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THE CONCEPT OF SMART PRESTRESSED COMPOSITES. NUMERICAL MODELLING AND EXPERIMENTAL EVALUATION.

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Summary. *The paper introduces the concept of prestressing selected layers of laminated composite in order to increase its stiffness and improve its overall mechanical response. Both analytical and numerical models of prestressed composites are proposed and utilized to optimize applied prestressing forces. Further, dedicated laboratory stand and developed methodology of experimental evaluation are described. The final part of the paper briefly discusses prospective applications of prestressed composites.*

1 INTRODUCTION

Initial prestressing is efficient technique used in reinforced concrete structures, such as girders and slabs, in order to minimize tensile stresses generated during bending and to overcome low tensile strength and brittle behaviour of concrete. Application of prestressed reinforcing bars or tendons results in favourable redistribution of internal forces in concrete, and, as a consequence, allows to increase load-capacity of the structure, extend its maximal span or decrease required thickness of the elements. Initial prestressing helps to avoid cracking of external surfaces and thus prevents corrosion of the entire structure. As a result, prestressed reinforced concrete structures are characterized by enhanced mechanical and functional properties and extended service life.

Herein, we propose to expand and generalize the concept of prestressing and to apply it to uni-directionally reinforced laminated composites. According to the proposed idea, prestress will be introduced by initial pre-tensioning of the reinforcing fibres of selected composite's layers. Due to the fact that laminates are composed of plies of various lamination angles, their mechanical behaviour is relatively complex and applied prestress will influence the entire structural response including in-plane forces, bending and twisting. Consequently, appropriately adjusted prestressing allows not only for increase of composite load capacity and reduction of cracking, but also for precise control of its overall state of deformation and state of stress. Prestressed composite can be precisely adjusted to expected working conditions and its total weight and fabrication cost can be significantly reduced.

The introductory results presented in this paper are the first stage of COMPRESS project conducted within SMART-NEST research cluster (Marie Curie Action). A long-term objective of this project is to study the relaxation behaviour, damage mechanisms and damage detection techniques for pre-stressed composites with the use of numerical models and experimental methods. The project is divided into the following modules:

1. formulation and validation of the numerical model for prestressed composite,
2. study of the relaxation process and its influence on composite properties,
3. study of the damage mechanisms of prestressed composites,
4. evaluation of prestressed composites damages by using acoustic techniques,
5. integration of all modules and building system demonstrator or prototype.

2 THE CONCEPT OF PRESTRESSED COMPOSITES

The term prestressing of fibre-reinforced composites indicates applying initial tensile stress to the fibres embedded in arbitrarily selected layers of the composite. By the similarity to reinforced concrete structures, initial pre-stressing of the fibres and their further release produces clamping load causing compressive stresses in the surrounding matrix and changes the state of equilibrium of the entire composite.

The process of fibres prestressing is conducted after moulding, before the curing process takes place. At a first step external tensile force is applied to the fibres causing their elongation and corresponding tensile stress (Fig. 1a). Since polymeric matrix remains in uncured state and it is not yet solidified, it is not affected by applied pre-load. Further, the laminate is cured in a standard manner, causing the matrix to gel and harden. At this stage matrix starts to adhere and bond to prestressing fibres. Moreover, residual tensile stresses are generated in matrix as a result of the curing process.

Once the composite is entirely cured and cooled down to room temperature the external tensile force is released. The pre-tensioned fibres tend to contract and static friction force causes that compressive stresses are locally induced to the matrix. In final situation reinforcing fibres still remain tensioned, while the surrounding matrix is compressed (Fig.1b). The entire procedure generates initial self-equilibrated state of stress of the composite, which substantially influences its mechanical properties and its response to external loading.

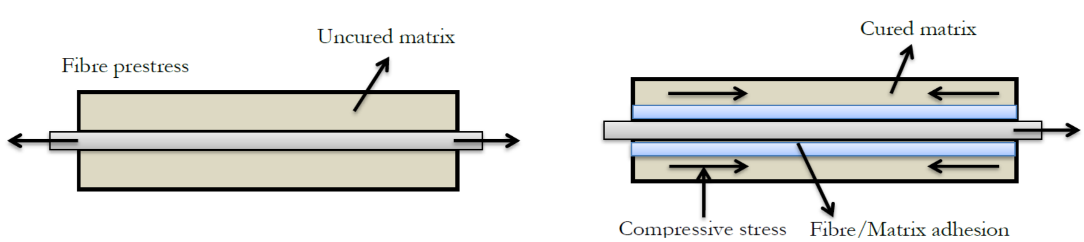


Figure 1: Two stages of prestressing of a single layer of fibre-reinforced composite

Manufacturing of prestressed composites and applying a pre-load to the laminate can be performed with the use of various methods including dead weight method, filament winding method and biaxial loading frame method [1,3]. Different techniques together with their characteristics and drawbacks are presented by Krishnamurthy [3].

A special case of prestressed composite is uni-directionally reinforced laminate with initial prestressing of one external layer. In such case applied prestress causes additional bending moment acting on the composite and causing its deformation into a camber shape, Figure 2a. Created bending moment effectively counteracts self-weight of the structural member and vertical service loads, which it typically has to carry, Figure 2b. Superposition of the response to prestress and response to external loading results in reduction of internal stresses and decrease of vertical deflection. Accordingly, prestressed composite may be considered as stiffer than the initial one.

The above example illustrates only the simplest possible, classical case of applying prestress, which resembles application in reinforced concrete structures. However, in contrast to a single reinforced-concrete element a composite may have a complex construction (as composed of many layers of various mechanical properties and lamination angles) and typically they may be subjected to external forces of various direction and magnitude (airplane airfoil, helicopter propeller, etc.). Therefore, applied loading may cause a very complex mechanical behaviour including coupling between tension, bending and twisting. The problem of optimal design of prestressed composites is to select prestressed plies and adjust the value of prestressing force such that structural response to expected external loading is mitigated in a desired way.

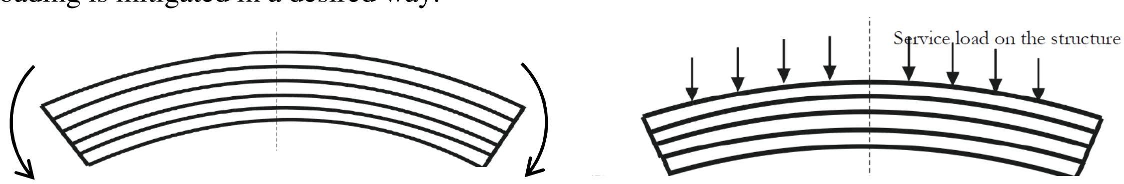


Figure 2: (a) Structure after prestressing, (b) Prestressed structure during service load

Apart from above described major objective, prestressing of fibre-reinforced composites has several important additional advantages. At first, compressive stresses generated in the matrix reduce both cure-induced and thermally-induced residual tensile stresses resulting from the manufacturing cycle. Consequently, creation of compressive stresses prevents or, at least, impedes formation and propagation of crack in the matrix [1]. As it was suggested by Dvorak et al [2], due to the fact that fibres are maintained prestressed during the entire curing process, the misalignment of fibres, also called fibre waviness, is substantially reduced. It was also proved that pres-stressed composites are characterized by improvement of fatigue life and resistance to stiffness degradation in the matrix-dominated fatigue region [5,6].

The problems of damage of prestressed composites were already preliminarily investigated in the scientific literature. Krishnamurthy [3] describes damage mechanisms in uni-directionally reinforced laminates, which were tested under quasi-tension, compression and fatigue. Similar problems were analyzed by Haris [6] and Fuwa [7] who suggested that damage mechanisms depend strongly on the relationship between composites static failure strains. The above cited literature also describes other damage mechanisms as fibre breakage, interfacial debonding, matrix cracking and interfacial shear failure. Basic mechanical properties of prestressed composites as resistance to tensile, flexural loading and impact, as well as their advantages over classical composites are presented in references [1-5].

In the broader context, design of prestressed laminated composites can be treated as a first step to development of 'smart prestressed composites', which are additionally equipped with:

- system of sensors serving for load detection and damage identification,
- active fibres allowing for changing prestressing of selected layers,
- embedded software controller which optimally adjust actual prestressing force to recognized loading and possible damage.

The examples of sensing and actuating systems are presented in Fig. 3. Two innovative methodologies of damage detection utilize electrical grid embedded in composite. In the first approach composite damage is identified by using response of damaged RLC circuit to a given excitation [8] (ELGRID system, Fig. 3a), while in the second one by change of thermal field observed by long-wave thermovision camera [9] (THERM-GRID system, Fig. 3b). In turn, two possible methods of system actuation are based on fibers made of shape-memory alloys (Fig. 3c) and piezoelectric fibers (Fig. 3d). The entire system is expected to provide comprehensive control of composite structural response including actual deformation and state of stress. As a result, on the similarity of systems of Adaptive Impact Absorption [8], proposed intelligent composite could automatically adapt to actual excitation and possible defect and acquire temporarily optimal mechanical characteristics.

