ASSESMENT OF CORTICAL BONE THICKNESS USING CEPSTRUM ANALYSIS. SIMULATION STUDY

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Assessment of cortical bone thickness is important from a medical point of view because bone-layer thickness has a diagnostic value. The thinning of the cortical bone layer reduces the mechanical strength of the bone and exposes it to an increased risk of osteoporotic fractures [1]. The hip bone (proximal femur) is the most critical fracture site. The thickness of the cortical layer in the proximal femur is often too thin to be detected from ultrasonic echoes using traditional peak detection methods (for example the envelope method). In such a case the cepstrum analysis technique may be very useful. In this study the cepstrum method was applied to analyze numerically simulated echoes reflected from the layer and to determine layer thickness. In simulation, the transducer operated at 1 MHz and pulses of a 1.5 μs duration were assumed. The thickness of the thinnest layer for which the applied cepstrum analysis gave, the correct result equaled 1 mm, which was ¼ λ (λ – wavelength of an ultrasonic wave). That value of the d/λ ratio is sufficient for future measurements performed in-vivo conditions.

INTRODUCTION

The goal of our research is to develop a noninvasive method that could be used to assess cortical bone thickness.

The cortical bone makes up the main mass of the skeleton, which means about 80 %, so it supports most of the load of the body and is mainly involved in many osteoporotic fractures [2]. Cortical thickness is related to fracture risk. During osteoporosis cortical bone thickness usually decreases [3]. Thus the bone thickness has a diagnostic value.

There is another important reason for the assessment of cortical bone thickness and ultrasonic measurements of the trabecular bone. Backscatter parameters derived from the pulse-echo ultrasound measurement of the trabecular bone also may be used for the diagnostics of osteoporosis. In this case the attenuation coefficient of the trabecular bone is very important. This coefficient is determined from the spectra of ultrasonic echoes. These spectra are significantly distorted by the cortical bone. In Fig.1, results of simulation are presented. The short ultrasonic pulse was transmitted through a parallel layer of varying thickness. It can be clearly seen that the pulse amplitude spectrum depends very much on layer thickness.

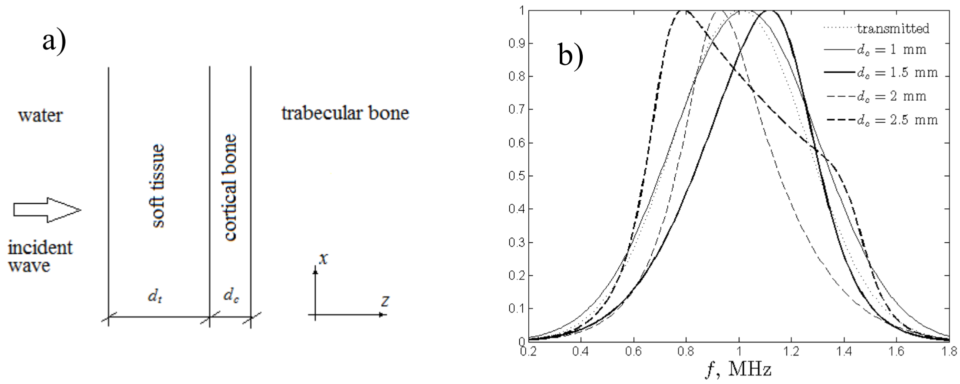

Fig. 1. The model of a layer consists of soft tissue, cortical bone and trabecular bone (a) and the amplitude spectra of the transmitted ultrasonic pulse (b). d_o – denotes the cortical bone thickness and "transmitted" stands for $d_o = 0$.

For a reliable assessment of ultrasound backscatter from the trabecular bone, the effect of the overlying cortical layer must be taken into account. This can be achieved only when the thickness of the cortical bone layer is known.

Several methods have been used to measure layer thickness using reflected waves. The envelope method [1] or autocorrelation method [3] can be applied in the case when the reflections from layer interfaces are separated in time. In the case when the internal reflections overlap each other, the layer reflection signal can be subjected to cepstral analysis, which is presented in this paper. There are also other methods like the parametric method [5] or the maximum entropy analysis method [6], which allows for the determination of layer thickness in the case of the layer reflection consisting of interfering components. Good results can also be achieved by fitting the theoretically determined transfer function of the layer system to the experimental one [7].

1. BONE STRUCTURE

Bones are a significant element of our body. They have important functions, for instance sustaining loads from external actions (gravity), allowing movement and the protective role of the vital organs (which is the case of the thorax, skull and pelvis).

