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Structural damage identification using random response based on Virtual Distortion Method *

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Abstract—**Structural damage identification plays a critical role in structural health monitoring on evaluating structural safety and maintaining structural integrity. This paper presents a damage identification approach based on Virtual Distortion Method (VDM) using random response. VDM is a fast structural reanalysis method in which virtual distortions are introduced to simulate structural damages or modifications. Via VDM, responses of damaged structure can be computed quickly without reanalysis of the whole structure. In this paper, firstly the frequency response of damaged structure is constructed efficiently using VDM, and then damage extents are optimized using the objective function which is computed using the MAC (modal assurance criterion) of the power spectrum of theoretical response and measured responses. At last, a plane truss is proposed to verify the proposed method.**

I. Introduction

Structural health monitoring is a hot research topic, where damage identification plays an important role on structural safety evaluation and maintaining its integrity. Although there exist many effective methods on damage identification, it is still difficult on accurate structural damage identification in civil engineering due to the large and complexity of structures, as well as the limitations of measuring points.

Damage identification methods based on vibration responses are categorized into two mainly kinds in frequency domain and in time domain. Based on dynamic signature, many identification methods have been discussed ([2], [3], [4]) like natural frequency, modal shape etc. However when the adjacent modes are very close, it is hard to identify the natural frequencies and modes, then the measured or identified frequency response function (FRF) can be used instead. Damage identification methods in frequency domain are robust to noise. Structural random response is easily to be obtained in practice, which is used often for damage identification in frequency domain. However structural modal parameters are insensitivity to local damages, it takes amounts of work on optimizing the accurate results, which costs lots of computation work.

Methods in time domain [4] use measured responses directly for identification. The classical methods include least square method and its improved algorithms. Virtual distortion method (VDM) ([5]) introduces certain virtual distortions to simulate structural damages, and then responses of damaged structure in time domain are estimated quickly which avoids reanalysis of the whole structure. However methods in time domain are sensitivity to measurement noise, which is aviodless in practice.

This paper extends VDM into frequency domain and based on it presents a damage identification method using random response. First frequency response of the damaged structure is constructed using the basic concept of VDM. Then structural damages are optimized and identified by comparing the relativity between power spectrum density (PSD) of measured responses and that of the computed frequency response to given damage extents. A numerical truss model is used to verify the proposed idea. The application of VDM is limited to small deformation structure [5]. In order to express the proposed method simply, this paper takes truss as model, however it can be easily expanded to the application of structural with complex elements like beam or plate[6].

II. Responses construction using Virtual Distortion Method

Virtual Distortion Method (VDM) introduces virtual distortions to simulate structural damages. Denote by $y_\alpha^{\mathrm{L}}(t)$ the response of the intact structure at measure point α, and the corresponding dynamic response of damaged structure $y_\alpha(t)$ can be expressed as

$$y_\alpha(t) = y_\alpha^{\mathrm{L}}(t) + \sum_\beta \sum_\tau D_{\alpha\beta}(t-\tau)\,\varepsilon_\beta^0(\tau) \tag{1}$$

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where $\varepsilon_{\beta}^{0}(t)$ is the time-varying virtual distortion of the damaged element. $D_{\alpha\beta}(t)$ is the impulse responses at measure point α of the intact structure to the unit impulse virtual distortion which occurred in the β th damaged element. The latter equals to a pair of equivalent force applied on the element to make it occur a unit impulse distortion.

From (1), it can be seen that if response of the intact structure is known, responses of damaged structure can be obtained quickly without reanalysis of whole structure. Virtual distortions are coupled with structural responses and dependent on damage extents. Denoted by μ_i the damage extent of the *i*th element, which is the ration between modified stiffness and original stiffness. Then the actual distortion of the *i*th damaged element $\varepsilon_i(t)$ has the following relation with damage extent μ_i and the corresponding virtual distortion $\varepsilon_i^{0}(t)$ [5]:

$$\varepsilon_i^0(t)=(1-\mu_i)\varepsilon_i(t) \tag{2}$$

Furthermore, $\varepsilon_i(t)$ is the distortion of the damaged structure, similarly to (1), there is

$$\varepsilon_i(t)=\varepsilon_i^{\mathrm{L}}(t)+\sum_{\beta}\sum_{\tau}D_{i\beta}(t-\tau)\varepsilon_{\beta}^{0}(\tau) \tag{3}$$

