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ELECTROMAGNETIC DAMPING OF A MECHANICAL HARMONIC OSCILLATOR WITH THE EFFECT OF MAGNETIC HYSTERESIS

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The paper presents a method of damping of mechanical vibrations by application of electromagnetic actuators. The physical phenomena that take place in such elements are related to magnetic induction and hysteresis. Induction, which consists in generation of electric current in a conductor placed within a variable magnetic field, has a precise quantitative description in form of the Faraday law. At the same time, magnetic hysteresis in ferromagnetic materials reflects the relation between magnetization of a given sample and magnitude of the external magnetic field that had been active during its whole history. Unfortunately, there is no at present a single commonly accepted model of this phenomenon. In this paper, some considerations by Bertotti (1998) are incorporated to this subject. The authors use a bilinear magnetization curve, which depicts the phenomenon of magnetic saturation. Analysis of the derived equations of motion of the coupled electromagneto-mechanical model revealed that the effect of hysteresis on reduction of vibration amplitude is rather of minor impact, however noticeable. At the same time, the electromagnetic damping itself is powerful. To support the conclusions, appropriate time histories, graphs of the logarithmic damping decrement as well as resonant characteristics are given here as in the previous authors' paper.

Key words: damping of vibration, electromagnetic actuator, magnetic hysteresis, saturation

1. Introduction

One of the methods of damping of mechanical vibrations is making use of electromagnetic actuators incorporating magnetic induction and hysteretic phenomena. The induction was discovered in the first half of the 19-th century independently by Faraday and Henry, and was precisely quantitatively described in form of the Faraday law. Magnetic hysteresis in ferromagnetic materials means a relationship between magnetization density in a given sample and history of the surrounding magnetic field the sample has been continuously exposed to. The hysteresis is also vulnerable to mechanical and thermal treatment of the sample as well as to its geometry, stress state and temperature. Still, physicists permanently look for a general theory explaining the phenomenon of magnetic hysteresis. Considerations on that subject can be found in, above all, a monograph by Bertotti (1998) as well as in Knoepfel (2000), Nicolaide (2001) and papers by Ossart and Meunier (1990) and Fallah and Moghani (2006).

Recently, Przybyłowicz and Szmidt (2008) examined vibrations of a mechanical harmonic oscillator suspended between two electromagnets, however without the effect of magnetic hysteresis taken into consideration. In this paper, the authors analyse the influence of hysteresis on damping of vibrations in an analogous system. A simple model describing the phenomenon of hysteresis is introduced. It depicts the hysteresis as a result of induced eddy currents in a ferromagnetic material subject to a variable magnetic field. The model was formulated by Bertotti (1998) and further employed by Dziedzic and Kurnik (2002) and Dziedzic (2005) who analysed vibrations of rotating shafts supported on journal bearings stabilised by electromagnetic actuators. Some research on journal bearing systems was also attempted by Przybyłowicz (2001) who employed piezoceramic actuators to reach the stabilisation goal.

2. Analysed system

Between two identical electromagnets, a mechanical harmonic oscillator of mass $m = 1.5\,\mathrm{kg}$ is suspended on springs whose resultant stiffness equals $k = 15000\,\mathrm{N/m}$, see Fig. 1. Inside the oscillator there are steel cores embedded, which together with the electromagnets create a magnetic circuit with length $l = 200\,\mathrm{mm}$ and diameter $2a = 3\,\mathrm{mm}$. Around each of the electromagnetic cores, $N = 320$ wire coils having electric resistance $R = 1.22\,\Omega$ are wound (copper with wire diameter $0.25\,\mathrm{mm}$) to which a constant voltage U is supplied. The gaps between the armature and electromagnets are: $z_1 = \delta + x$, $z_2 = \delta - x$, where $\delta = 1\,\mathrm{mm}$, and $x \in [0, \delta]$ is a mechanically constrained displacement of the armature in the direction of the right-hand electromagnet. The parameter being changed in the investigations is the supply voltage U.

