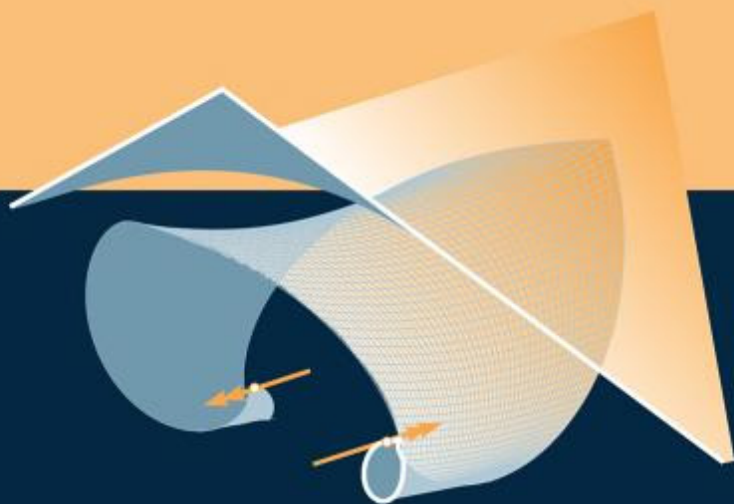


# Shell Structures

## *Theory and Applications*

*Volume 3*

Editors: W. Pietraszkiewicz & J. Górski



# Shell Structures: Theory and Applications

VOLUME 3

*Editors*

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## A plane stress elastic-plastic analysis of sheet metal cup deep drawing processes

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**ABSTRACT:** The paper presents a new yield criterion for the transversal isotropy of metal sheets under plane-stress conditions which is an extension of the isotropic yield function proposed by Burzyński (Burzyński W. 1928). Studium nad hipotezami Burzyński's doctoral dissertation "Study on material effort hypotheses", Engng. Trans., 2009, t. 57, nr 3–4, s. 185–215). Two additional coefficients have been introduced in order to allow a better representation of plastic behavior of metal sheets. The proposed yield condition includes the influence of first invariant of the stress tensor and also the strength differential effect. The system of equations describing the sheet metal forming process is solved by algorithm using the return mapping procedure. Plane stress constraint is incorporated into the Newton-Raphson iteration loop. The proposed algorithm is verified by performing a numerical test using shell elements in commercial FEM software ABAQUS/EXPLICIT with a developed VUMAT subroutine. It is shown that the proposed approach provides the satisfactory prediction of material behavior, at least in the cases when anisotropy effects are not advanced. To perform FE simulations of cup deep drawing processes, three independent yield stresses ( $\sigma_Y^T, \sigma_Y^C, \sigma_Y^{CC}$ ) are required. Those yield stresses can be obtained from: directional uniaxial tensile test, directional uniaxial compression test and equibiaxial compression tests. In the paper the formability of two metal sheets are analysed. First the influence of strength differential effect on the cup height profile is shown. Then the comparison between the Huber-Mises-Hencky yield condition and the proposed yield condition is presented.

### 1 INTRODUCTION

The anisotropy of a metal sheet during sheet forming is a combination of the initial anisotropy due to its previous history of thermomechanical processing and to the plastic deformation during the stamping operation. The former leads to symmetry with the orthotropic character while the latter, called deformation-induced anisotropy, can destroy this symmetry when principal material symmetry and deformation axes are not superimposed. Therefore, modelling of plastic anisotropy itself and its implementation in finite element (FE) code can be complex. For practical purposes, the assumption that the change of anisotropic properties during sheet forming is small and negligible, when compared to the anisotropy induced by rolling and heat treatment, has been widely adapted in the analysis of sheet metal forming. This is particularly important for industrial applications, where computation times are important factors to consider. In this case, it is convenient to use the concepts of anisotropic yield functions and isotropic hardening.

Finite element method is an efficient numerical tool to analyse such problems of shell deformation as for

instance the sheet metal forming processes including cup drawing and stamping. Proper description of material properties is crucial for the accurate analysis. In particular, the anisotropy and asymmetry of elastic range of considered materials play an important role in the finite element simulation. For metal forming analysis many experimental tests are needed to obtain the proper description of anisotropic behaviour of metal sheets. There are some attempts to account for both, anisotropy and the elastic range asymmetry, e.g. Plunkett et al. (2006). However, according to our opinion, there is still lack of workable description of these effects, which could allow to analyse practical problems effectively. For an anisotropic sheet material acceptable calibration of an appropriate yield function requires numerous mechanical testing procedures involving different loading modes such as directional uniaxial tensile tests and an equibiaxial tensile or bulge test. Realization of the mentioned loading modes requires the usage of different testing devices that are not always available. In the case of sheet materials we usually have the yield stress in uniaxial tension for different angles of loading directions. For metal sheets with transversely isotropic symmetry which are commonly observed it is very

important to have some experimental data in the normal direction.