Then virtual distortion $\varepsilon_i^{0}(t)$ can be computed via solving (3) and (4) to given damage extent μ_i, that is, via the following equations,

$$\varepsilon_i^0(t)=(1-\mu_i)\varepsilon_i^{\mathrm{L}}(t)+(1-\mu_i)\sum_{\beta}\sum_{\tau}D_{i\beta}(t-\tau)\varepsilon_{\beta}^{0}(\tau)d\tau \tag{4}$$

It is a linear system of equations consisting of *n* equations, where *n* is the number of damaged elements, and it can be solved step by step through discretizing time *t* into proper length. It is second kind of Volterra integral equation with unique solution. Substitute the computed virtual distortion $\varepsilon_i^{0}(t)$ into (1), responses of the damaged structure to given damage extents can be obtained quickly without repeatedly analysis of the whole structure.

Using VDM, responses of damaged structure can be computed efficiently, which takes an advantage in damage identification. However in practice, measured responses are usually contaminated with noise, and identification in time domain is sensitivity to noise. In addition, in time domain it is hard to employ random responses or free responses which are easily to be obtained in practice especially for large structure.

III. CONSTRUCTION OF FREQUENCY RESPONSE OF DAMAGED STRUCTURE

Via Fourier Transform, structural responses in time domain can be converted into responses in frequency domain. Therefore (1) is turned into the following expression via Fourier Transform,

$$y_{\alpha}(\omega)=y_{\alpha}^{\mathrm{L}}(\omega)+\sum_{\beta}D_{\alpha\beta}(\omega)\varepsilon_{\beta}^{0}(\omega) \tag{5}$$

where $y_{\alpha}(\omega)$ is responses of damaged structure frequency domain, which is molded as the linear combination of corresponding responses of the intact structure $y_{\alpha}^{\mathrm{L}}(\omega)$ and virtual distortions $\varepsilon_{\beta}^{0}(\omega)$ in frequency domain.

Assume that the excitation is unit impulse applied on the structure along its *j*th degree of freedom (Dof), response $y_{\alpha}(\omega)$ in (5) is the frequency response of the damaged structure, denoted it by $h_{\alpha j}(\omega)$,

$$h_{\alpha j}(\omega)=h_{\alpha j}^{\mathrm{L}}(\omega)+\sum_{\beta}D_{\alpha\beta}(\omega)\varepsilon_{\beta}^{0}(\omega) \tag{6}$$

where $h_{\alpha j}^{\mathrm{L}}(\omega)$ is the frequency response of the intact structure to unit impulse applied along its *j*th Dof. The corresponding virtual distortion $\varepsilon_{\beta}^{0}(\omega)$ can be computed by performing Fourier Transform on (4), that is

$$\varepsilon_i^0(\omega)=(1-\mu_i)\varepsilon_i^{\mathrm{L}}(\omega)+(1-\mu_i)\sum_{\beta}D_{i\beta}(\omega)\varepsilon_{\beta}^{0}(\omega),(i=1,2,\cdots n) \tag{7}$$

So given damage extents, virtual distortion $\varepsilon_i^0(\omega)$ can be obtained by solving (7), which is a linear system equation consisting of n equations. The corresponding computation work is very low.

Assume that frequency response $h_{\alpha j}^{L}(\omega)$ is known in advance, given damage extent, virtual distortion $\varepsilon_\beta^0(\omega)$ can be computed quickly, then substitute $\varepsilon_\beta^0(\omega)$ into (6), frequency responses of damaged structure $h_{\alpha j}(\omega)$ can be computed fast.

IV. DAMAGE IDENTIFICATION

Let $y_\alpha^M(t)$ be measured random responses of damaged structure. Then the response power spectrum $S_\alpha^M(\omega)$ can be obtained by Fourier Transform of its self-correlation:

$$S_\alpha^M(\omega) = \int_{-\infty}^{\infty} E\left[y_\alpha^M(t)\, y_\alpha^M(t+\tau)\right] e^{-j\omega\tau} d\tau \tag{8}$$

According to the analysis in above section, the frequency response of the damaged structure can be constructed quickly to given damage extents using VDM. Denoted by $S_\alpha^H(\omega,\mu)$ the power spectrum of the frequency response $h_{\alpha\alpha}(\omega,\mu)$ of the damaged structure, similarly to (8), there is

$$S_\alpha^H(\omega,\mu) = \left\| h_{\alpha\alpha}^2(\omega,\mu) \right\| \tag{9}$$