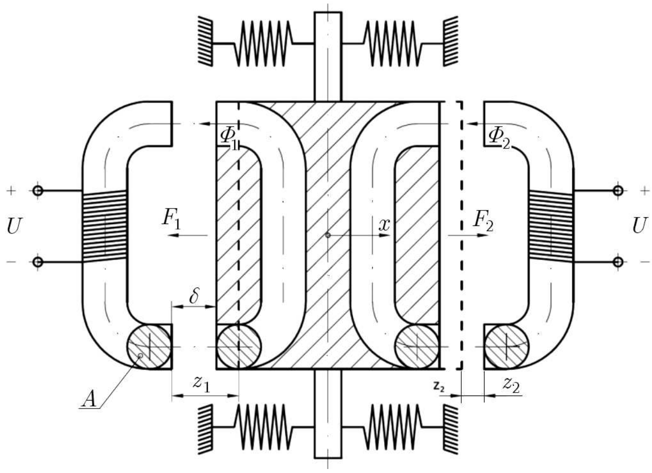

Fig. 1. Analysed system – harmonic oscillator suspended between two electromagnets

Electric conductivity of the steel the cores are made of (Si-Fe alloy, see Bertotti, 1998) amounts to $\sigma = 2 \cdot 10^6\,(\Omega\mathrm{m})^{-1}$. Although, to describe its magnetic properties a bilinear characteristic of magnetization is assumed

$$B = \phi(H) = \begin{cases} \dfrac{B_s}{H_s}H & 0 \leqslant H \leqslant H_s \\ B_s + \mu_0(H - H_s) & H > H_s \end{cases} \tag{2.1}$$

where $H_s = 500\,\mathrm{A/m}$ denotes the electromagnetic field intensity at which the cores become magnetically saturated, $B_s = 1.5\,\mathrm{T}$ is the magnetic induction corresponding to saturation and $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{Tm/A}$ is the magnetic permeability of vacuum. The $B - H$ curve and issuing curve of relative magnetic permeability $\mu = \mu(H) = \phi(H)/(\mu_0 H)$ are shown in Fig. 2.

Actually, the true magnetization curve has a more complex shape, see Bertotti (1998). However, if the vibrations are not excessive and the supplied voltage not very high, then the assumption of a linear character of the core material does not considerably affect the behaviour of the system (Przybyłowicz and Szmidt, 2008). The bilinear magnetization characteristic additionally allows one to take into account the effect of magnetic saturation, which leads to the appearance of new mechanical equilibrium points. It was thoroughly explored by Dziedzic (2005).

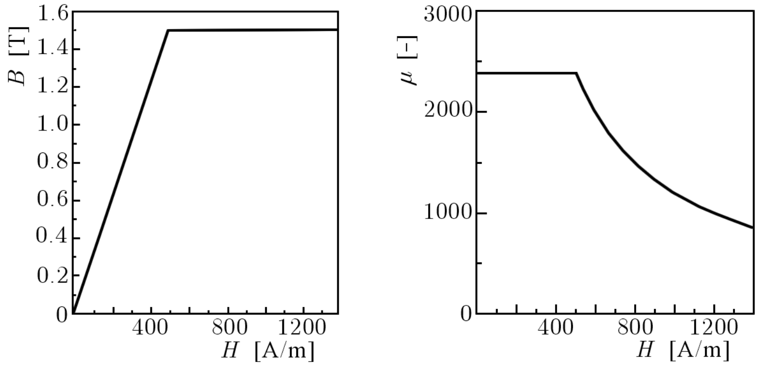

Fig. 2. Primary magnetization curve and relative magnetic permeability of steel

3. Model of magnetic hysteresis

In this paper, a model proposed by Bertotti (1998) that explains the magnetic hysteresis phenomenon via losses brought about by eddy currents is applied to the examinations. Its complete formulation enables description of the static and dynamic hysteresis loop. For a given sample, the static loop depends only on the amplitude of variable magnetic field, though the dynamic loop is also affected by frequency and form of such variations (sine, triangle, etc.).