Therefore, a calibration procedure for advanced yield functions involving only a single tensile or compression tests seems attractive.

For a plane stress state, integration of stress needs to satisfy the condition that the out-of-plane components of stress are zero, cf. Lee et al. (1998), Simo and Taylor (1985) and Simo and Taylor (1986). Due to this condition, which is called the plane stress constraint, particular schemes have been developed for plane stress elastic-plastic finite element analysis by Souza Neto et al. (2008) and Ohno et al. (2013). Simple schemes usually are based on plane stress projected constitutive models with in-plane stress  $\sigma_{xx}$ ,  $\sigma_{yy}$ ,  $\sigma_{xy}$  and strain  $\varepsilon_{xx}$ ,  $\varepsilon_{yy}$ ,  $\varepsilon_{xy}$  components. For complex constitutive models, when it is not so easy to derive plane stress-projected models, we can use other ideas, see Souza Neto et al. (2008).

## 2 CONSTITUTIVE EQUATIONS

Our material is described by a modified J2 plasticity which includes influence of the first invariant of stress tensor and strength differential effect.

We presume that the strain rate is additively decomposed into the elastic part obeying the isotropic Hooke's law and the plastic part governed by the associated flow rule:

$$\begin{aligned}\dot{\varepsilon} &= \dot{\varepsilon}^e + \dot{\varepsilon}^p \\ \dot{\sigma} &= \mathbf{C} : (\dot{\varepsilon} - \dot{\varepsilon}^p)\end{aligned}\quad (1)$$

$$\mathbf{C} = 2G\mathbf{I}' + K\mathbf{1} \otimes \mathbf{1}$$

where  $G$  is the shear modulus and  $K$  is the bulk modulus.

The flow rule has the form:

$$\dot{\varepsilon}^p = \dot{\lambda} \frac{\partial f}{\partial \sigma}\quad (2)$$

Finally, loading/unloading may be simply formulated in the Kuhn-Tucker form, that is:

$$\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda} f = 0$$

## 3 YIELD CONDITION

Plastic deformations in numerical analysis of sheet metal forming operations can be described by means of analytical yield functions. The proposed yield condition is based on the analysis of limit condition for transversally isotropic solids. The yield criterion for the transversal isotropy of metal sheets under plane-stress conditions is an extension of the isotropic

yield function proposed by Burzyński (1928), cf. Zyczkowski (1999), Nowak et al. (2013). In case of plane stress state the yield condition is in the following form:

$$\begin{aligned}f &= \frac{1}{2k_1} \{3(k_1 - 1)p + \\ &+ \sqrt{9(k_1 - 1)^2 p^2 + 4k_1 q^2}\} - \sigma_Y^T(\bar{\varepsilon}_p) = 0\end{aligned}\quad (3)$$

where

$$p = \frac{\sigma_x + \sigma_y}{3}$$

$$q = \sqrt{\sigma_x^2 + \sigma_y^2 + R_B \sigma_x \sigma_y + (2 - R_B) \sigma_{xy}^2}$$

$$R_B = 2 - \frac{1}{k_1 k_2^2} - \frac{2}{k_2} + \frac{2}{k_1 k_2}$$

$$k_1 = \sigma_Y^C / \sigma_Y^T$$

$$k_2 = \sigma_Y^{CC} / \sigma_Y^C$$

and  $\sigma_Y^C$  is the initial yield stress in uniaxial compression,  $\sigma_Y^T$  is the initial yield stress in uniaxial tension and  $\sigma_Y^{CC}$  is the initial yield stress in biaxial compression. When the applied material becomes isotropic,  $k_1 = 1.0$  and  $k_2 = 1.0$ , the yield condition in the form of Eq. (3) is transformed to the Huber-Mises-Hencky yield condition.