The following objective function is built using modal assurance criterion taking damage extent μ as optimization variables,

$$f(\mu) = \sum_\alpha \left\| 1 - MAC(\bar{\mathbf{S}}_\alpha^M, \bar{\mathbf{S}}_\alpha^H(\mu)) \right\| \tag{10}$$

where μ is the modification factor to be identified, $\bar{\mathbf{S}}_\alpha^M = \{S_\alpha^M(\omega_1), S_\alpha^M(\omega_2), S_\alpha^M(\omega_3), \cdots\}$, $\bar{\mathbf{S}}_\alpha^H(\mu) = \{S_\alpha^H(\omega_1,\mu), S_\alpha^H(\omega_2,\mu), S_\alpha^H(\omega_3,\mu), \cdots\}$. ω_i is the identified ith frequency of damaged structure.

V. NUMERICAL EXAMPLE

Fig.1 shows a supported steel 6-span plane truss, which is used to verify the proposed idea. The length is 3m with height of 0.3m, and the truss consists of 21 bars, with 12 nodes. The density is 7800kg/m3, and the young's modulus is 2.0Gpa. The weight of the node is 10kg.

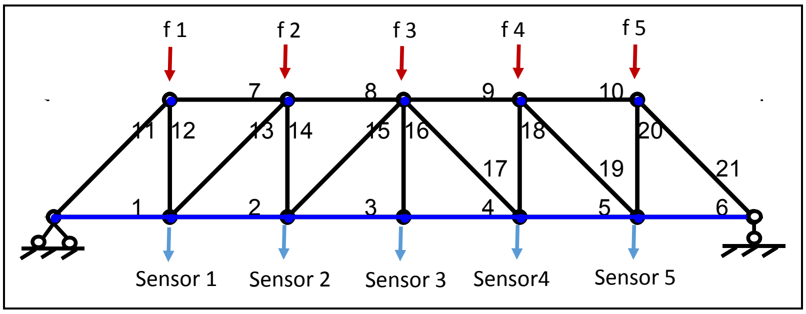

Figure 1. Model of the steel truss

Assume that six bottom chord bars, which are No. 6, 10, 14, 18, 22, 26, are probably damaged, while the rest bars are intact. The damage extents are shown in 错误!未找到引用源。. Five accelerometers are placed on bottom nodes of chord bars, see Fig.1, to measure the vertical accelerations.

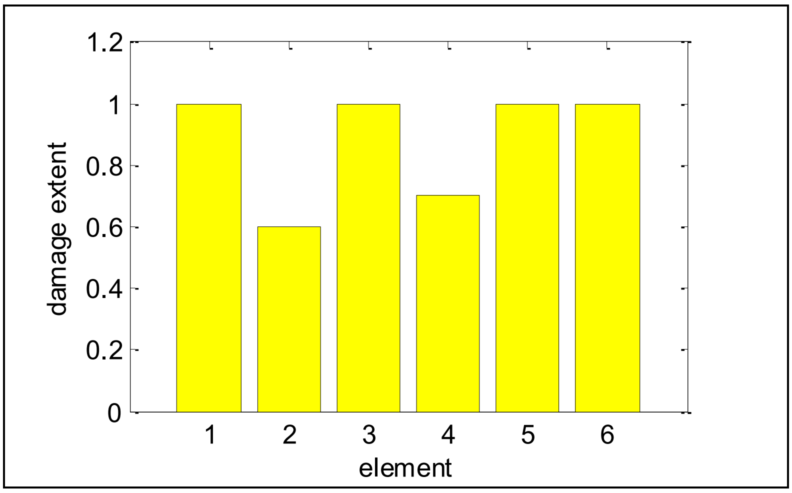

Figure 2. Damage extents

Five random excitations are applied respectively on nodes of top chords, named as f1~f5. The first excitation f1 can be found in Fig.3, and the corresponding response of Sensor 5 is shown in Figure with 5% Gaussian noise pollution to simulate the actual measurement.