An accurate description of both types of hysteresis loops requires identification of several physical parameters pertaining exclusively to the given sample. Because of that, the study will be confined to the model of a dynamic loop with other effects related to the structure of magnetic domains neglected.

In the first subsection, the decomposition of energy losses depending on the scale in which eddy currents are induced will be described. This will enable us to consider the type of hysteresis we are exactly interested in. Its description requires reaching back to Maxwell's equations, so let then refer to them with the simplifying assumption. Next, we derive the constitutive equation for the steel core placed in a variable magnetic field. This will relate the external magnetic field with the average intensity of the field inside the core. Let us repeat Bertotti's calculations for the sample analysed in our case, and then formulate the assumption for which the obtained equation well approximates the reality. At the end of this section, a set of non-linear ordinary differential equations will be derived, which links motion of the mechanical oscillator with electromagnetic phenomena occurring in the cores and winding.

3.1. Energy losses

Having known the eddy current density $\boldsymbol{j}(\boldsymbol{r},t)$ in any point of the sample and any time instant t, it would be possible to calculate the thermal energy lost during one cycle per unit volume ($\mathrm{J/m^3}$, being quantitatively equal to the hysteresis loop area) from the equation

$$\frac{P}{f} = \frac{1}{V}\int\limits_V dV \int\limits_0^{1/f} \frac{|\boldsymbol{j}(\boldsymbol{r},t)|^2}{\sigma}\,dt \tag{3.1}$$

where σ is the electric conductivity of the material of the sample, V – its volume, f – frequency of the field variability.

Unfortunately, description of the eddy current distribution is very difficult as it depends not only on geometry of the sample and some material constants, but on the structure of magnetic domains as well. The proposed by Bertotti (1998) model depicts the induced eddy current as a sum of currents brought about by randomly appearing Barkhausen's jumps, i.e. local changes in the magnetic domains. The averaging of the effect of individual jumps leads to decomposition of energy losses (3.1) into three elements: hysteresis loss, classical loss, and the so-called excess loss

$$\frac{P}{f} = C_0 + C_1 f + C_2\sqrt{f} \tag{3.2}$$

The first element is responsible for the static – independent of the frequency of field variability – hysteresis loop. The other two describe the dynamical loop. The constants C_0 and C_2 require identification, while C_1 can be found from geometry of the given sample, its electric conductivity and the function describing time variability of the magnetic field.

According to Bertotti (1998), for a metal sheet made of Si-Fe alloy of thickness 0.21 mm subject to sinusidally changing magnetic field with the maximum induction 1.5 T, these constants are: $C_0 = 33$, $C_1 = 0.058$, $C_2 = 1.4$.

To the oscillating system considered in this paper, a model describing the second, linear with respect to frequency, element of the hysteresis will be applied. It results from the eddy current flowing in the macro-scale, when the sample material is treated as homogeneous. Since then, this current will be denoted by $\boldsymbol{j}$, having in mind that the total current induced in the sample is greater.

3.2. Maxwell's equations

In the preceding analysis, Maxwell's equations will be incorporated. They include respectively: Faraday's, Ampère's and Gauss's laws for electric and magnetic fields in differential or integral forms. In Eq. (3.1) there appears one component only since in mechanical systems we usually face with relatively low frequencies, for which Maxwell's correction to Ampère's law is negligible

$$\begin{aligned}
&\operatorname{rot}\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad && \int\limits_{\gamma} \boldsymbol{E}\, d\boldsymbol{l} = -\frac{d\varPhi}{dt}\\
&\operatorname{rot}\boldsymbol{H} = \boldsymbol{j} && \int\limits_{\gamma} \boldsymbol{H}\, d\boldsymbol{l} = I\\
&\operatorname{div}\boldsymbol{D} = \rho && \int\limits_{\varGamma} \boldsymbol{D}\, d\boldsymbol{S} = \int\limits_{V} \rho\, dV\\
&\operatorname{div}\boldsymbol{B} = 0 && \int\limits_{\varGamma} \boldsymbol{B}\, d\boldsymbol{S} = 0
\end{aligned} \tag{3.3}$$