## 4 INTEGRATION OF THE ELASTO-PLASTICITY EQUATIONS

Utilizing the normality rule, the associated plastic strain increment  $\Delta \varepsilon^p_{ij}$  is obtained from the effective stress. The numerical procedure to obtain  $\Delta \varepsilon^p_{ij}$  is to find the unknown  $\Delta \bar{\varepsilon}^p$  (the equivalent plastic strain increment) from nonlinear equations. Using  $\Delta \bar{\varepsilon}^p$ , all kinematics variables and stresses are updated at the end of every step.

Let us consider a thin plate element in the plane stress state. We then have:  $\sigma_z = 0$ ,  $\sigma_{zx} = \sigma_{zy} = 0$  and  $\varepsilon_z \neq 0$ ,  $\gamma_{zx} = \gamma_{zy} = 0$ , where  $\gamma_{ij}$  ( $i \neq j$ ) indicates the engineering shear strain. Our calculation bases on assumption that the transversally isotropic thin shells made of the transversally isotropic, homogeneous material have the shear modulus which in the transverse direction is much smaller than the modulus in the tangential directions. After increment of time the stress is defined as:

$$\begin{aligned}\sigma_{(n+1)} &= \sigma_{(n+1)}^{trial} - \mathbf{C} : \Delta \varepsilon^p = \\ &= \sigma_{(n+1)}^{trial} - (2G\mathbf{I}' + K\mathbf{1} \otimes \mathbf{1}) : \Delta \varepsilon^p\end{aligned}\quad (4)$$

where the trial stress is:

$$\sigma_{(n+1)}^{\text{trial}} = \sigma_{(n)} + \mathbf{C} : \Delta \epsilon$$

After the updated stresses caused by material deformation are calculated from the equation (4), the thickness strain is updated.

## 5 CUP DEEP DRAWING PROCESS

In order to verify the implementation of the new yield function as well as its performance, cup drawing test simulations were carried out for the steel sheet. The square cup deep drawing process is selected. This is one of the benchmark problems provided by NUMISHEET93 and also analysed by Lee et al. (1998).

Figure 1 shows a schematic description of the tools for the square cup deep drawing. The geometric parameters of the deep drawing operation are: the punch diameter = 70.0 mm with rounded-off corner radius = 8.0 mm, rounded-off corner radius = 5.0 mm, the die diameter = 74.0 mm and the initial sheet thickness = 0.78 mm. The friction coefficient between the interface and the punch is set to 0.1, while that between the die and the blank holder is taken as 0.1 accounting for a certain degree of lubrication. The characteristic dimensions of the square cup drawing process are as follows: size of the blank  $150 \times 150$  mm; thickness of the blank 0.78 mm and the blankholding force 19.6 kN.

The blank is modelled by four-nodes shell element (ABAQUS elements library type – S4R), whereas the die, punch and holder are modelled by rigid elements (ABAQUS elements library type – R3D4).

In numerical simulations of the deep drawing process we used AISI 4330 steel. AISI 4330 is a heat treatable steel alloy (for high strength) containing chromium (0.85 wt. %), nickel (1.8 wt. %) and molybdenum (0.24 wt. %). Carbon content is 0.34 wt. % and in the heat treated condition the alloy has good toughness and fatigue strength as well as good strength. The formability is good, especially for sheet material in the annealed condition. Mechanical properties of the material of the blank 4330 steel used in the simulation are as follows: Young's modulus 198 GPa, Poisson ratio 0.29,  $\sigma_Y^T = 1437$  MPa,  $\sigma_Y^C = 1535$  MPa and  $\sigma_Y^C = 1842$  MPa (cf. Spitzig et al. 1975 and Hu & Wang 2009). For steel AISI 4330 the material parameters  $k_1 = 1.06$  and  $k_2 = 1.2$ .

The uniaxial true stress–strain data measured in the tension test for steel were fit to the power law equation

$$\sigma_Y^T(\epsilon_p) = A + B(\epsilon_p)^C \quad (5)$$

and the obtained coefficients are:  $A = 1435$  MPa,  $B = 824.9$  MPa,  $C = 0.3$ .

The blank is composed of 1849 elements with 1936 nodes.

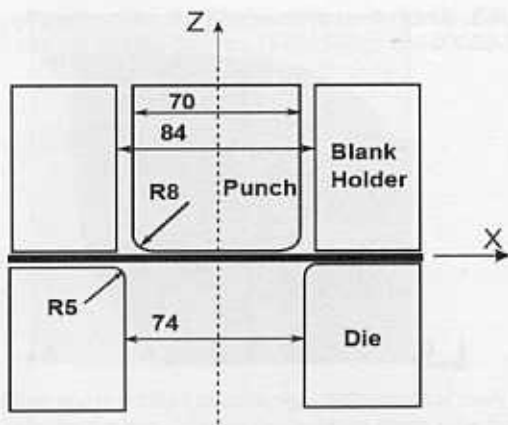


Figure 1. Schematic illustration of process setup in the square cup drawing tests, unit: mm.