Bone is a living material. It evolves during life. In the sixth week of prenatal life a skeleton develops. We are born with about 300 soft bones. During childhood and adolescence, the cartilage grows and is slowly replaced by hard bone. Some of these bones later fuse together, meaning that the adult skeleton has 206 bones.

Bone is composed of two main components which are also visible macroscopically: cortical (or compact) bone and cancellous (or trabecular) bone. These are presented in Fig. 2. Their names reflect their structural nature; the cortical bone presents a dense structure of low porosity, whereas cancellous bone is porous, rather like a sponge. Compact bone composes an external envelope of all bones: long (such as, for instance, the femur or tibia), short bones (the vertebra or calcaneus) and flat bones (the skull). The thickness of the cortical layer is the largest in the diaphysis (about 1cm) and diminishes up to about 1mm in the epiphyses. The cancellous bone is located in the inner parts of bones, especially in the epiphyses of long bones where the thickness of the cortical bone layer is relatively small. As an example, in the femur neck bone (see Fig. 3), cortical bone thickness varies from 1mm to about 3mm.

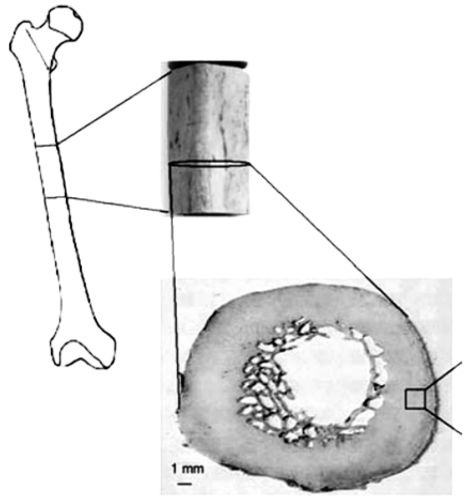

Fig. 2. Mid-diaphysis of a femur; cross section at the mid-diaphysis illustrating the outer cortical shell and the inner cancellous bone compartment at the periphery of the medullary canal [2].

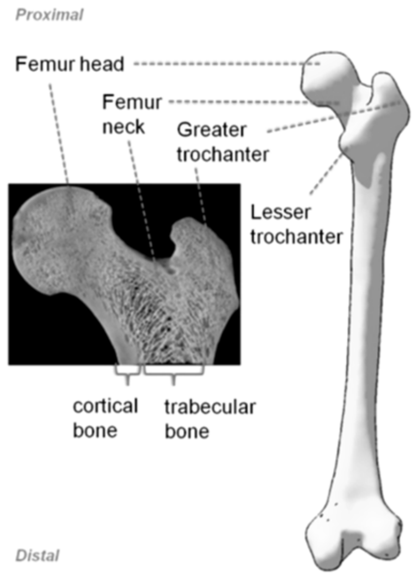

Fig. 3. Cross-section of a human proximal femur [8].

2. REFLECTION FROM A PLANE LAYER

The model of the bone consists of three layers simulating tree media: soft tissue – skin, cortical bone and trabecular bone, and are indicated in Fig. 4 by numbers 3, 2 and 1, respectively.

The resulting wave reflected from layer 2 thickness *d* (Fig. 4) may be regarded as a superposition of: a) the wave reflected from the front surface of the layer (the boundary between media 3 and 2); b) the wave transmitted through the front surface of the layer, passed through the layer and reflected from its back surface (the boundary between media 2 and 1), passed through the layer again and finally leaving it through its upper boundary between media 2 and 3; c) the wave penetrating the layer, undergoing two reflections at the back surface and one at the upper surface, passing twice through the layer back and forth, and then again leaving the layer, etc.

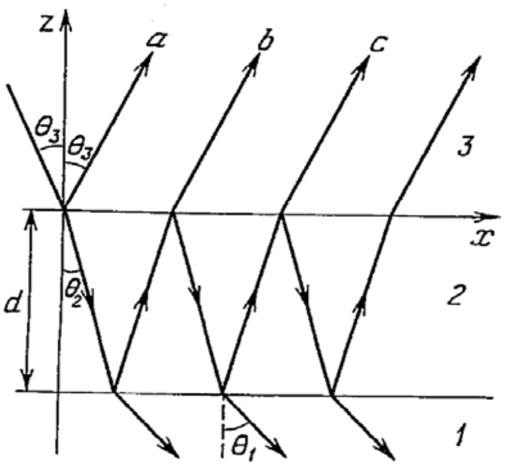

Fig. 4. Reflected and transmitted waves for a plane wave incident on a layer (2) located between two media (1 and 3). θ_3 is an angel of the incident wave and the angles θ_2, θ_1, are respectively diffraction angles for the transmission from medium 3 to 2 and from 2 to 1 [9].