In the above equations the following denote: $\boldsymbol{D}$ [C/m^2] – electric induction, $\boldsymbol{B}$ [T] – magnetic induction, $\boldsymbol{E}$ [V/m] – electric field intensity, $\boldsymbol{H}$ [A/m] – magnetic field density, $\varPhi$ [Wb] – magnetic flux, $\boldsymbol{j}$ [A/m^2] – current density, I [A] – resultant current, ρ [C/m^3] – charge density. In the integrals in Eq. (3.3), the symbol γ denotes any arbitrary spatial curve, while $\varGamma$ a surface enclosing the space V (the curve and the surface are closed and regular). Additionally, the following relationships hold

$$\boldsymbol{D} = \epsilon \boldsymbol{E} \qquad\qquad \boldsymbol{j} = \sigma \boldsymbol{E} \tag{3.4}$$

where ϵ is electric permeability of the medium (with linear electrisation of the core material assumed).

3.3. Constitutive equation of the core

In Fig. 3, an infinitely long steel core is shown, along which an external, time-variable magnetic field $\boldsymbol{H}_a = [0,0,H_a]$, $H_a = H_a(t)$ is applied. Let $\boldsymbol{H} = [0,0,H]$, $H = H(x,y,t)$ be the magnetic filed in the core, $\boldsymbol{B} = [0,0,B]$, $B = B(x,y,t)$ – the induction corresponding to that field, and $\boldsymbol{j} = [j_x, j_y, 0]$, $j_x = j_x(x,y,t)$, $j_y = j_y(x,y,t)$ – the induced eddy current density. If the cores of magnetic circuits are sufficiently long, then the above model can be applied to the considered system. Then $\boldsymbol{H}_a$ will be the magnetic field generated by the electromagnet winding.

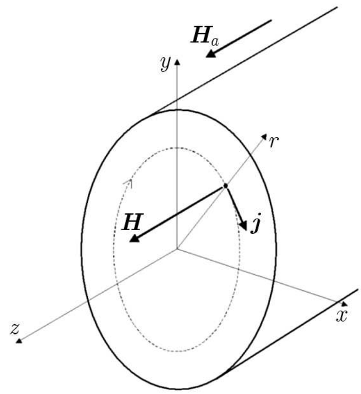

Fig. 3. The steel core placed in an axially directed variable magnetic field

Maxwell's equations in their differential form (3.3) and formulas (3.4) lead to the following problem of the magnetic field occurring within the core

$$
\begin{aligned}
&\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = \sigma \frac{\partial B}{\partial t} \\
&H\Big|_{\partial A}(t) = H_a(t) \qquad \frac{\partial H}{\partial x}(0,0,t) = 0 \qquad \frac{\partial H}{\partial y}(0,0,t) = 0
\end{aligned}
\tag{3.5}
$$

Expressing the Laplacian in polar coordinates, one obtains:

$$
\begin{aligned}
&\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} = \sigma \frac{\partial B}{\partial t} \\
&H(a,t) = H_a(t) \qquad \frac{\partial H}{\partial r}(0,t) = 0
\end{aligned}
\tag{3.6}
$$

Assume now that B and $\partial B/\partial t$ only slightly vary with distance r from the center of the core. Hence, these quantities can be replaced by average values in the core cross-section, denoted by $\overline{B}$ and $d\overline{B}/dt$, respectively. The solution to $(3.6)_1$ is then given by the formula

$$
H = H_a - \frac{1}{4}\sigma(a^2 - r^2)\frac{d\overline{B}}{dt} \tag{3.7}
$$

The obtained parabolic distribution of the magnetic field in the core cross-section is approximate. The real distribution was presented by Knoepfel (2000).