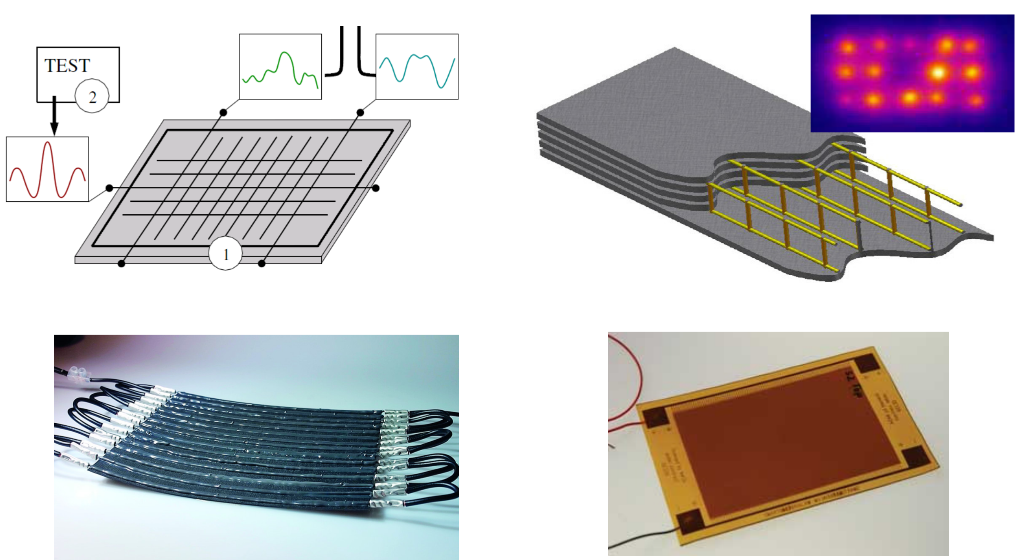


Figure 3: Components of smart prestressed composite : a) ELGRID damage identification system, b) THERM-GRID damage identification system, c) actuator based on shape memory alloy fibers (Institute for Verbundwerkstoffe, Germany), d) piezolelectric fibers actuators (Smart-Material Company, Germany)

Let us note that simulation of smart prestressed composites is associated with additional research challenges. At first, it requires multi-physical modelling of shape memory alloy or piezoelectric phenomenon and prestress induced by such effects. At second, simulation of active response of adaptive composite requires additional coupling between load (or damage) identification system and applied system of actuation. At last, final design of the smart prestressed composite involves optimization of layout and mechanical properties, elaboration of methods for load and damage identification, development of control strategies and system integration.

3 ANALYTICAL MODELLING OF PRESTRESSED COMPOSITES

Preliminary evaluation of the prestressing concept will be conducted with the use of numerical analysis. The evaluation procedure requires development of mathematical models of uni-directionally reinforced prestressed composites and application of optimization algorithms in order to adjust prestress according to assumed optimality criteria.

The problem of modelling prestressed composites will be considered in micro and macro scale, initially with the use of analytical models and, further, the numerical ones. The analysis will be limited to the case of linear models based on assumptions of small deformation and elastic constitutive relations. An elementary structural part considered in composite theories is inhomogeneous layer constructed of matrix and reinforcing fibres. Although such element is initially analysed by micromechanical models, it is further homogenized into a single orthotropic layer. A composite constructed as stack of orthotropic layers with various lamination angles can be analysed on macro-mechanical level by applying assumptions of plate theories. Two basic possibilities are: i) using layer-wise (LW) models where each homogenous layer is considered separately and ii) using equivalent single layer (ESL) models where the constitutive and equilibrium equations are set for the mid-surface only.

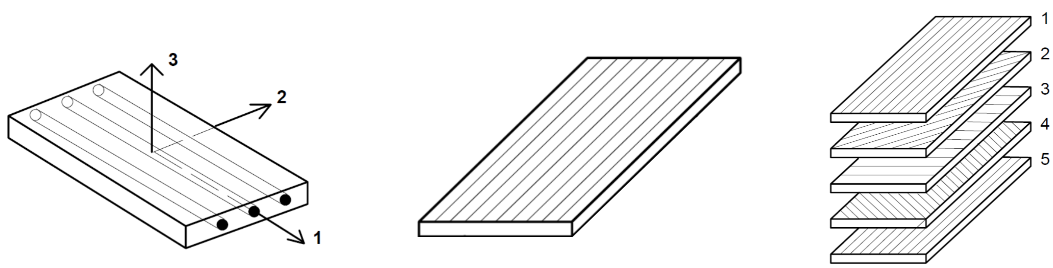


Figure 4: Three stages of modelling fibre-reinforced composites: a) inhomogeneous layer of the composite, b) homogenized isotropic model of the layer, c) layered model of the composite.

The above subsequent steps of derivation of classical laminate plate theory (CLPT) will be methodologically followed taking into account initial prestressing of selected layers. Consequently, this section will be composed of three parts dedicated to analytical models of prestressed plies, methods of plies homogenization and analytical models of prestressed composites. The following sections will concern numerical implementation of such models in commercial finite element code and formulation and examination of the corresponding optimization problems.

3.1 Analytical models of prestressed plies

The first step of derivation of the mathematical model of prestressed composite is idealized analytical micro-mechanical model of a single ply reinforced with fibres parallel to one of the edges and subjected to action of longitudinal, transverse and shear stresses on the boundaries ($\sigma_{11},\sigma_{22},\sigma_{12}$). Material data for the model are Young modulus, Poisson's ratio and volume fraction of the fibres E_z,ν_z,v_z and the matrix E_m,ν_m,v_m. The fundamental assumption of the model is perfect bonding between the fibres and the matrix. As a consequence, both longitudinal and transverse loading induces equal longitudinal strains in fibres and matrix: $\varepsilon_{11}^z=\varepsilon_{11}^m$. It is also assumed that transverse loading causes equal transverse stresses in fibres and matrix $\sigma_{22}^z=\sigma_{22}^m$, which can be treated as a result of geometrical simplification. Finally, a plane state of stress is considered.

Derivation of the analytical model of the prestressed ply requires taking into account state of stress and strain caused by and prestressing of the fibres with longitudinal force F_0. Let us note that in considered method of composite manufacturing applied prestress causes two-directional deformation of the fibres before solidification of the matrix. Consequently, two initial strains of the fibres have to be introduced into the model. This is in contrast to classical longitudinal prestress, when only longitudinal initial strain should be applied. State of stress and strain in fibres and matrix can be determined directly from the equilibrium equations, constitutive relations taking into account initial deformation of the fibres and generalized Hook's law. The corresponding system of equations takes the form:

$$\sigma_{11}^z\mathrm{v}_z+\sigma_{11}^m\mathrm{v}_m=\sigma_{11},\quad \sigma_{22}^z=\sigma_{22}^m=\sigma_{22}, \tag{1a,b}$$

$$\varepsilon_{11}^z-\varepsilon_{11}^{z(0)}=\frac{\sigma_{11}^z}{E_z}-\frac{\nu_z}{E_z}\sigma_{22},\quad \varepsilon_{11}^m=\frac{\sigma_{11}^m}{E_m}-\frac{\nu_m}{E_m}\sigma_{22}, \tag{1c,d}$$

$$\varepsilon_{22}^z-\varepsilon_{22}^{z(0)}=-\frac{\nu_z}{E_z}\sigma_{11}^z+\frac{\sigma_{22}}{E_z},\quad \varepsilon_{22}^m=-\frac{\nu_m}{E_m}\sigma_{11}^m+\frac{\sigma_{22}}{E_m}, \tag{1e,f}$$

$$\gamma_{12}^z=\frac{2\sigma_{12}(1+\nu_z)}{E_z},\quad \gamma_{12}^m=\frac{2\sigma_{12}(1+\nu_m)}{E_m}. \tag{1g,h}$$

where: $\varepsilon_{11}^{z(0)}=-F_0/(E_z A\mathrm{v}_z)$, $\varepsilon_{22}^{z(0)}=\nu_z F_0/(E_z A\mathrm{v}_z)$