The first tree complex amplitudes (taking into account the phase factor) of these waves (a, b and c in fig.4) are, respectively:

$$\begin{gathered} R_{32} \\ T_{32}\,R_{21}T_{23}\exp(2i\varphi) \\ T_{32}\,R_{21}R_{23}R_{21}T_{23}\exp(4i\varphi) \end{gathered} \tag{1}$$

where: R_{ij} – the reflection coefficient at the ij (i, j = 1,2,3) boundary, and T_{ij} – the transmission coefficient. Reflection coefficients are equal to [10]:

$$R_{ij} = \frac{Z_j - Z_i}{Z_j + Z_i}, \tag{2}$$

where: Z_i, Z_j – acoustic impedances. Summing up all the waves which form the total reflected signal and using the expression for the sum of infinite geometrical progression, and after some transformations that involve the equalities: $R_{ij} = R_{ji}$, $T_{ij} = 1 + R_{ij}$, we obtain a formula for the reflection coefficient from the layer of thickness d in the case of the normal incident of the plane wave of the united amplitude [11]:

$$R(f) = \frac{R_{32} + R_{21}\exp\left(i2\pi f\frac{2d}{c}\right)}{1 + R_{32}R_{21}\exp\left(i2\pi f\frac{2d}{c}\right)} \tag{3}$$

The dependence of the reflection coefficient on frequency is presented in Fig. 5.

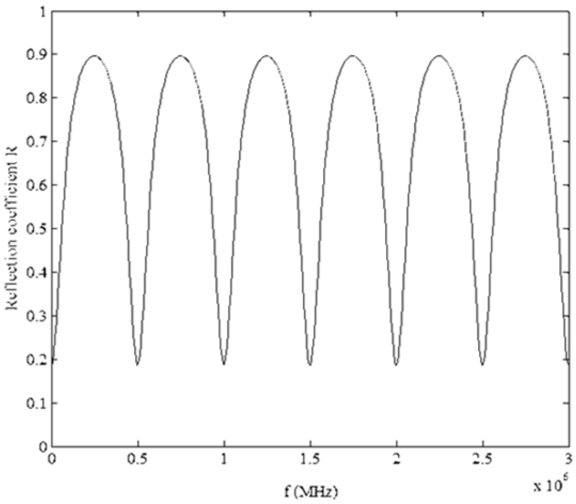

Fig. 5. Reflection coefficient (for a layer of a 4mm thickness) calculated from equation (3).

It can clearly be seen that some frequencies are much less reflected than the others. That gives rise to a significant deformation of the reflected pulse compared to the transmitted one.

3. CEPSTRUM METHOD

It is relatively easy to measure layer thickness when the reflected waves are separated over time (Fig. 8a). In this case the traditional peak detection method gives satisfactory results. For the thin layer, the waves reflected from the top of the layer and from the bottom overlap each other, creating an interference signal which makes any direct separation of constituent reflections impossible. In this case, the information of the arrival times of partial reflections may by obtained using the cepstrum method. Cepstrum analysis is a way to separate and analyze signals when several signals are added together [12, 13, 14].

The cepstrum function of signal *y(t)* is the Inverse Fourier Transform (IFT) of the logarithm of the Fourier Transform (FT) of that signal, which gives equation (4)

$$c(t) = \mathrm{IFT}\left(\log\left(\left|\mathrm{FT}(y)\right|\right)\right). \tag{4}$$

The cepstrum method has been used for a variety of purposes. Originally it was used to find echoes in seismic signals from earthquakes or explosions, so that the depth of the seismic source might be determined from the echo delay [15]. In acoustics, it has been applied, for example, to analyze speech sound [15]. The cepstrum method has also been applied to determine the thickness of a layer made of a variety of materials, for instance ceramics, semiconductors, and soft tissues [16, 17]. It may be also used for the assessment of cortical bone thickness.