Ampère's law in the differential form implies that density of the eddy current induced in the core is

$$j = -\frac{\partial H}{\partial r} = -\frac{1}{2}\sigma r \frac{d\overline{B}}{dt} \tag{3.8}$$

where the negative sign results from the Lenz law – the induced by a change in the magnetic field current generates a field counteracting that change. Calculating the average values of both sides of equation (3.7) for the core cross-section, we obtain

$$\overline{H} = H_a - \frac{1}{8}\sigma a^2 \frac{d\overline{B}}{dt} \tag{3.9}$$

The average value of magnetic field $\overline{H}$ in the core must be consistent with the magnetization law for the material $\overline{H} = \phi^{-1}(B)$, which leads to the following constitutive equation of the core

$$\frac{1}{8}\sigma a^2 \frac{d\overline{B}}{dt} + \phi^{-1}(\overline{B}) = H_a \tag{3.10}$$

An analogous equation for geometry of a plate was presented by Bertotti (1998). It only differs by the constant standing at $d\overline{B}/dt$ – if the plate has thickness $2a$, then 1/3 appears in (3.9) instead of 1/8.

To answer the question in which conditions the assumption of negligible variations of B and $\partial B/\partial t$ with respect to r holds, one should introduce the notion of a skin layer of the core (Knoepfel, 2000). In the skin layer, most of the heat coming from the induced eddy currents is generated. Deeper, down the skin, the amplitude of magnetic field is much lesser than within it. This effect of shielding of the skin layer is harnessed e.g. in induction furnaces for surfacial heat treatment of metals.

The thickness of the skin layer (skin depth) of the core subject to a harmonically variable with frequency magnetic field amounts to

$$d = \frac{1}{\sqrt{\pi\sigma\mu\mu_0 f}} \tag{3.11}$$

It is seen in Fig. 4 that the increasing frequency of the magnetic field makes the skin layer thinner. One can also make a simplifying assumption when its thickness is less than the core radius. For the given geometry and electric and magnetic properties of the core material, this assumption states that $f \leqslant 24\,\mathrm{Hz}$.

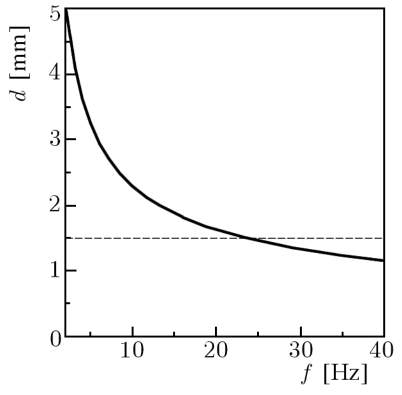

Fig. 4. Skin depth in the core undergoing harmonically variable magnetic field

3.4. Dynamical equations

To fix the attention, let us consider the left electromagnet (see Fig. 1). Derive now an equation describing electromagnetic phenomena taking place in the core. Consider a curve γ passing normally through the center of any cross-section of the core. Applying to it Ampère's law in the integral form, we obtain

$$lH(0) + 2(\delta + x)H_z(0) = I \tag{3.12}$$

where H and H_z are magnetic field intensities in the core and gap, respectively, and I is the resultant current flowing through the surface enclosed by the curve γ.

Form Gauss's law for magnetic fields in the integral form it ensues that in each cross-section of a magnetic circuit there is the same magnetic flux. Since the cross-section A of the core is constant, then the magnetic induction in the core remains the same as in the gap. Thus

$$H = \phi^{-1}(B) \qquad\qquad H_z = \frac{B}{\mu_0} \tag{3.13}$$

The resultant current I is a sum of the current flowing through the electromagnet winding and the eddy current in the core

$$I = Ni + l\int\limits_0^a j(r)\,dr = Ni - l(H_a - H(0)) \tag{3.14}$$

where the second equality originates from (3.8). Substituting H, H_z and I into (3.12) and replacing $H(0)$ and $B(0)$ by the average values in the core, we arrive at

$$l\phi^{-1}(\overline{B}) + 2(\delta + x)\frac{\overline{B}}{\mu_0} = Ni - l(H_a - \phi^{-1}(\overline{B})) \tag{3.15}$$