The equations 1a,c-d can be solved independently in order to find quantities corresponding to longitudinal deformation: $\sigma_{11}^z,\sigma_{11}^m,\varepsilon_{11}^{z/m}$ and the remaining quantities corresponding to transverse and shear deformation can be determined from equations 1e-h. Finally, total state of strain and stress in a prestressed ply is defined as:

$$\varepsilon_{11}^z=\varepsilon_{11}^m=\frac{\sigma_{11}}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}-\frac{\nu_z\mathrm{v}_z+\nu_m\mathrm{v}_m}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}\sigma_{22}-\frac{F_0}{A(E_m\mathrm{v}_m+E_z\mathrm{v}_z)}, \tag{2a}$$

$$\varepsilon_{22}^z=-\frac{\nu_z\sigma_{11}}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}-\frac{\sigma_{22}\nu_z\mathrm{v}_m(-E_z\nu_m+E_m\nu_z)}{E_z(E_z\mathrm{v}_z+E_m\mathrm{v}_m)}+\frac{\sigma_{22}}{E_z}+\frac{\nu_z F_0}{A(E_m\mathrm{v}_m+E_z\mathrm{v}_z)}, \tag{2b}$$

$$\varepsilon_{22}^{m}=-\frac{\nu_m\sigma_{11}}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}+\frac{\sigma_{22}\nu_m\mathrm{v}_z(-E_z\nu_m+E_m\nu_z)}{E_m(E_z\mathrm{v}_z+E_m\mathrm{v}_m)}+\frac{\sigma_{22}}{E_m}+\frac{\nu_m F_0}{A(E_m\mathrm{v}_m+E_z\mathrm{v}_z)}, \tag{2c}$$

$$\gamma_{12}^{z}=\frac{2\sigma_{12}(1+\nu_z)}{E_z},\quad \gamma_{12}^{m}=\frac{2\sigma_{12}(1+\nu_m)}{E_m}. \tag{2d}$$

$$\sigma_{11}^{z}=\frac{E_z\sigma_{11}}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}+\sigma_{22}\mathrm{v}_m\left[\frac{-E_z\nu_m+E_m\nu_z}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}\right]-\frac{E_zF_0}{A(E_m\mathrm{v}_m+E_z\mathrm{v}_z)}+\frac{F_0}{A\mathrm{v}_z}, \tag{2e}$$

$$\sigma_{11}^{m}=\frac{E_m\sigma_{11}}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}-\sigma_{22}\mathrm{v}_z\left[\frac{-E_z\nu_m+E_m\nu_z}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}\right]-\frac{E_mF_0}{A(E_m\mathrm{v}_m+E_z\mathrm{v}_z)}, \tag{2f}$$

The above given formulae provide complete knowledge about micro-mechanical response of the prestressed ply subjected to in-plane loading. Calculation of total longitudinal, transverse and shear deformation requires considering strains calculated for the matrix and the fibres and taking into account their partial fractions:

$$\varepsilon_{11}=\varepsilon_{11}^{z}=\varepsilon_{11}^{m} \tag{3a}$$

$$\begin{aligned}\varepsilon_{22}=-\sigma_{11}\frac{\nu_z\mathrm{v}_z+\nu_m\mathrm{v}_m}{E_z\mathrm{v}_z+E_m\mathrm{v}_m}+\sigma_{22}\left(\frac{\nu_m}{E_m}-\frac{\nu_z}{E_z}\right)\left[\mathrm{v}_z\mathrm{v}_m\frac{(-E_z\nu_m+E_m\nu_z)}{E_z(E_z\mathrm{v}_z+E_m\mathrm{v}_m)}\right]\\+\sigma_{22}\left(\frac{\mathrm{v}_z}{E_z}+\frac{\mathrm{v}_m}{E_m}\right)+\frac{F_0(\nu_z\mathrm{v}_z+\nu_m\mathrm{v}_m)}{A(E_m\mathrm{v}_m+E_z\mathrm{v}_z)}\end{aligned} \tag{3b}$$

$$\gamma_{12}=2\sigma_{12}\left[\frac{(1+\nu_z)v_m}{E_z}+\frac{(1+\nu_z)v_z}{E_z}\right] \tag{3c}$$

In turn, the above formulae define global mechanical response of the prestressed ply. Let us note that in micromechanical model transversal loading σ_{22} causes generation of longitudinal stresses in fibres and matrix, which influence transversal deformation. Only in a special case when $\nu_z/E_z=\nu_m/E_m$, applied transverse loading causes the same deformation of fibres and matrix in longitudinal direction, which leads to substantial simplification of the formulae defining strains and stresses fields (Eq.2b,c and Eq.2e,f).

3.2 Homogenization of prestressed plies

Elaboration of simple and effective model of prestressed composite requires homogenization of a single layer, i.e. replacing heterogeneous layer composed of matrix and fibres with equivalent homogenous layer, whose mechanical response resembles global response of the micromechanical model. Due to the fact that considered layer has three planes of material symmetry, the homogeneous layer will be orthotropic and in considered plane state of stress it will be characterized by four independent material constants E_1,E_2,ν_{12},G_{12}. Since original layer is prestressed the equivalent macro-mechanical model will be described by the following constitutive relation:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix}-\begin{bmatrix}\varepsilon_{11}^{0}\\ \varepsilon_{22}^{0}\\ 0\end{bmatrix}=\begin{bmatrix}1/E_1 & -\nu_{21}/E_2 & 0\\ -\nu_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix} \tag{4}$$

The symmetry of the matrix of the above matrix implies the equality: $\nu_{21}/E_2 = \nu_{12}/E_1$, which confines the number of independent material constants. In classical homogenization method considered layer composed of matrix and fibres, as well as homogeneous orthotropic layer are subsequently subjected to three types of allowable in-plane loadings. The equivalence of micro and macro-mechanical model should hold for all components of strain when a single loading of a certain type is applied. Recalling micro-mechanical quantities defined by Eq.3a,b the equations used for calculation of macro-mechanical constants of homogenized material take the form:

$$\frac{\sigma_{11}}{E_z \mathrm{v}_z + E_m \mathrm{v}_m} = \frac{\sigma_{11}}{E_1}, \tag{5a}$$

$$-\frac{\nu_z \mathrm{v}_z + \nu_m \mathrm{v}_m}{E_z \mathrm{v}_z + E_m \mathrm{v}_m}\sigma_{22} = -\frac{\nu_{21}}{E_2}\sigma_{22} = -\frac{\nu_{12}}{E_1}\sigma_{22} \tag{5b}$$

$$-\sigma_{11}\frac{\nu_z \mathrm{v}_z + \nu_m \mathrm{v}_m}{E_z \mathrm{v}_z + E_m \mathrm{v}_m} = -\frac{\nu_{12}}{E_1}\sigma_{11}, \tag{5c}$$

$$\sigma_{22}\left(\frac{\nu_m}{E_m}-\frac{\nu_z}{E_z}\right)\left[\mathrm{v}_z \mathrm{v}_m \frac{(-E_z \nu_m + E_m \nu_z)}{E_z(E_z \mathrm{v}_z + E_m \mathrm{v}_m)}\right] + \sigma_{22}\left(\frac{\mathrm{v}_z}{E_z}+\frac{\mathrm{v}_m}{E_m}\right) = \frac{\sigma_{22}}{E_2}, \tag{5d}$$

$$\frac{\sigma_{12}}{G_z}\mathrm{v}_z + \frac{\sigma_{12}}{G_m}\mathrm{v}_m = \frac{\sigma_{12}}{G_{12}}. \tag{5e}$$

Utilizing symmetry of macro-mechanical compliance matrix in equation (5b) leads to the equation identical to (5c), which allows to find four unknown material constants expressed as:

$$E_1 = E_z \mathrm{v}_z + E_m \mathrm{v}_m, \quad \nu_{12} = \nu_z \mathrm{v}_z + \nu_m \mathrm{v}_m, \tag{6a,b}$$

$$E_2 = \left[\frac{\mathrm{v}_z}{E_z}+\frac{\mathrm{v}_m}{E_m}+\left(\frac{\nu_m}{E_m}-\frac{\nu_z}{E_z}\right)\left(\mathrm{v}_m \mathrm{v}_z \frac{(-E_z \nu_m + E_m \nu_z)}{(E_z \mathrm{v}_z + E_m \mathrm{v}_m)}\right)\right]^{-1}, \tag{6c}$$

$$G_{12} = \left[\frac{\mathrm{v}_z}{G_z}+\frac{\mathrm{v}_m}{G_m}\right]^{-1}, \quad \nu_{21} = \frac{E_2}{E_1}\nu_{12} \tag{6d,e}$$

Let us note that in homogenized model of a single ply, in contrast to exact analytical one, a transverse loading does not cause generation of longitudinal stresses. Since in micro-mechanical model stresses in matrix and fibres are in equilibrium, the homogenization procedure averages them to zero. Nevertheless, derived formulae for transversal Young modulus E_2 provide that the above effect of longitudinal stress generation is taken into account and that exact value of transversal deformation is obtained. A similar situation occurs during considering macro-mechanical model including initial prestress of the layer.