In the case of the cortical bone of the femur neck bone, the cortical layer is so thin that ultrasound waves reflected from the periosteum and endosteum are overlaid. The studied system (cortical bone layer) may be regarded as an LTI (linear time invariant) system, so the output signal (echo signal) can be presented as the convolution of an input signal *x(t)* and the impulse response *h(t)*:

$$y(t) = x(t) * h(t) \tag{5}$$

This can be represented in the frequency domain as:

$$|Y(f)| = |X(f)| \cdot |H(f)| \tag{6}$$

The logarithmic representation of equation (6) will be

$$\log|Y(f)| = \log|X(f)| + \log|H(f)| \tag{7}$$

In our case, the transmission function *H(f),* which characterizes the studied system, represents the reflection coefficient *R(f),* defined by the formula (3).

The expression for R has oscillating components that are clearly visible on the plot of the logarithm of the Fourier Transform of the echo signal (Fig. 6a). The frequency *f* of these oscillations is equal to

$$f = c/2d. \tag{8}$$

After the application of the Inverse Fourier Transform (the final step toward computing the cepstrum), this periodicity will manifest itself as a peak at a value of time equal to:

$$t = 2d/c, \tag{9}$$

which is shown in Fig. 6b.

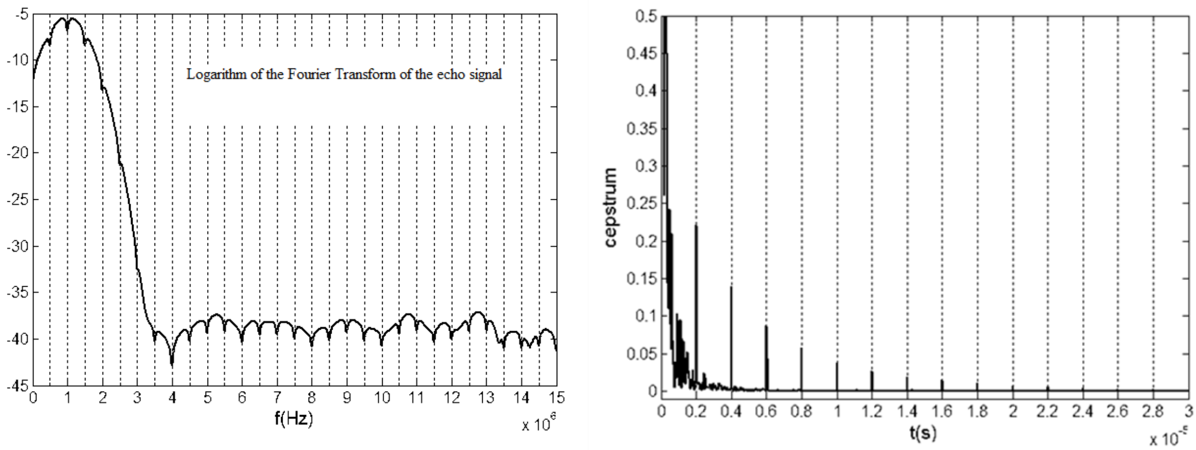

Fig. 6. a) The logarithm of the Fourier Transform of the echo signal, b) Cepstrum function.

Both plots present two approaches that enable the determination of layer thickness. At the plot of the cepstrum function (Fig. 3 b) the successive echoes are represented by single peaks located at positions *t*, and at multiples of *t*. The value of *t* may be determined from the position of the peak of the maximum amplitude or from the distance between the peaks. This time, *t,* describes the time over which the wave travels the distance between the layer's borders (forward and backward) and for the assumed velocity, which allows the calculation of the thickness of the layer.

In the alternative plot (Fig. 3a), the determination of the period of oscillations (in the frequency range) allows us to calculate layer thickness. In this example, this period equals 0.5 MHz.

4. NUMERICAL SIMULATION OF LAYER REFLECTION

The ability of the cepstral technique to measure layer thickness was verified using simulated echoes. The transmitted signal was numerically generated by windowing the

1 MHz continuous wave with the Gaussian function. The resulting pulse was 1.5 μs long and is presented in Fig. 7.

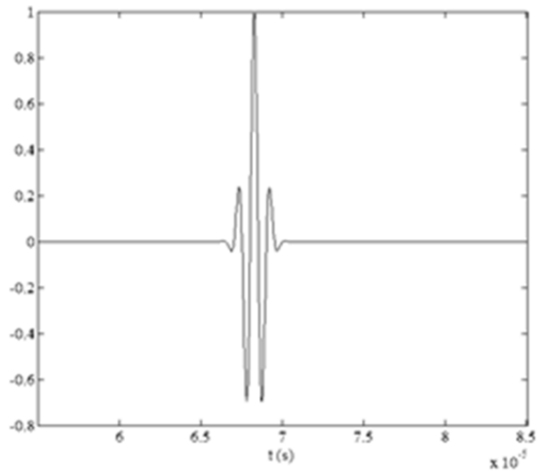

Fig. 7. The simulation of the emitted impulse.