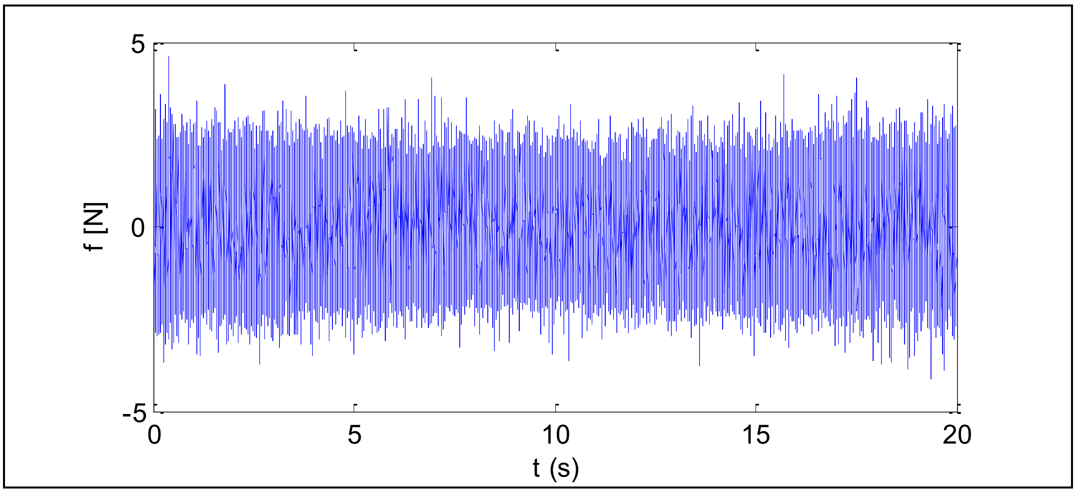

Figure 2 Random excitation f1

The first 4 order frequencies of the intact and damaged structure are shown in Table , and the first 4 order mode shapes are shown in Fig.5 computed via the finite element model.

Fig.6 (PSD) shows the power spectrum density (PSD) of measured random response, and it is compared with the PSD of frequency responses of the damaged structure which is constructed via VDM (Fig.6, VDM). It can be seen that they have high relativity, and the frequency of the peak point is very close to those of actual damaged frequencies, which implies the feasibility of damage optimization using(10).

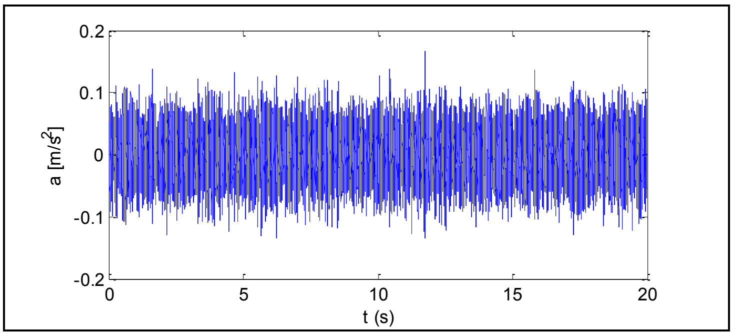

Figure 4. Random acceleration response of Sensor 5 to excitation f1

TABLE I. FREQUENCIES (HZ)

order	intact	damaged
1	51.942	48.587
2	122.949	117.636
3	164.699	164.129
4	237.833	234.787