Homogenization of the prestressed layer requires taking into account additional strains and stresses induced by initial prestress of the fibres. By considering total values of prestress-induced longitudinal strains and transversal strains (last terms of Eq. 3a,b) and by applying homogenized material constants we obtain:

$$\varepsilon_{11}(F_0) = -\frac{F_0}{A(E_m \mathrm{v}_m + E_z \mathrm{v}_z)} = -\frac{1}{E_1}\frac{F_0}{A}, \qquad \varepsilon_{22}(F_0) = \frac{(\nu_z \mathrm{v}_z + \nu_m \mathrm{v}_m)F_0}{A(E_m \mathrm{v}_m + E_z \mathrm{v}_z)} = \frac{\nu_{12}}{E_1}\frac{F_0}{A}. \tag{7a,b}$$

In turn, summing up prestress-induced longitudinal stresses (cf. Eqs. 2e,f) gives

$$\sigma_{11}^{total}(F_0) = \sigma_{11}^{z}(F_0) + \sigma_{11}^{m}(F_0) = 0 \tag{7c}$$

The above equalities reveal the difference between exact and homogenized model of the prestressed ply. In exact model applied prestress of the fibers causes both state of deformation and longitudinal state of stress, while in homogenized model applied prestress causes exclusively state of deformation and stresses are averaged to zero. This indicates that in homogenization procedure applied prestress cannot be considered as additional external loading since it always induces simultaneous state of deformation and state of stress. Instead, deformation caused by prestress should be considered and modelled as initial two-directional strain. By comparing prestress-induced deformations in micromechanical model (Eq. 7a,b) and initial strains in homogenized model (Eq. 4) we obtain:

$$\varepsilon_{11}^0 = \varepsilon_{11}(F_0) = -\frac{1}{E_1}\frac{F_0}{A}, \qquad \varepsilon_{22}^0 = \varepsilon_{22}(F_0) = \frac{\nu_{12}}{E_1}\frac{F_0}{A}. \tag{8a,b}$$

Thus, similarly as for micro-mechanical model, in homogenized model unidirectional prestress is represented by two-directional initial strain of a specific ratio. The equilibrium under zero external loading holds for deformed configuration of the ply, when it is shortened in longitudinal direction and elongated in the transverse one. In case when the ply is in the initial state $\varepsilon_{11} = 0, \varepsilon_{22} = 0$ only the longitudinal tensile stress is generated (cf. Eq.4). Although the above approach is simple and straightforward, it requires introducing the notion of initial strain and considering it during constructing a model of the composite.

3.3 Analytical modelling of the entire prestressed composite

The following step in derivation of the model of prestressed composite is embedding prestressed layer in the composite. The proposed model of the prestressed composite will be based on equivalent single layer (ESL) approach and it will utilize classical laminate plate theory (CLPT). Classical steps of deriving ESL model of the composite will be conducted for a composite containing single prestressed layer in order to observe the influence of initial prestress on the model of the composite. Ultimately, the proposed analytical method of modelling composite with prestressed layer will be based on application of additional external loading and the principle of superposition.

In the first step of derivation of the ESL model of the composite an micro-mechanical model of inhomogeneous layer composed of fibres and matrix is replaced by orthotropic macro-mechanical model. As it was shown in Sect. 3.2 on the macro-mechanical level prestress of the layer is equivalent to two-directional state of initial strain. Consequently, in local coordinate system the constitutive relations take the following forms:

$$\boldsymbol{\varepsilon}'-\boldsymbol{\varepsilon}'^{\mathbf{0}}=\mathbf{S}\boldsymbol{\sigma}', \quad \boldsymbol{\sigma}'=\mathbf{Q}(\boldsymbol{\varepsilon}'-\boldsymbol{\varepsilon}'^{\mathbf{0}}) \tag{9}$$

In above equations $\boldsymbol{\varepsilon}'$ and $\boldsymbol{\sigma}'$ indicate engineering strain and stress vectors (Voigt notation) referring to the local coordinate system defined by the orientation of the fibres, previously marked with Latin numbers. Additionally, $\boldsymbol{\varepsilon}'^{\mathbf{0}}$ is a vector of initial strains which contains two non-zero components ε'^{0}_{11} and ε'^{0}_{22} corresponding to longitudinal and transverse direction and defined by the Eq. 8a,b. Moreover, **S** and **Q** are classical compliance and stiffness matrices of the orthotropic layer.

The second step of the procedure is transformation of the local strain tensor, local stress tensor and the constitutive relation into global coordinate system. Transformation of the strain and stress vectors between the coordinate systems, as well as transformation between engineering and tensorial strain is conducted in a classical way, however considered transformations take into account vector of initial strains:

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \tfrac{1}{2}\gamma_{12}\end{bmatrix}-\begin{bmatrix}\varepsilon^{0}_{11}\\ \varepsilon^{0}_{22}\\ 0\end{bmatrix}=\mathbf{T}\left(\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \tfrac{1}{2}\gamma_{xy}\end{bmatrix}-\begin{bmatrix}\varepsilon^{0}_{xx}\\ \varepsilon^{0}_{yy}\\ \tfrac{1}{2}\gamma^{0}_{xy}\end{bmatrix}\right), \qquad \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}=\mathbf{T}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} \tag{10}$$

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}-\begin{bmatrix}\varepsilon^{0}_{xx}\\ \varepsilon^{0}_{yy}\\ \gamma^{0}_{xy}\end{bmatrix}=\mathbf{R}\left(\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \tfrac{1}{2}\gamma_{xy}\end{bmatrix}-\begin{bmatrix}\varepsilon^{0}_{xx}\\ \varepsilon^{0}_{yy}\\ \tfrac{1}{2}\gamma^{0}_{xy}\end{bmatrix}\right)$$

where **T** is classical transformation matrix and **R** is the Reuters matrix. After standard derivation, the constitutive equations defining relation between engineering strain vector and stress vector in global coordinate system take the form:

$$\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{0}=\overline{\mathbf{S}}\,\boldsymbol{\sigma}\ , \quad \boldsymbol{\sigma}=\overline{\mathbf{Q}}(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{0}) \tag{11a,b}$$

where the compliance and stiffness matrices are given by classical formulae, while $\boldsymbol{\varepsilon}^{0}$ is vector of initial strain transformed into a global coordinate system:

$$\overline{\mathbf{S}}=\mathbf{R}\mathbf{T}^{-1}\mathbf{S}\mathbf{R}^{-1}\mathbf{T}\ , \quad \overline{\mathbf{Q}}=\overline{\mathbf{S}}^{-1}=\mathbf{T}^{-1}\mathbf{R}\mathbf{Q}\mathbf{T}\mathbf{R}^{-1}, \quad \boldsymbol{\varepsilon}^{0}=\mathbf{R}\mathbf{T}^{-1}\mathbf{R}^{-1}\boldsymbol{\varepsilon}'^{0} \tag{12a,b,c}$$

The following step is derivation of the macro-mechanical model of the entire composite constructed as a stack of plies of various lamination angles. We will apply classical laminate plate theory (CLPT), which can be treated as extension of Kirchhoff-Love theory for homogeneous plates. A composite is assumed to be thin, to have constant thickness and to undergo small deflection in comparison to the thickness. Kirchhoff hypothesis about the normal to midsurface states that the normal remains straight, perpendicular and of the same length, which results in vanishing of selected strain components: $\gamma_{xz} = 0,\ \gamma_{yz} = 0,\ \varepsilon_{zz} = 0$. According to the above kinematic assumption displacements of an arbitrary point can be expressed in terms of displacements of the midsurface u^M, v^M and its rotation angle with respect to particular axes:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u^M \\ v^M \end{bmatrix} - z \begin{bmatrix} w^M_{,x} \\ w^M_{,y} \end{bmatrix} \tag{13}$$

Consequently, strains at arbitrary point of the composite can be defined with the use of strains of the midsurface and its curvature:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^M + z\boldsymbol{\kappa}^M, \quad \boldsymbol{\varepsilon}^0 = \boldsymbol{\varepsilon}_0^M + z\boldsymbol{\kappa}_0^M \tag{14}$$

Let us note that the above kinematic hypothesis holds both for total strains $\boldsymbol{\varepsilon}$, as well as for the initial strains $\boldsymbol{\varepsilon}^0$. Stresses in particular layers of the composite are expressed in classical manner with the use of stiffness matrix in global coordinate system $\overline{\mathbf{Q}}_k$, however definition of stress in a single prestressed layer additionally takes into account initial strain:

$$\boldsymbol{\sigma}_k = \overline{\mathbf{Q}}_k \boldsymbol{\varepsilon} = \overline{\mathbf{Q}}_k (\boldsymbol{\varepsilon}^M + z\boldsymbol{\kappa}^M), \qquad \boldsymbol{\sigma}_p = \overline{\mathbf{Q}}_k (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^0) = \overline{\mathbf{Q}}_k (\boldsymbol{\varepsilon}^M + z\boldsymbol{\kappa}^M) - \overline{\mathbf{Q}}_k \boldsymbol{\varepsilon}^0 \tag{15a,b}$$