Figure 2. Deformation of the square blank at the punch stroke 40 mm for the cup deep drawing of AISI 4330 steel with application of ABAQUS/EXPLICIT for modified Burzyński yield condition and isotropic hardening law for the punch stroke 40 mm of AISI 4330 steel.

In order to minimize the influence of the blank-holding force in this calculation, a cup formed with a minimum blank-holder force to prevent buckling under a well-lubricated condition was simulated. The influence of the yield stress only in uniaxial tension  $\sigma_Y^T$  and  $k_2 = \sigma_Y^C / \sigma_Y^T$  anisotropy parameter, i.e., the signature of the yield function, on the cup height profile was investigated.

The Figure 2 shows deformation of the square blank at the punch stroke 40 mm for the cup deep drawing with application of ABAQUS/EXPLICIT for proposed modified Burzyński yield condition Eq. (3) with the isotropic hardening law Eq. (5).

Figure 3(a) shows the deformed shape at the punch stroke 40 mm of AISI 4330 steel. Figure 3(b) shows the distance between points BB' as a function of time along the O–B direction of the cutting sections OB shown in Figure 3(a) for modified Burzyński yield condition Eq. (3) and Huber-Mises-Hencky yield condition. It is seen in Figure 3(b) that at the same level of punch stroke the results of the simulations for modified Burzyński yield condition Eq. (3) are not so advanced as for Huber-Mises-Hencky yield condition.

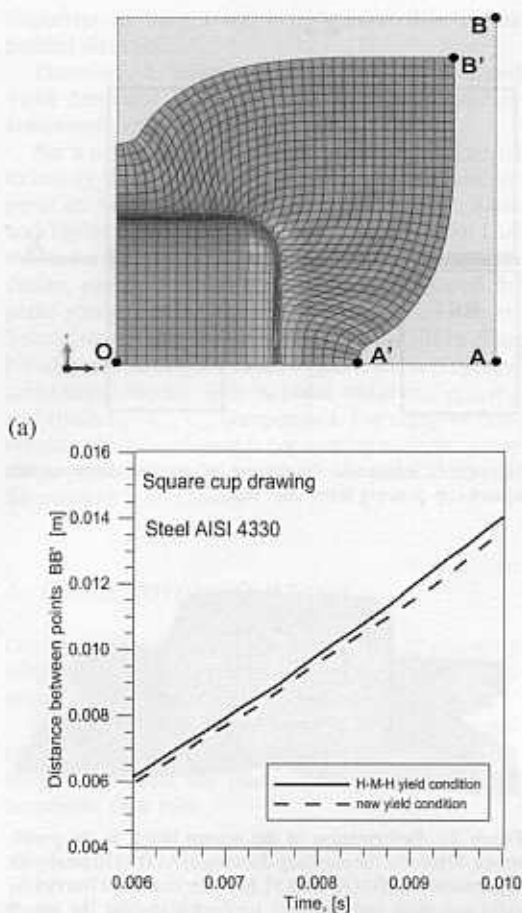


Figure 3. (a) Definition of the cutting sections O-B and the draw-in  $BB'$ ; (b) the distance between the points  $BB'$  of the cutting sections OB for AISI 4330 steel as a function of time with modified Burzyński yield condition Eq. (3) and Huber-Mises-Hencky yield condition.

## 6 CONCLUSIONS

Using a newly-developed yield condition, plastic deformation of a square steel blanket has been studied during cup deep drawing process. The main conclusions drawn from this study are:

It has been demonstrated that the proposed yield function calibration involving two directional uniaxial tests for tension and compression and one equibiaxial compression test gives satisfactory results.

The comparison of the deformation predictions with the proposed yield condition results shows a good agreement from an engineering point of view; however, the evaluation of the local strain and stress

histories indicates that the yield condition needs to be improved further.

The new criterion has an increased flexibility to make the prediction of proposed model more approaching reality due to the fact that it uses five coefficients in order to describe the yield surface with pressure and strength-differential dependence.

## ACKNOWLEDGEMENT

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