Taking advantage of (3.10), we obtain the following equation

$$\frac{1}{8}\sigma l a^2 \frac{d\overline{B}}{dt} + \frac{2(\delta + x)}{\mu_0}\overline{B} + l\phi^{-1}(\overline{B}) = Ni \tag{3.16}$$

Proceed now to the equation describing the induction in the electric circuit. According to Faraday's law, the induced electromotive force is proportional to the rate of change of the magnetic flux. Applying the second Kirchhoff law, we may write down

$$i = \frac{U}{R} - \frac{NA}{R}\frac{d\overline{B}}{dt} \tag{3.17}$$

Derive now a formula for the magnetic force. Let us assume, for a while, the average induction in the core as a function of the gap size $\overline{B} = \overline{B}(z)$ exclusively. Shifting the oscillator away from the attracting electromagnet by a distance z, one accumulates in the magnetic field filling up the gap some potential energy of density

$$u(z) = \frac{\overline{B}(z)^2}{2\mu_0} \tag{3.18}$$

This expression is derived for an ideal solenoid, but it remains true for an arbitrary homogeneous magnetic field. Thus, the amount of energy accumulated in the gap of size z is

$$U(z) = \int_0^z u(y)A\ dy = \frac{A}{2\mu_0}\int_0^z \overline{B}(z)^2\ dz \tag{3.19}$$

And, consequently, the magnetic force the electromagnet attracts the oscillator equals

$$F = 2\frac{dU}{dz} = \frac{A}{\mu_0}\overline{B}^2 \tag{3.20}$$

Substituting (3.17) into (3.16) to eliminate the current, and then putting down an analogous equation for the right electromagnetic circuit, we obtain the following mathematical description of dynamics of the analysed system

$$
\begin{aligned}
&\dot{x} = v \\
&m\dot{v} = \frac{A}{\mu_0}(\overline{B}_2^2 - \overline{B}_1^2) - kx + F_0 \sin(2\pi f t) \\
&\Big(\frac{AN^2}{R} + \frac{1}{8} l\sigma a^2\Big)\frac{d\overline{B}_{1,2}}{dt} + 2\frac{(\delta \pm x)}{\mu_0}\overline{B}_{1,2} + l\phi^{-1}(\overline{B}_{1,2}) = \frac{NU}{R} \\
&x(0) = x_0 \qquad v(0) = v_0 \qquad \overline{B}_{1,2}(0) = \phi(\overline{H}_{01,2})
\end{aligned}
\tag{3.21}
$$

where, by virtue of (3.12) and by assuming the zero initial eddy current, $\overline{H}_{0_{1,2}}$ is a solution to the equation

$$
\overline{H}_{0_{1,2}} = \frac{\frac{NU}{R}}{2\mu(\overline{H}_{0_{1,2}})(\delta \pm x_0) + l}
\tag{3.22}
$$

By index 1 all quantities in the left circuit are denoted, by 2 – all in the right one.

4. Damping of vibrations

Investigate at the beginning, by means of the logarithmic decrement, the effect of magnetic hysteresis on damping of free vibration started with an initial displacement of the oscillator. Then, analyse the case of forced vibration by plotting resonant characteristics of the system. At the end, the balance of power will be calculated together with comparison of energy dissipation due to induction of a current with losses in the core.

4.1. Free vibration

In Fig. 5, a vibration history of the oscillator initially displaced by $x_0 = 0.5$ mm at the supply voltage $U = 1.7$ V is shown. The hysteresis causes a slight but noticeable growth of damping of the vibration.

The logarithmic decrement is defined as $\ln[x(nT)/x((n+1)T)]$, $n = 0, \dots, 29$, where T is the vibration period (Fig. 6). Its value with hysteresis taken into account is by 15% up to 19% greater than in the case with no hysteresis effect considered.