Further in-plane forces **N** and bending moments **M** are defined as a sum of integrals of layer's stresses ($\boldsymbol{\sigma}_k$ or $\boldsymbol{\sigma}_p$) over layer's thicknesses:

$$\mathbf{N} = \int_{-h/2}^{h/2} \boldsymbol{\sigma}\, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \boldsymbol{\sigma}_{k/p}\, dz, \qquad \mathbf{M} = \int_{-h/2}^{h/2} \boldsymbol{\sigma} z\, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \boldsymbol{\sigma}_{k/p}\, z dz \tag{16a,b}$$

By using definition of stress in standard layers and single prestressed layer we obtain:

$$\mathbf{N} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{\mathbf{Q}}_k \boldsymbol{\varepsilon}^M\, dz + \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{\mathbf{Q}}_k z \boldsymbol{\kappa}^M\, dz - \int_{z_p}^{z_{p+1}} \overline{\mathbf{Q}}_k \boldsymbol{\varepsilon}^0\, dz, \tag{17a}$$

$$\mathbf{M} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{\mathbf{Q}}_k \boldsymbol{\varepsilon}^M\, z dz + \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{\mathbf{Q}}_k \boldsymbol{\kappa}^M\, z^2 dz - \int_{z_p}^{z_{p+1}} \overline{\mathbf{Q}}_k \boldsymbol{\varepsilon}^0\, z dz \tag{17b}$$

Finally, by eliminating terms independent on 'z' off before the integral sign we obtain the following relations between the internal forces and deformation of the middle surface of the composite:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^M \\ \boldsymbol{\kappa}^M \end{bmatrix} - \begin{bmatrix} \mathbf{N}_p \\ \mathbf{M}_p \end{bmatrix}, \qquad \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa}^0 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}^{-1} \left(\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} + \begin{bmatrix} \mathbf{N}_p \\ \mathbf{M}_p \end{bmatrix} \right) \tag{18a,b}$$

where stiffness matrix of the composite (ABD matrix) is composed of matrix in-plane stiffness matrix **A**, bending stiffness **D** and coupling stiffness **B** defined by the formulae:

$$\mathbf{A}=\sum_{k=1}^{N}\overline{\mathbf{Q}}_k(z_{k+1}-z_k)\ ,\qquad \mathbf{D}=\sum_{k=1}^{N}\overline{\mathbf{Q}}_k(z_{k+1}{}^3-z_k{}^3)\ ,\qquad \mathbf{B}=\sum_{k=1}^{N}\overline{\mathbf{Q}}_k(z_{k+1}{}^2-z_k{}^2) \tag{19}$$

while additional tractions and bending moments resulting from the prestress are equal:

$$\mathbf{N}_p=\overline{\mathbf{Q}}_k(z_{p+1}-z_p)\boldsymbol{\varepsilon}^0=\mathbf{T}^{-1}\mathbf{RQTR}^{-1}(z_{p+1}-z_p)\boldsymbol{\varepsilon}'^0 \tag{20}$$

$$\mathbf{M}_p=\tfrac{1}{2}\overline{\mathbf{Q}}_k(z_{p+1}{}^2-z_p{}^2)\boldsymbol{\varepsilon}^0=\mathbf{T}^{-1}\mathbf{RQTR}^{-1}\tfrac{1}{2}(z_{p+1}+z_p)(z_{p+1}-z_p)\boldsymbol{\varepsilon}'^0$$

Eq.18b allows to determine strains and curvatures of the midsurface of the composite in terms of in-plane forces **N** and moments **M** resulting from external loading and additional in-plane forces $\mathbf{N}_p$ and moments $\mathbf{M}_p$ resulting from prestress. In general case of prestressing ply of an arbitrary lamination angle, initial prestress causes generation of both longitudinal, transverse and shear forces, as well as bending and twisting moments. In contrast, when prestress is applied in layer with 0 or 90 degrees lamination angle, it causes exclusively longitudinal in-plane forces and uni-directional bending.

The following step of the procedure is aimed at computations of strains and stresses for particular layers. In-plane strains in global coordinate system are calculated according to kinematical hypothesis, while in-plane stresses are determined from modified constitutive relation and further transformed to local coordinate system:

$$\boldsymbol{\varepsilon}_k=\boldsymbol{\varepsilon}^M+z\boldsymbol{\kappa}^M\ ,\quad \boldsymbol{\sigma}_k=\overline{\mathbf{Q}}_k(\boldsymbol{\varepsilon}_k-\boldsymbol{\varepsilon}^0)\ ,\quad \boldsymbol{\sigma}'_k=\mathbf{T}\boldsymbol{\sigma}_k \tag{21}$$

Finally, the above formulae allow for calculation of longitudinal stress in fibres and matrix.

4 NUMERICAL SIMULATION OF PRESTRESSED COMPOSITES

Numerical, Finite Element Method - based modelling of a prestressed composites is required when applied boundary conditions or external loading causes inhomogeneous state of strain and stress of the middle surface. In such case the equation of equilibrium is no longer trivial and it requires numerical solution by finite element method. All numerical models of prestressed composites presented in this section will be implemented in ABAQUS software. Initially, two classical macro-mechanical methods of modelling non-prestressed composites, equivalent single layer method and layer-wise method, will be briefly compared with each other. These basic models will be used to propose three methodologies of FEM modelling of prestressed composites: applying additional boundary forces, applying initial thermal strains and using prestressed smeared layer of rebar.

Finite Element Method - based macro-mechanical modelling with **equivalent single layer (ESL)** method can be treated as generalization of previously presented analytical ESL method. The basic difference is that in case of FEM modelling analytical dependencies leading to derivation of the equivalent single layer hold on the level of finite element. Implementation of the model in ABAQUS utilizes the following features:

- modelling space 3D, shape: shell, type: composite,
- shell section: homogeneous, alternatively: composite layup,

- material: elastic, type: lamina (6 material constants),
- loading: shell edge load.

At the first step applied boundary conditions and loading were uniform at each boundary, which enabled comparison with the analytical solution. The numerical model had revealed good agreement with the analytical model in terms of strains and stresses in the layers, as well as in-plane and out of plane displacements for various properties of the plies and various lamination angles.

In turn, Finite Element Method - based macro-mechanical modelling with the use of **layer wise (LW)** method utilizes separate geometrical modelling of each layer of the composite. Typically, each layer is modelled by classical solid finite elements. Consequently, kinematic hypotheses of theory of plates are not assumed and degrees of freedom are defined as displacements of each node of the solid element. In proposed implementation of the method in ABAQUS software the following features were used:

- modelling space 3D, shape: solid,
- solid section: homogeneous (6 sections defined for 6 layers),
- material: elastic, type: engineering constants (9 material constants),
- loading: pressure at the boundaries.

Since the model does not take into account classical assumptions of the theory of plates, obtained results may differ from results obtained from the ESL model. In plane displacements are larger than in ESL model (up to 14%), while out of plane displacements are smaller (up to 22%) and they depend on application of finite elements with reduced integration scheme. The differences in strains and stresses result from the above discrepancies in deformations. Moreover, obtained results clearly do not comply with classical in plate theory assumption of plane state of stress ($\sigma_{13} \neq 0, \sigma_{23} \neq 0, \sigma_{33} \neq 0$).

Although described classical ESL and LW models can be directly implemented in ABAQUS software, **modelling initial pre-stress** of particular layers is not natively supported. Following the methodology elaborated for the analytical model of the composite we conclude that the easiest method of applying prestress is application of additional loading or introduction of initial thermal strains in prestressed layer.

In the method based on application of additional loading a compressive / shear forces and a bending / twisting moments resulting from prestress are applied to the structure. In case of numerical ESL model the values of these forces are exactly the same as in case of analytical model and they are defined by Eqs 20a,b. However, in numerical model the above forces can be applied at the boundaries of the considered region only in case when the entire structure is subjected to uniform state of stress (situation analogous as in analytical model). In general loading case with non-uniform state of stress, the conditions of equilibrium of particular finite elements indicate that additional forces and moments have to be applied at boundaries of each finite element.

In the method based on introduction of two-directional initial strains in prestressed layer we will utilize the feature of orthotropy of thermal properties of the material. The main concept is to replace initial strains by gradually applied thermal deformation of the prestressed layer. The parameters of thermal expansion are chosen in such a way that they cause the same deformation of the layer as initial strains (cf. Eq. 8a,b):

$$-\frac{1}{E_1}\frac{F_0}{A}=-\alpha_1 T_{\max}, \quad \frac{\nu_{12}}{E_1}\frac{F_0}{A}=-\alpha_2 T_{\max} \tag{22}$$

Consequently, the coefficients of thermal expansion are equal:

$$\alpha_1=\frac{F_0}{A}\frac{1}{E_1 T_{\max}}, \quad \alpha_2=-\frac{F_0}{A}\frac{\nu_{12}}{E_1 T_{\max}} \tag{23}$$

The entire procedure of modelling prestressed composite with the use thermal orthotropy involves the following steps:

- classical macro-mechanical modelling of all composite layers (ESL or LW method),
- definition of orthotropy of thermal properties of the prestressed layer,
- application of temperature to prestressed layer.