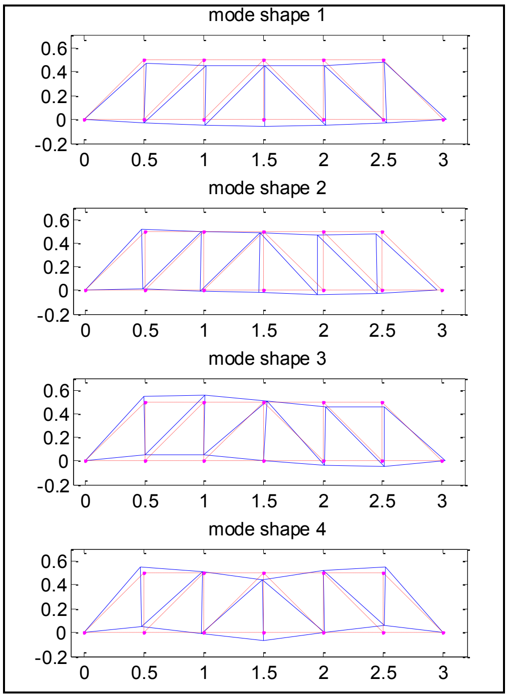

Figure 5. Mode shapes of intact structure

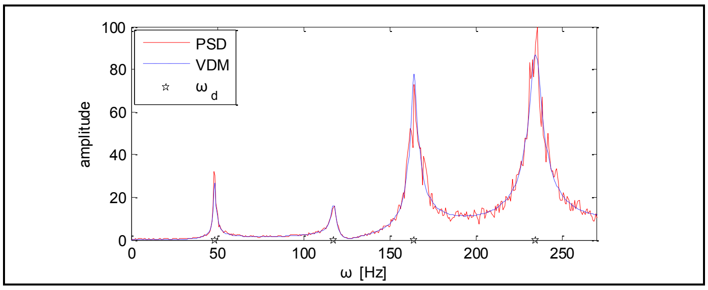

Figure 6. Power specturm of measured response and constructed frequency response of damaged structure

Based on above analysis, structural damages are optimized via the objective function. During the optimization, the frequency responses of the structural regard to given damage extents are constructed, and then the damage extents which minimize the objective function are the identified structural damages. Fig.7 shows the identified results. It shows that both the damage extents and the locations can be identified precisely even under the 5% noise pollution.

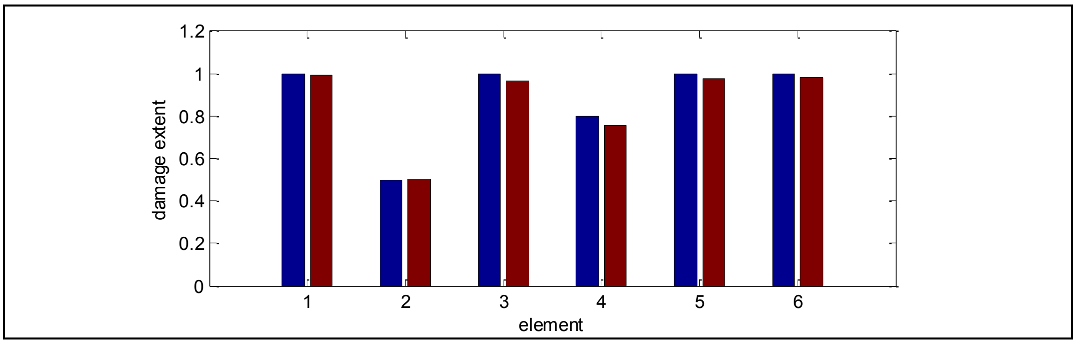

Figure 7. Identified damage extents

VI. CONCLUSION

This paper proposes a damage identification method, which is verified using a supported steel 6-span plane truss. The identification is performed using measured structural random responses, which is easy to be obtained in practice. Damage extents are taken as optimization variables and optimized by the MAC between power spectrum of measured responses and that of computed frequency responses of damaged structure to given damage extents. The corresponding frequency responses are constructed efficiently using Virtual Distortion Method in frequency domain, and the whole structural reanalysis is avoided during the optimization to given damage extents. In numerical example, both damage extents and the locations are identified precisely even under 5% noise pollution.

REFERENCES

[1] S. W. Doebling, C. R.Farrar, M. B. Prime, and D.W. Shevitz, "Damage identification and health monitoring of structural and mechanical system from changes in their characteristics: a literature review", New Mexico: Los Alamos National Laboratory, 1996.
[2] C. R.Farrar, and K. Worden, "An introduction to structural health monitoring ", Phil. Trans. R. Soc. A, 365, 303–315, December 2006.
[3] Z. Zong, J. Niu, and H. Wang, "A review of structural damage identification methods based on the finite element model validation", China Civil engineering Journal,45(8), 121–130,August 2012 (in Chinese).
[4] L.R. Zhou, G.R. Yan, and J.P. Ou, Response Surface Method Based on Radial Basis Functions for Modeling Large-Scale Structures in Model Updating. Computer-Aided Civil and Infrastructure Engineering, 2013, 28(3):210-226.
[5] J. Holnicki-Szulc, (Ed.). "Smart Technologies for Safety Engineering", John Wiley & Sons, Chichester, 2008
[6] Q.Zhang, L.Jankowski, and Z. Duan, "Simultaneous identification of excitation time histories and parametrized structural damages" , Mechanical Systems and Signal Processing, 33:56-68,2012