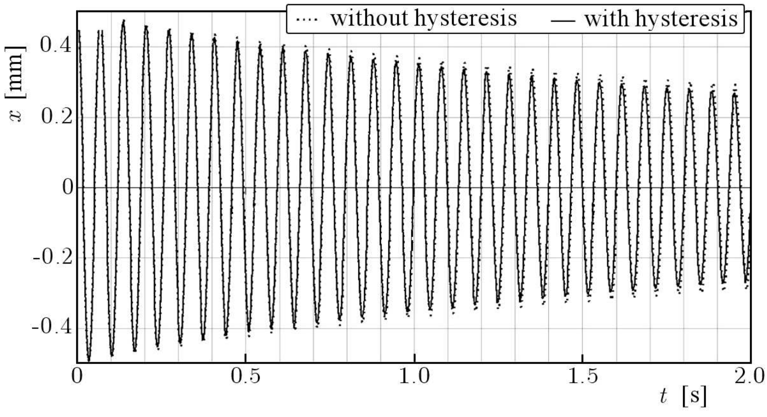

Fig. 5. Free vibration $(x_0 = 0.5\,\mathrm{mm},\ v_0 = 0,\ U = 1.7\,\mathrm{V})$

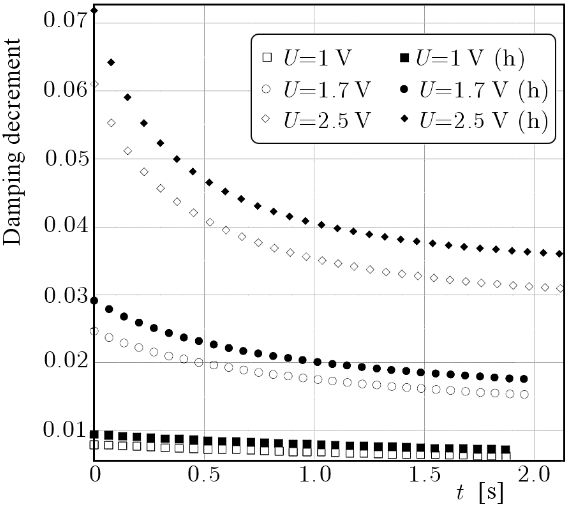

Fig. 6. Logarithmic decrement of damping $(x_0 = 0.5\,\mathrm{mm},\ v_0 = 0,\ U = 1,\, 1.7,\, 2.5\,\mathrm{V})$

4.2. Forced vibration

The resonant characteristic shows the relation between the maximum registered vibration amplitude and frequency of the excitation. As in linear systems, it is plotted for steady-state vibrations. It is yet to be remembered that in this paper a special numerical approach has been employed as near the resonance there appears the beating phenomenon, which because of small efficiency of the magnetic damping for the assumed parameters decays too long in typical numerical simulation procedures.

Taking into consideration the effect of magnetic hysteresis, one observes a slight smoothing of the resonant characteristics for all the applied levels of voltage supplied to the electromagnets (Fig. 7).

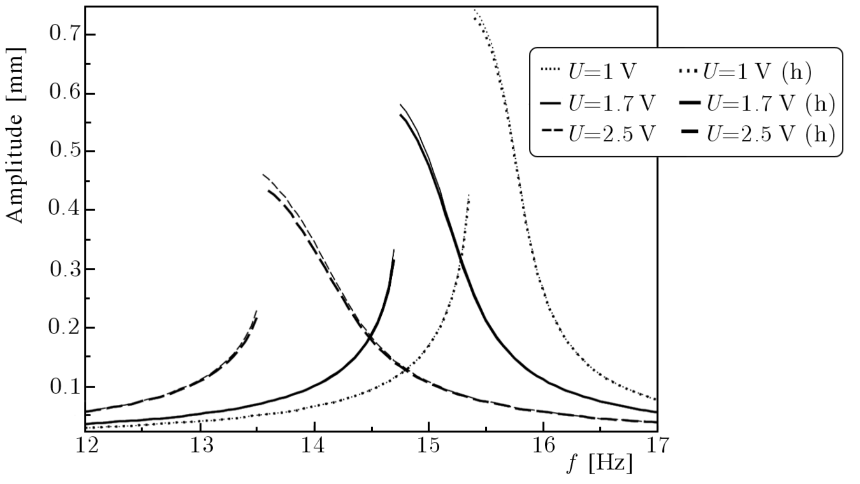

Fig. 7. Resonant characteristics $(x_0 = 0,\ v_0 = 0,\ F_0 = 0.1\,\mathrm{N},\ U = 1,\ 1.7,\ 2.5\,\mathrm{V})$