The proposed methodology allows for gradual increase of layer's prestress by gradual increase of applied temperature. It can be applied for both equivalent single layer and layer-wise macro-mechanical models of the composite with almost the same implementation method. Moreover, it can be applied for prestressed layers of an arbitrary lamination angle without any complications in implementation.

Another investigated possibility utilizes embedding in composite a layer of reinforcing bars ('rebar layer'). The method was primarily developed in ABAQUS for modelling reinforcement in concrete elements and it allows for introduction of initial prestress of rebar. A rebar layer is defined by micro-mechanical parameters (Young modulus and Poisson coefficient, rebar spacing, total area and orientation angle) and it is embedded in homogeneous matrix. Before finite element analysis a rebar layer and adjacent matrix are converted by FEM program into a smeared layer with modified material properties (Fig. 5).

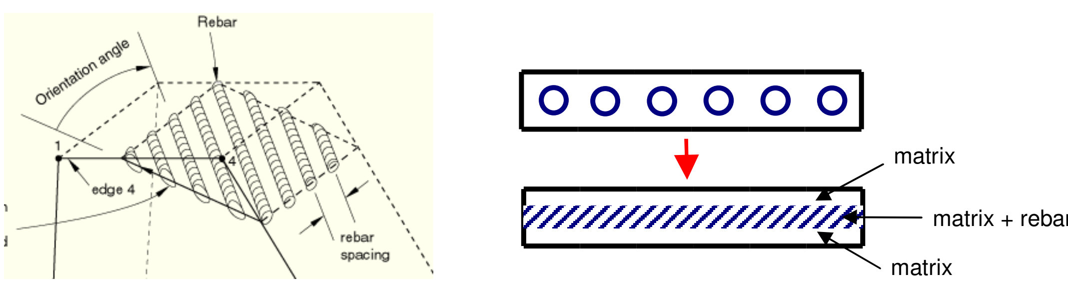


Figure 5: Application of the 'rebar layer' for creating orthotropic layers of the composite: a) general concept [10], b) transformation of non-homogeneous ply into a layered element

In basic numerical example concerning non-prestressed structure a single layer was subjected to unidirectional stretching. Since in part of the region rebar covers with matrix, Young modulus of the rebar was reduced by Young modulus of the matrix and Poisson's coefficient of the matrix was set to be equal to weighted average of coefficients for matrix and rebar. Such modifications provide agreement of results with analytical model when reinforcement is parallel to the layers edges and applied loading does not involve shear force.

Proposed implementation of the 'rebar layer' method for modelling prestressed composites involves the following steps:

- standard modelling of non-prestressed layers (ESL or LW approach),
- modelling of the matrix of prestressed layer,
- ESL model: creation of the rebar layer at appropriate height of the section,
- LW model: creation of the embedded (non-structural) surface element in prestressed layer, creation of the rebar layer in embedded element,
- introducing prestress of the rebar layer.

The entire methodology is more convenient for the ESL model due to more direct introduction of the rebar layer, which is incorporated into equivalent single layer. In turn, in LW models a layered element is created such that entire composite is composed of classical orthotropic elements modelling non-prestressed plies and layered element modelling prestressed ply.

Basic verification of above methods of introducing prestress was conducted for both ESL and LW models with prestressed layer of zero lamination angle and an arbitrary external loading. Comparison of analytical ESL model, numerical ESL model with additional external loads and numerical ESL model with thermal orthotropy had shown good conformance of results in terms of global strains and curvatures of the composite, as well as, in terms of stresses and strains in the layers. In turn, in ESL model with prestressed rebar an acceptable agreement of results was obtained in case when fibres were located in parallel to the composite edges. Regarding LW models, certain discrepancies of obtained results were caused by the same reasons as for the non-prestressed composite.

Figure 6a,b presents exemplary deformation of the composite modelled with ‘rebar layer approach’ subjected exclusively to initial prestress of the fibres located at the bottom and at the top of the structure. In both cases intuitive deformation of the composite was obtained. Moreover, similar deformation was achieved with the use micromechanical model (Fig. 6c,d) where the matrix and the fibres were modelled and meshed as distinct geometrical regions.

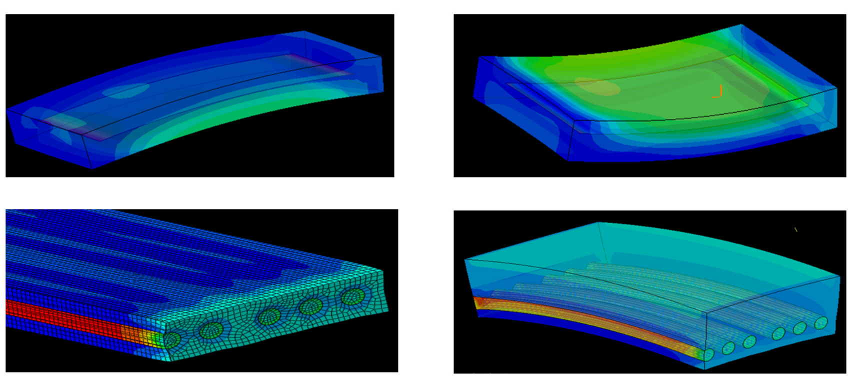


Figure 6: Deformation of the prestressed composite: a, b) 'rebar layer' method: pre-tension of bottom and top fibres; c,d) micro-mechanical models: detailed view and deformation caused by prestress.

5 OPTIMIZATION OF APPLIED PRESTRESS

Presented above analytical and numerical models can be effectively used to optimize initial prestress of the composite in order to improve its global mechanical characteristics or to obtain desired structural response. Herein, optimization of prestress will be aimed at minimization of strain/stress components in particular layers, which can be treated as initial step before and maximization of the composite load capacity. In the following examples the simplest of developed models, i.e. analytical ESL model, will be applied. Initially, optimization problems will be formulated and the dependence of the objective functions on prestress parameters will be examined. Further, methods of solving particular problems will be proposed and implemented.

The first considered optimization problem was minimization of the selected strain or stress component in terms of prestress parameters. Three different objective functions were defined:

- longitudinal deformation of the middle surface of the plate: $\left|\varepsilon_{xx}^{M}\right|$,
- maximal longitudinal deformation of the layer: $\max\left(\left|\varepsilon_{11}\right|\right)$,
- maximal longitudinal stress in the layer: $\max\left(\left|\sigma_{11}\right|\right)$.

In each case the optimization variables were: the number of the layer which is prestressed n_p and value of the prestressing force N_p. Consequently, the optimization problem assumes the form:

$$\text{Find } \left\{n_p, N_p\right\} \text{ such that } \left|\varepsilon_{xx}^{M}\right|,\ \max\left(\left|\varepsilon_{11}\right|\right),\ \max\left(\left|\sigma_{11}\right|\right) \text{ is minimal} \tag{24}$$

Although analyzed composite was composed of plies of various lamination angles, in solution of the optimization problem prestressed layer was rotated such that fibres were parallel to the edges and prestress caused exclusively compressive force and bending moment. Such operation causes change of the stiffness matrix of the layer and change of global stiffness matrix of the composite, which makes the optimization procedure more complicated. Moreover, the number of the prestressed layer is a discrete variable, while the value of prestressing force is a continuous one, which causes that optimization problem can be considered as semi-discrete.

Assuming a fixed number of the prestressed layer, we can analyze the influence of the prestressing force value on the value of considered objective function. Longitudinal strain in the middle layer of the composite subjected to external loading and prestressing force can be expressed as:

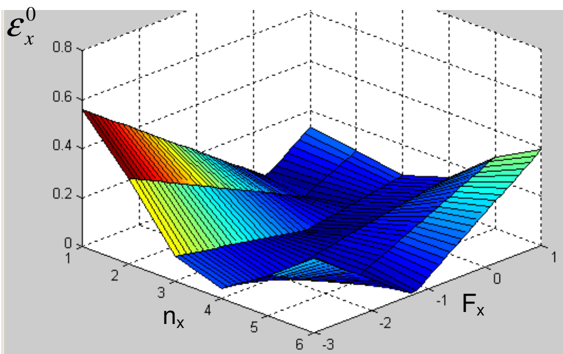
$$\begin{aligned} \varepsilon_{xx}^{M} = {} & S_{11}N_x + S_{12}N_y + S_{13}N_{xy} + S_{14}M_x + S_{15}M_y + S_{16}M_{xy} + \\ & S_{11}N_x^p + S_{12}N_y^p + S_{13}N_{xy}^p + S_{14}M_x^p + S_{15}M_y^p + S_{16}M_{xy}^p \end{aligned} \tag{25}$$

where initial six terms result from external loading and remain constant while the next six terms result from prestress and each of them depends linearly on the value of prestressing force. Thus, longitudinal strain is linearly dependent on the value of prestressing force and it reaches zero providing that adequately large range of force is considered. Consequently, prestress of arbitrarily chosen layer allows to minimize the objective function to zero. Since definition of the objective function contains absolute value, for each selected prestressed layer the objective function is bilinear in terms of prestressing force, Fig.7a.