After performing the Inverse Fourier Transform of the product on the emitted pulse spectrum and the reflection coefficient, the reflected signal from the layer of a given thickness was obtained. An example of the simulated echoes are presented in Fig. 8. In Fig. 8a – for a layer thickness equalling 10mm, the successive reflections from the inner boundary layer of compact bone are clearly separated from each other. In Fig. 8b, the layer is much thinner (d = 4mm, and the thickness of the layer is equal to the wavelength for a sound velocity equal to 4000 m/s) and the successive reflections are overlapped.

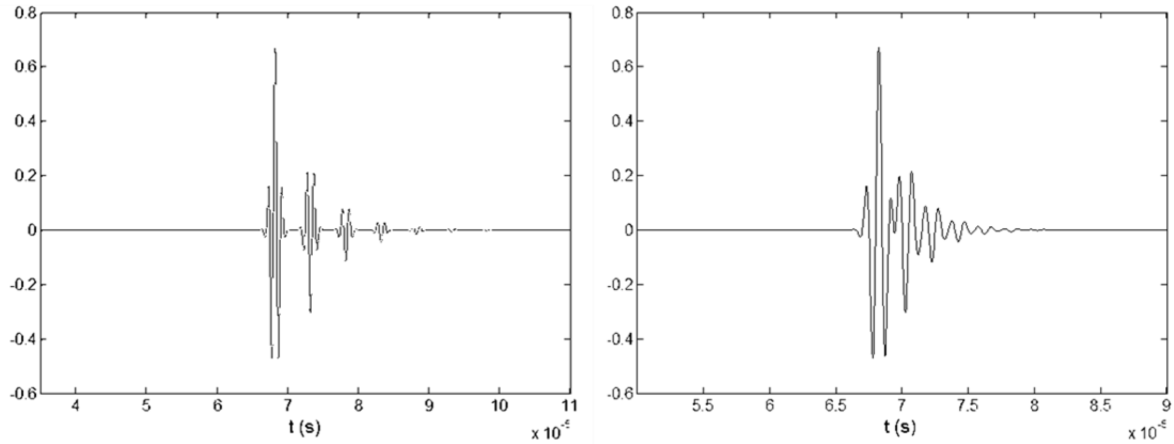

Fig. 8. Signals reflected from the layer of a thickness equal to: a) – 10mm, b) – 4mm, (assumed wave velocity – 4000 m/s).

5. RESULTS

The cepstrum analysis was applied to simulated echo signals reflected from a layer with a thickness varying from 1mm to 10mm. The value of time t required to pass the layer was determined from the cepstrum plots. The results are presented in the third column in table 1, together with the time values calculated from the equation (9) – the second column in table 1.

Tab. 1. The values of time calculated using cepstrum analysis for a particular thickness of a layer

d(mm)	t(s)	t(s)
	t = 2d/c	t - cepstrum
1	5,00E-07	5,00E-07
2	1,00E-06	1,00E-06
3	1,50E-05	1,50E-05
4	2,00E-06	2,00E-06
5	2,50E-06	2,50E-06
6	3,00E-06	3,00E-06
7	3,50E-06	3,50E-06
8	4,00E-06	4,00E-06
9	4,50E-06	4,50E-06

It is shown in table 1 that both values of *t* (calculated and obtained from cepstrum analysis) are equal. So, the values of *t* received from cepstrum may be used for the assessment of layer thickness. Figure 9 presents examples of cepstrum plots, for a thickness of layers equal to 2mm and 1mm.

