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ESTIMATION OF THE EFFECTIVE PROPERTIES OF COMPOSITES WITH INCLUSIONS OF DIVERSE SHAPES AND PROPERTIES

P. Sadowski, S. Stupkiewicz

Institute of Fundamental Technological Research, Warsaw, Poland

1. Introduction

Multi-phase composites with inclusions which cannot be characterized by one representative shape and geometry, e.g., when different carbon allotropes (desired or not) can be found in the matrix (polymer or metal), are becoming more popular. Estimation, even rough, of the properties of such composites, is inevitably desired in the process of their design and production. Three classical methods, based on work of Eshelby [1] are used in this work, however the assumption concerning isotropy and similar shape of inclusions, which is commonly adopted in practice, is skipped in our analyses.

2. Classical estimation of the effective properties

Estimation of the effective elastic properties of the composites is summarized in e.g. [2]. Using the average strain theorem and the Hill's concept of the strain/stress concentration tensor [3], the effective stiffness tensor can be expressed as

$$C = C^m + \sum_{n=1}^{N} v_n (C_n^i - C^m) A_n^i \,, \tag{1}$$

where C_n^i and C^m are the stiffness tensors of the n-th type inclusions and the matrix respectively, v_n is the volume fraction of the n-th type inclusions, N – the number of types of inclusions and A_n^i is the strain concentration tensor.

Perfect bonding between inclusions and the matrix is assumed. Each family of inclusions can be characterized by a chosen shape which in turn can be approximated by an ellipsoid in order to use the concept of a single inclusion embedded in an infinite matrix. The effective elastic properties of the composite can be estimated once the strain concentration tensor for each type of inclusions is known. There are many micromechanical approaches to find A_n^i and results for three of them, namely Mori-Tanaka method[1)] (MT), the self-consistent scheme[2)] (SC) and the effective-medium-field approximation[3)] (EMF) [2] are presented. Although these classical equations are well known, they were mainly applied for single-phase composites. Equation (1) is solved without any simplifications in the considered problem, however its application is not trivial. Thanks to its general form, all isotropic as well as anisotropic elastic parameters can be evaluated from the obtained effective stiffness tensor. In the MT scheme the effective stiffness is given explicitely by Eq. (1). This is not the case in the other two approximations where Eq. (1) must be solved iteratively, e.g., by recurrent scheme. Furthermore, the well known simplified formulae for the Eshelby's tensor must be replaced by their general form, see [4],

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1) $A_n^{\mathrm{MT}} = A_n^{\mathrm{dil}} \left[v_m I + \sum_{n=1}^{N} v_n A_n^{\mathrm{dil}} \right]^{-1}$, $A_n^{\mathrm{dil}} = [I + S_n (C^m)^{-1} (C_n^i - C^m)]^{-1}$, I – the unit tensor, S_n – the Eshelby's tensor.

2) $A_n^{\mathrm{SC}} = [I + S_n C^{-1} (C_n^i - C)]^{-1}$.

3) $A_n^{\mathrm{EMF}} = A_n^{\mathrm{SC}} \left[v_m I + \sum_{n=1}^{N} v_n A_n^{\mathrm{SC}} \right]^{-1}$.

to have the micromechanical approach consistent. As a consequence, additional numerical integration is needed. Another difficulty connected with the general form of Eq. (1), which is due to diverse shapes of inclusions, is the lack of convergence of the SC scheme due to a high contrast in the material properties of composite constituents. Relevant details will be discussed in the paper.

3. Numerical example

The effective elastic properties of the composite with three types of carbon inclusions embedded in the copper matrix have been estimated. These types of inclusions are following: #1 modelled as sphere has the properties of fullerene, #2 modelled as oblate spheroid with the properties of graphite and #3 modelled as disc with the properties of graphene. All constituents of the composite are assumed to have isotropic properties. Due to the Eshleby's tensor specified in the local coordinate system of the inclusion, transversely isotropic effective properties are obtained. Assumption of the isotropic effective properties [5] or averaging over orientations (for random distribution of inclusions) leads to the isotropic effective stiffness tensor, and an example of this kind of results is presented below.

The effective Young modulus of the composite with respect to Young modulus of the matrix is shown in Fig. 1 for (a) only one type of inclusions (volume fraction of other types is equal to 0) and (b) the mixture of all considered types of inclusions, with $v_1 = v_2 = v_3$. The influence of the considered approximation methods is also presented.

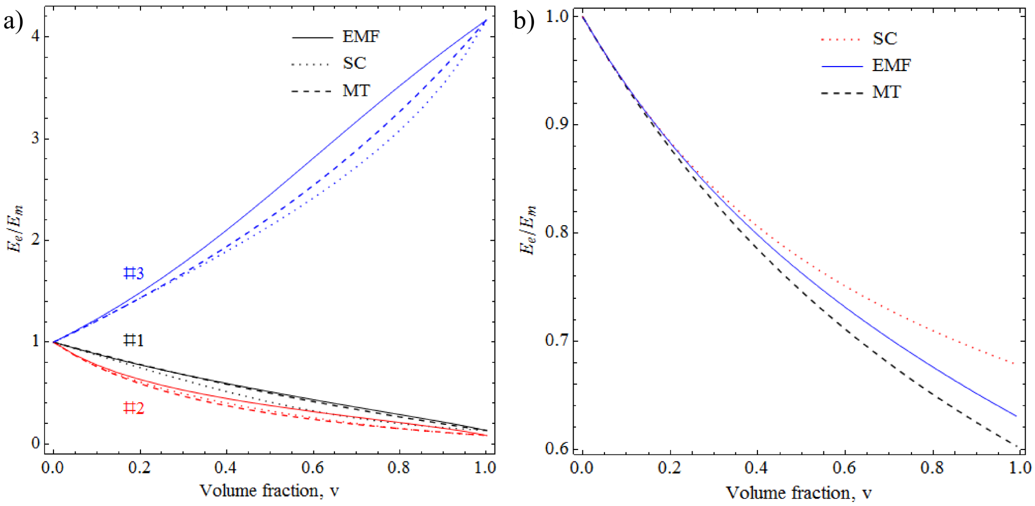

Fig. 1. Relative effective Young modulus E_e/E_m for three micromechanical approximations (MT – dashed line, SC – dotted line, EMF – solid line) for: a) only one type of inclusions ($v = v_n$) and b) three types of inclusions ($v = v_1 + v_2 + v_3$ and $v_1 = v_2 = v_3$).

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