In case of the second objective function, deformation of the layer in global coordinate system $\boldsymbol{\varepsilon}$ and local coordinate system $\boldsymbol{\varepsilon}'$ can be expressed as:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^M + z\boldsymbol{\kappa}^M \quad \text{and} \quad \boldsymbol{\varepsilon}' = \mathbf{RTR}^{-1}\boldsymbol{\varepsilon} \tag{26}$$

Each component of the midsurface strain $\boldsymbol{\varepsilon}^M$ and curvature $\boldsymbol{\kappa}^M$ depends linearly on the value of prestressing force and the same holds for layer's strains in global coordinate system $\boldsymbol{\varepsilon}$ and their transformation into local coordinate system $\boldsymbol{\varepsilon}'$. Therefore, response to a given prestressing force can be computed by scaling response to prestressing force of a unit value. Consequently, total longitudinal strain in selected layer can be minimized to zero providing that adequately wide range of force is considered. However, since the objective function is formulated as maximal value of longitudinal strain, considered optimization problem assumes the form of min-max problem and strains in particular layers can not be considered separately. This causes that for each selected prestressed layer the objective function is multi-linear in terms of prestressing force and its minimal value can not be reduced to zero, Fig. 9b.



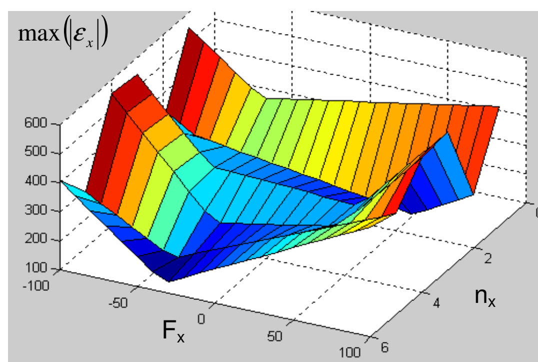


Figure 7: Shape of the objective functions for exemplary 6-ply composite: a) longitudinal strain in the middle surface, b) maximal longitudinal strain in the layers.

Similar situation holds for the third considered optimization problem where the maximal longitudinal stress in the layers σ_{11} is minimized. Stresses tensor in global coordinate system $\boldsymbol{\sigma}$ and its transformation to local coordinate system $\boldsymbol{\sigma}'$ can be expressed as:

$$\boldsymbol{\sigma} = \overline{\mathbf{Q}}\boldsymbol{\varepsilon} \quad \text{where} \quad \overline{\mathbf{Q}} = \mathbf{T}^{-1}\mathbf{RQTR}^{-1}, \tag{27}$$

$$\boldsymbol{\sigma}' = \mathbf{T}\boldsymbol{\sigma} \quad \text{or} \quad \boldsymbol{\sigma}' = \mathbf{Q}\boldsymbol{\varepsilon}'$$

The above formulae indicate that both global and local longitudinal stresses are linear in terms of prestressing force and considered objective function and optimization problem are similar as in case when maximal longitudinal strain was minimized. Moreover, considering longitudinal stress in reinforcement as the objective function requires only multiplication of the layer stress by a constant and adding terms indicating prestress so it does not significantly affect the type of formulated optimization problem.

Due to the fact that applied optimization software does not support problems of considered semi-discrete type, the optimization with respect to prestressing force was conducted separately for each layer and further minimal value of the objective function was selected. Solution of the optimization problem with the second and third objective function is expected to be obtained either at the boundary of the feasible domain or at points where the number of the layer with maximal value of the objective function changes (vertices in plot 7b). Since development of the effective procedure for finding these points for the case of large number of layers is challenging, a minimal value was found in direct way by systematic search of admissible range of prestressing force.

The second optimization problem concerns minimization of exactly the same quantities (longitudinal strain of the middle surface, maximal longitudinal strain and stress in the layer) conducted with respect to two prestressing forces applied in different layers of the composite. Consequently, the optimization problem takes the form:

$$\text{Find } \{N_p^1, N_p^2\} \text{ such that } \left|\varepsilon_{xx}^M\right|,\ \max\left(\left|\varepsilon_{11}\right|\right),\ \max\left(\left|\sigma_{11}\right|\right) \text{ is minimal} \tag{28}$$

In order to simplify the problem, a six-ply composite was considered and the prestressing force was always applied at second and fourth layer, where the fibres are located in parallel to the edges of the composite, perpendicularly to each other ($\alpha_2 = 0^0, \alpha_4 = 90^0$).

The current optimization problem is substantially simpler than the previous one since the objective function depends exclusively on prestressing forces, which change the vector of external loading, but does not affect the stiffness matrix of the composite. Recalling that the dependence on prestressing force is linear, relatively simple shapes of the objective functions are expected. Moreover, both optimization variables are continuous and the problem can be approached with classical optimization solver.

The dependence of the first considered objective function on the values two prestressing forces takes the form:

$$\begin{aligned}\varepsilon_{xx}^M = {} & S_{11}N_x + S_{12}N_y + S_{13}N_{xy} + S_{14}M_x + S_{15}M_y + S_{16}M_{xy} + \\ & S_{11}N_x^{p2} + S_{14}M_x^{p2} + S_{12}N_y^{p4} + S_{15}M_y^{p4}\end{aligned} \tag{29}$$

Initial six terms result from external loading and they always remain constant. In turn, prestressing force in the second layer causes longitudinal stress and bending in longitudinal direction, while prestressing force in fourth layer causes transversal stress and bending in transversal direction. The objective function is linear in terms of both prestressing forces. Consequently, the shape of the surface representing objective function is bilinear and it is composed of two or half-planes joining in zero (Fig. 8a). Thus, longitudinal strain of the middle surface can be reduced to zero for infinitely many combinations of prestressing forces. Providing that adequately large range of force is considered, for each value of longitudinal prestressing force the transversal force can be adjusted in a way that ε_{xx}^M vanishes.

For the following optimization problems the methodology of calculating the subsequent objective functions is identical as in previous case (cf. Eq. 26-27). Therefore, the shape of the objective functions defined as maximum over the layers is analogous as for the case of single prestressing force in Fig.7b, however it is extended into two dimensions. Both in case of local strains and stresses the objective function is composed of several intersecting planes of a various inclination angles, Fig. 8b. Definition of the objective function as a maximum over the layers causes that minimized quantity can not be reduced to zero.

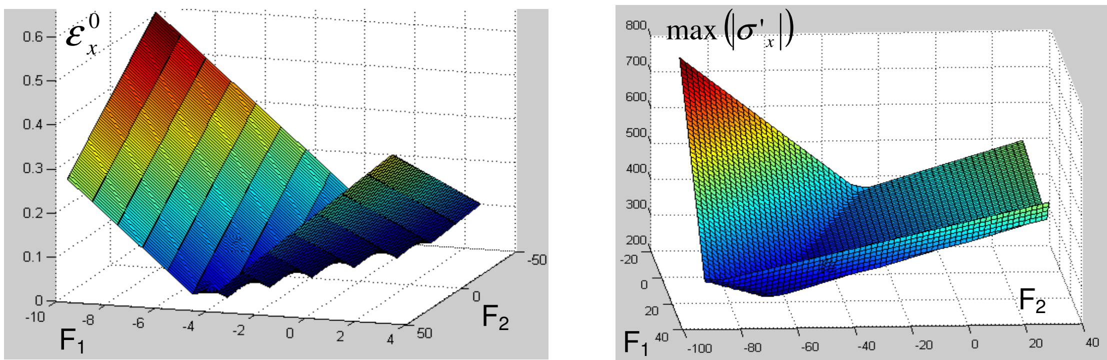


Figure 8: Shape of the objective functions for exemplary 6-ply composite: a) longitudinal strain in the middle surface, b) maximal longitudinal stress in the layers.

Since the problem is entirely continuous it can be successfully solved with the use of classical optimization software. In considered case MATLAB optimization toolbox was utilized and the optimization problem was solved both with the use of gradient-based method and pattern search method. In gradient-based optimization the 'interior point' method was utilized as one of the most efficient optimization algorithms. In turn, in pattern search method a 'complete search' options and 'latin hypercubes' were utilized. In the first optimization problem, a specific shape of the objective function causes that optimization results depend on starting point of the optimization procedure. In turn, in the second problems gradient-based method was faster than pattern-search method and it has always lead to correct results, i.e. identical as the ones obtained from direct search of the feasible domain.