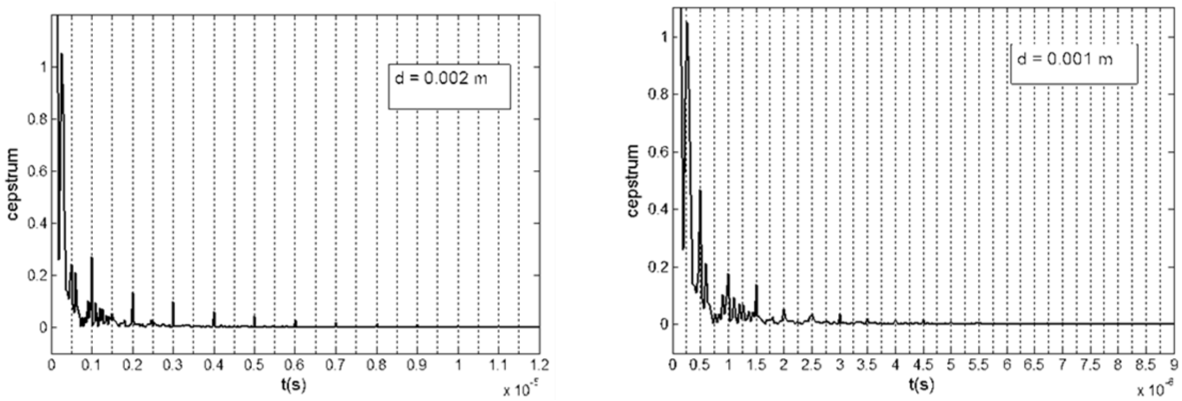

Fig. 9. The cepstrum function of an echo signal reflected from a 2mm- and 1mm-thick layer.

For thinner layers – with a thickness of the order of fractions of millimeter – the shape of the cepstrum function becomes very noisy and any determination of the thickness is not possible.

An alternative way involves an analysis of the logarithm of the Fourier Transform of the echo signal, also enabling an assessment of layer thickness for thickness values of the order of millimeters. Also, this method is limited to a thickness of 1mm for 1MHz signals and a velocity of 4000 m/s assumed in the layer material.

6. IMPACT OF NOISE

So far, the simulated echo signals have been clear from noise. In practice, noise is always a part of real signals. We have checked how noise may influence the operation of the cepstral method. To this end, a white noise of varying amplitude was added to the simulated echoes and layer thickness was determined using the same procedures as described above. It was observed that even a relatively high amplitude of noise (compared to the amplitude of the emitted ultrasound impulse) did not affect the resolution of the cepstrum function (see fig.10).

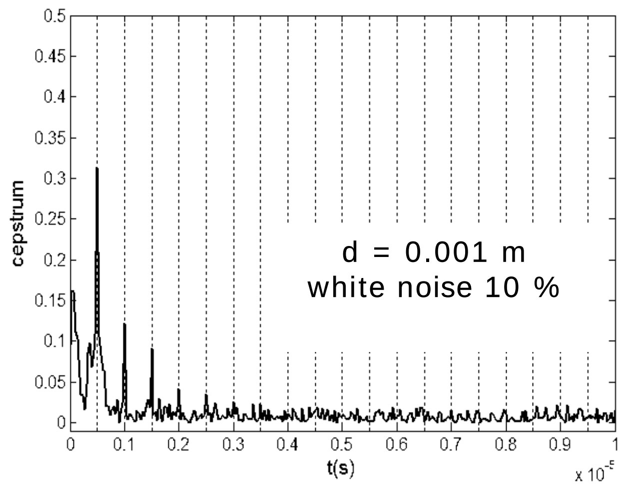

Fig. 10. The cepstrum function of an echo signal reflected from a 1mm thick layer. The white noise of an amplitude equal to 10 % of the emitted impulse amplitude was added prior to calculations.

7. CONCLUSION

The cepstral analysis of simulated signals of ultrasonic echoes reflected from the endosteal and periosteal surfaces of the cortical bone gives excellent results for thin layers. The thinnest layer we could measure using the cepstrum method equaled 1mm, which is ¼ λ (λ – the wavelength of an ultrasonic wave). These results are very promising. At the measurement in vivo the minimal thickness of the cortical bone of the femur neck is about 1mm, which at a frequency of 1MHz (in the range of the standard frequencies used in the ultrasonic densitometry of bones), gives a similar d/λ ratio.

The value of sound velocity in cortical bone varies from 3600 m/s to 4000 m/s [18]. During in vivo measurements, this velocity is unknown, which results in the additional error of the determined value of bone thickness. So the uncertainty of the assessment of thickness is connected with the uncertainty of the assumed value of the velocity. In calculations a velocity of 3800 m/s was used - the central value of the theoretical range. In this case the maximum uncertainty of velocity Δc is 200m/s, so the absolute uncertainty $\Delta d/d$ of the thickness assessment is 5 %, where Δd is the relative uncertainty equal to

$$\Delta d = \frac{\partial d}{\partial c}\Delta c \tag{10}$$

An accuracy of 5 % is quite satisfactory in biological measurements.

During simulation, a relatively high amplitude white noise was added to simulated echoes and the resolution of the method remained unchanged. So we suppose that for real signals good results will also be obtained. The cepstrum method seems to be a suitable technique for the assessment of the thickness of the cortical bone in vivo.

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