4.3. Effect of hysteresis

The instantaneous power lost in a single core due to hysteresis is determined from the formula:

$$P_h(t) = \int\limits_V \frac{|j(r,t)|^2}{\sigma}\, dV = \frac{1}{8}\pi a^4 l\sigma\Big(\frac{d\overline{B}}{dt}\Big)^2 \tag{4.1}$$

While the instantaneous power of induced current in a single winding is, see (3.17)

$$P_i(t) = R\Big(-\frac{NA}{R}\frac{d\overline{B}}{dt}\Big)^2 = \frac{1}{336}\pi^2 a^5 N\widetilde{\sigma}\Big(\frac{d\overline{B}}{dt}\Big)^2 \tag{4.2}$$

where $\widetilde{\sigma}$ is the electric conductivity of the material of winding (here copper). While deriving formula (4.2), it was assumed that the electromagnet winding consists of a wire having 12-fold less diameter than the core wound in two layers. After substituting the accepted numerical values of the parameters, we obtained that the instantaneous power losses due to hysteresis constitute roughly 19% of the losses brought about by the current induced in the winding.

5. Concluding remarks

The obtained results enable one to conclude that the effect of magnetic hysteresis itself on vibration damping is small but noticeable. The model incorporated does not reflect hysteretic losses coming from eddy currents induced in the core within the scale corresponding to magnetic domains, hence in real conditions this effect is stronger. On the other hand, for higher frequencies the shielding effect becomes visible, which lowers oscillations of the magnetic field within the core, thus confines hysteretic losses and limits the efficiency of vibration damping at the same time.

The taking into account of the other two types of hysteretic losses requires identification of the model for physically existing cores. Additionally, the modelling of the shielding effect entails necessity of considering variable field distributions within cross-sections of the core, which, obviously, makes the mathematical description very complicated.

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Tłumienie drgań mechanicznego oscylatora za pomocą elektromagnetycznego układu z pętlą histerezy

Streszczenie

W pracy przedstawiono metodę tłumienia drgań mechanicznych za pomocą aktuatorów elektromagnetycznych. Zjawiskami fizycznymi zachodzącymi w takich elementach wykonawczych są indukcja magnetyczna i histereza. Indukcja, polegająca na wytwarzaniu prądu elektrycznego w przewodniku znajdującym się w zmiennym polu magnetycznym posiada precyzyjny opis ilościowy w postaci prawa Faradaya. Natomiast histereza magnetyczna w ferromagnetykach oznacza zależność namagnesowania próbki od wartości zewnętrznego pola magnetycznego, jakie występowało przez całą jej historię. Niestety, w chwili obecnej nie ma jednego powszechnie akceptowanego modelu opisującego to zjawisko. Autorzy tej pracy wykorzystali pewne refleksje zaczerpnięte z publikacji Bertottiego (1998) na ten temat. W rozważaniach użyto dwuliniową charakterystykę magnesowania, co pozwoliło odzwierciedlić zjawisko magnetycznego nasycenia rdzenia. Przeprowadzona analiza rozwiązań wyprowadzonych równań ruchu elektromagneto-mechanicznego modelu układu wykazała, że uwzględnienie zjawiska histerezy ma niewielkie znaczenie, jakkolwiek zauważalne. Jednocześnie skuteczność samej metody tłumienia elektromagnetycznego jest znaczna. Na potwierdzenie tych tez autorzy tej jak i poprzedniej pracy zaprezentowali odpowiedzi czasowe, wykresy logarytmicznego dekrementu tłumienia oraz charakterystyki rezonansowe badanego układu.

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