6 EXPERIMENTAL EVALUATION OF THE PRESTRESSSING CONCEPT

For the purpose of an experimental evaluation of the concept, a dedicated laboratory stand (Fig. 9a) was created. The main part of the stand was plane table, composed of two blocks: one was fixed to a base and the other was movable. The load screw was mounted in the moving block in such a way that the screw end was blocked in the fixed part of the stand and movable part was shifted during rotation of the screw. For alignment of the moving and fixed blocks two guide bars were used. The pre-stressed woven fabric was mounted by the clamps located at opposite ends of the table. Load cell, enabling controlled introduction of tensile force and monitoring of the stress state of the tensioned layer, was mounted in the fixed part of the table, so it was loaded by the screw. The force acting on the sensor was a reaction force due to tensioning of the layer. The idea of such stand and methodology of preparation the pre-stressed composite with the use of prepreg laminate was described in detail by Krishnamurthy in [3].

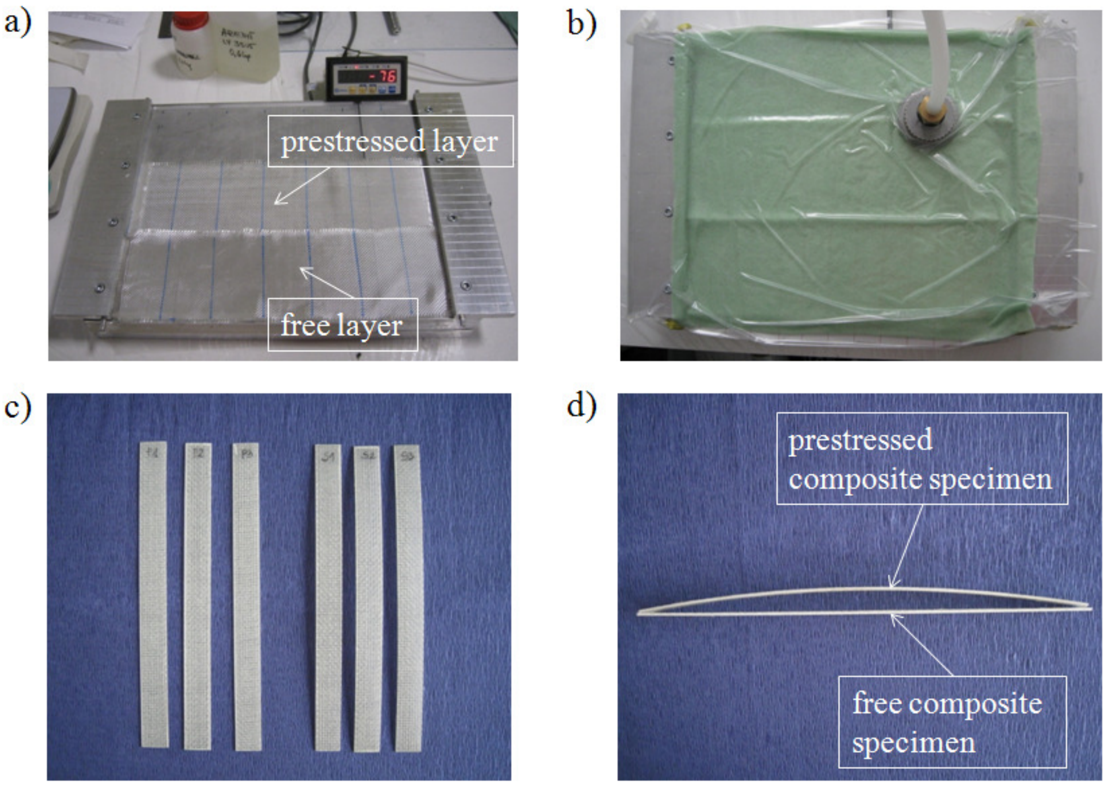


Figure 9. Preparation of a laminate reinforcement (a), experimental stand during composite curing (b), specimens after cutting (c), visual comparison between traditional and prestressed composite specimen (d).

The specimens were composed of:

- one layer of bidirectional glass fabric with basis weight 163g/m^2 (mass fraction: 8 %),
- three layers of bidirectional glass fabric with basis weight 280g/m^2 (mass fraction: 40 %),
- epoxy resin ARALDITE LY3505 with hardener XB 3403 (mass fraction: 52 %).

Only the first layer was tensioned by external force. Because this layer was bottom layer, one can observe characteristic bend of specimen made from pre-stressed composite (Fig. 9d). The level of the pre-stressing force was predetermined before specimen preparation by testing of fiber breaking force level. The external pre-stressing force level was assumed to be not higher than 1/3 breaking force and in the case of presented experiment it was set to 765 N. It was observed that after application of this force to the pre-tensioned layer of glass fabric, within 30 minutes value of pre-stressing force decreased to 706 N and stabilized. The composite was cured in room temperature by use of negative pressure during first hour of processing. The pre-stressing force was removed after five days.

The traditionally prepared composite element (without pre-stress) was produced simultaneously for the sake of comparison. Both elements were composed of the same materials and cured in identical conditions as presented on Fig. 9a,b. During production of specimens, the first layer of fabric for pre-stressed composite was placed at central part of the table and fabric for traditional specimen was placed next to it but it was not fixed in clamps.

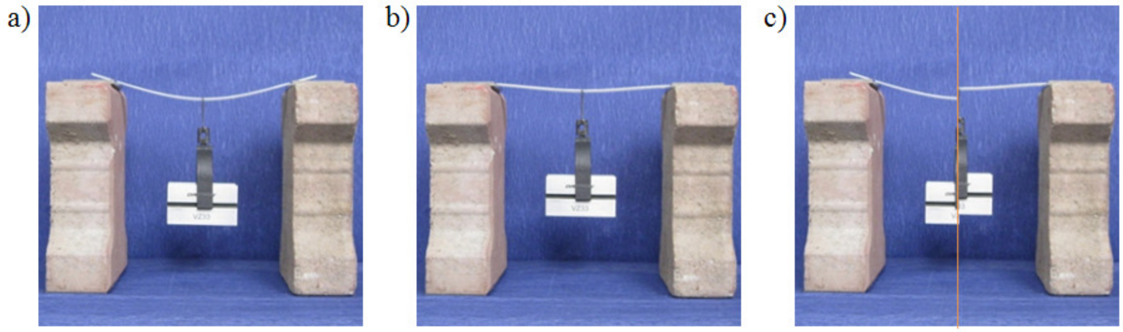


Figure 10. Composite static response: traditional specimen (a), pre-stressed specimen (b), comparison of responses for traditional composite (left part) and pre-stressed composite (right part) (d).

The other layers were added by wet lay-up technique and entire table was packed in the vacuum bag. After resin setting, each part of material (traditionally cured and pre-stressed) were cut into three beam specimens presented in Fig. 9c. Comparison of composite static response for pre-stressed composite material and for traditionally made specimen is presented in Fig.10. Specimens were loaded transversally with weight of 0.88 kg. Maximum deflection of specimen made from pre-stressed material, measured regarding its original shape was about 25% smaller than deflection of specimen produced with traditional method.

7 APPLICATIONS OF PRESTRESSED COMPOSITES

Prestressed composites have several promising applications in civil and mechanical engineering. Two applications are related to improvement of composite structures subjected to small deformations, while the third one concerns prestress-induced snap-through effect.

In the first concept, the usage of prestressed composites to construction of lightweight deck of the pedestrian bridge allows for reduction of stresses generated by self weight and human traffic, which leads to decrease of mass and overall cost of the bridge. In second application, prestressing of the composites enables reduction of stresses in cylindrical shell structures loaded by internal pressure (such as pressure tank or barrel of a canon). The last proposed application goes beyond previous linear models and concerns the case of finite deformations. As it will shown, prestressing of curved fibres of shells shaped as hyperbolic paraboloids allows to obtain controlled snap-through effect, which can be used for instance for design of fast pneumatic valves. The corresponding numerical examples will be skipped here for the sake of brevity, however they will be shown at the conference.

8 CONCLUSIONS

The concept of smart prestressed composites is based on prestressing selected layers of the laminated composite in order to improve its mechanical properties. Appropriate adjustment of applied prestress provides adaptation to actual loading conditions and allows to enhance global mechanical response of the composite.

Presented in the paper detailed and systematic theoretical approach to modelling prestressed composites involves the following steps: i) micromechanical modelling of the prestressed layers of the composite, ii) homogenization of the prestressed layers which leads to macromechanical model with additional initial strains modelling prestress, iii) elaboration of the analytical ESL model of the prestressed composite, which involves additional external forces, iv) proposition of various macro-mechanical FEM-based models of prestressed composites of various levels of complexity. Moreover, combination of the above models and proposed optimization methods aimed at minimization of selected strain or stress components constitutes a unique tool for design of the prestressed composites.

Described in the final part of the paper experimental evaluation of the concept shows that the manufacturing of the prestressed composites is feasible and it can successfully performed in laboratory conditions. Obtained sample of the prestressed composite shows improved mechanical properties in comparison to the classical composite, which ultimately positively verifies the concept of prestressing.

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