obtained 2D images of the multi-scatterers phantom is presented in Fig. 8.

The obtained 2D ultrasound images clearly demonstrate the advantage of using the Golay coded se-

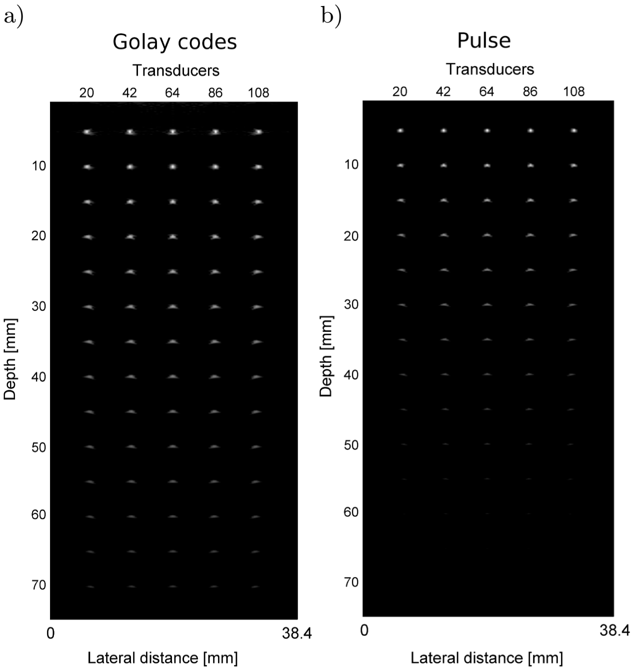

Fig. 8. 2D images of multi-scatterers phantoms for an 128-element linear array using: a) Golay sets with $M = 2$ and $L = 8$; b) one cycle pulse.

quences. With the elongation of the coded sequences the acoustical energy increases, yielding a higher SNR, which leads to an increase in the penetration depth while maintaining both axial and lateral resolutions. The latter depends on the transducer acoustic field and is discussed in (Nowicki *et al.*, 2007). When orthogonal 8-bits Golay sequences were applied the visualization depth is equal to about 7 cm (Fig. 8a), while in the case of applying one cycle this depth is equal to only 4 cm (Fig. 8b).

The cross-sectional fragment of the wire phantom (shown in Fig. 8) positioned at the depth of 40 mm is shown in Fig. 9. The normalization is performed with respect to the maximum values of the corresponding cross-section at the depth of 40 mm.

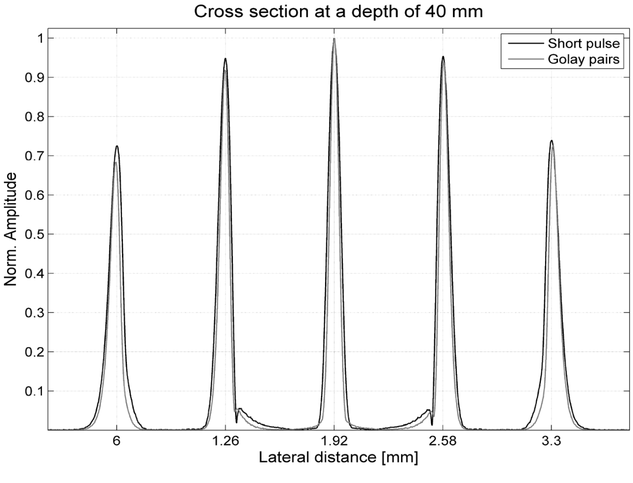

Fig. 9. Comparison of the lateral resolution at the depth of 40 mm. Black line – one cycle, gray line – Golay sets.

The resented results show that the penetration depth increases when Golay coded sets are used (Fig. 8), while the lateral resolution is comparable in both cases (Fig. 9).

7. Conclusion

Ultrasound imaging allows to visualize structures and organs in real-time, enabling an instantaneous evaluation of a clinical situation. But real problems appear when the reconstruction of the deeply located organs is needed. For that reason, coded excitation can be used making examination procedure more precise and allowing visualization of deeply located organs in 2D B-mode ultrasound imaging.

Another problem in medical ultrasound imaging is the image frame rate. To solve these problems, the mutually orthogonal complementary pairs of Golay coded sequences were defined and constructed.

A Golay-paired Hadamard matrix represents a special complementary sequence set in which the sequences can be grouped into Golay sequence pairs and they are orthogonal with each other. The mutually orthogonal Golay-paired Hadamard matrices represent a class of mutually orthogonal complementary sets of sequences.

The proposed imaging methods using mutually orthogonal Golay complementary sequences in the STA method are advantageous in that the frame rate is not sacrificed and that they are amenable to hardware implementation involving only binary insonifying pulsers while achieving a significantly better SNR over the conventional B-mode imaging method. Also a simultaneous transmit multi-zone focusing method using mutually orthogonal Golay provides a considerably improved lateral resolution without a sacrifice in the imaging frame rate. Applying of the coded transmission in the STA method in a standard ultrasound scanner could allow to increase the efficiency and quality of the ultrasound diagnostics.

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