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Surface Topography and Contact Stiffness After Laser-Mechanical Treatment

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Summary: The paper presents results of the new method applied for modification of surface layer of steel. The combining of the laser alloying with the burnishing process, both performed at the laser set up, was proposed to reduce surface roughness being formed during the laser treatment and improved contact stiffness. The experiments were done on stainless steel alloyed with Stellite 6 and simultaneously burnished in hot and cold condition. The alloying process was performed with continuous laser CO_2, with power of 2.5 kW, at different parameters. The influence of treatment parameters on surface topography and contact stiffness was examined.

1. INTRODUCTION

Laser Beam Machining has been successfully applied for surface layer properties improvement of machine and device elements. Surface treatment based on melting of surface with a laser beam such as cladding alloying and hardening, leads to changes of surface roughness and stresses in surface layer [1, 2].

The surface roughness formed in the laser alloying process are too high to apply elements without an additional abrasive machining, even at the optimal parameters of the laser treatment [3,4,5]. Therefore, a combining of the alloying process with the burnishing process, both performed simultaneously at the laser stage, was proposed in the work [6]. The aim of the hybrid treatment was to reduce surface roughness formed in the laser process and induce compressive stresses. A smoothing process of surface was carried out by plastic deformations of surface layer in high temperature, while reduction of the tensile stresses in the surface layer resulted from the cold work.

A detailed description of the hybrid processing and the results of preliminary tests for case of melted hardening have been presented in the work [6]. The current work presents the results of studies of the surface layer after laser alloying combined with dynamic burnishing. The experiments were conducted on stainless steel alloyed with Stellite 6.

The aim of this study was to assess the influence of hybrid processing parameters on the surface topography and the analysis of the impact of this treatment on contact stiffness, which is one of the basic functional properties of machine parts.

Contact stiffness depends on many properties of the surface layer, therefore is difficult to determine theoretically, and the results of calculations are vitiated by substantial errors. The contact stiffness is significantly influenced by: the height and shape of summits of roughness and the distribution of surface bearing area ratio, the value and distribution of micro-hardness of surface layer (SL), state and stress distribution of SL, plastic characteristic of SL, and several other factors [7]. All these features of surface layer undergo significant changes during the laser-mechanical machining, thus considerable differences in contact stiffness are also expected.

2. THE EXPERIMENTAL DESCRIPTION

2.1. The hybrid treatment

The surface burnishing treatment was realized by the special head for dynamic burnishing assembled to the laser station. The full description was presented in the work [6]. In present study the modified head with two rows of hammers, which allowed for the burnishing process in one pass at different temperatures: hot and cold, was applied. In addition, the oscillating motion was introduced in a direction perpendicular to the feed of sample at 15 osc / min. The aim was to achieve a more uniform surface cold work.

The experiments were performed with CO_2 laser with maximal power of 2.5 kW, which generates the axis-symmetrical beam in mode close to TEM_{10}. The beam was focused with the ZnSe lens that has focal length of 5". The system parameters such as the power of the laser, the diameter of the beam focused on metal surface, the kind of shielding gas and velocity of its blow, were selected to assure sufficient power density to reach melting and obtain the optimum effects. The surface of sample was covered by colloidal graphite before the treatment to raise absorption of laser beam. The hybrid process was performed on samples made of 304 steel, with a Stellite 6 layer, 0.02 mm thick, deposited prior to treatment. The surface of samples was melted at different velocities of sample motion in reference to the beam - feed rate (V_f). The burnishing process was performed simultaneously with alloying treatment. The force of burnishing element on treated surface was dependent on rotation speed of the head. The distance between burnishing element and laser beam axis was changed from 5 to 10 mm that allowed performing burnishing treatment at different temperatures. The hybrid process was conducted as single and multipath treatment.

2.2. The study of surface topography

The aim of the study was to define the impact of parameters of laser-mechanical machining on the surface topography. Studies of the surface layer state are very laborious. Therefore, the determination of the effect of processing conditions on the state surface layer was performed based on the theory of planned experiment which allows minimizing the number of experiments [8].

In the study the planned experiment, based on determined static program with multi factors and rotatibility with repetition PS/DS-λ, was carried out. Choice of input values and the ranges of their variation were based on the results of preliminary tests of laser-mechanical process assuming an additional criterion for their usefulness to control the hybrid treatment.

As input variables, measurable and controllable, that characterize the mathematical model of the object of research, the following quantities were assumed:

- Rotational speed of the burnishing head - V_o

- Feed speed of the sample - V_f
- Distance between the burnishing head and the axis of laser beam - X.

As output factors, that characterize the state of surface layer and the results of hybrid treatment, the following parameters were considered:

- 3D parameters for unfilted surface describing height features of topography: S_a, S_z, developed surface S_{dr}, surface bearing curve $S_{dc\ 20\text{-}80\%}$,
- roughness parameters of filtered profile: R_a, R_t,$R\ S_m$.

Based on preliminary studies, the areas of variations of treatment parameters and the ranges of variations of input data were determined. Denotations and ranges of input data variations, regarding to the experiment of hybrid treatment of steel alloyed with Stellite 6, contains Table 1. Five-level research program and the normalization interval [- α, α], α = 2, corresponding to the star arms of the PS/DS- λ plan, were considered.

Table 1. The values of input variables in the experiment of hybrid treatment of steel OH18N9 alloyed with Stellite 6

i	Input data	-2	-1	0	1	2
1	V_o [rev /min]	3500	4200	5000	5950	7100
2	V_f [mm/min]	150	235	367	574	900
3	X [mm]	5	6	7	8,5	10

After hybrid treatment the surface roughness was examined. Measurement of surface topography was conducted on a scanning profilometer after the alloying and hybrid process performed at different parameters. Surface roughness measurements were performed for each track of laser alloying, and the laser alloying with oscillations combined with micro-hammering. The 3D roughness measurements were conducted in the central area of melting path. Surface topography parameter values were determined for the field with dimensions 1.4 mm x 4 mm. The measurements were performed at dx=0.5 μm, dy =5, μm with the stylus radius of 2 μm. Profile measurements were carried out on the measuring section equal to 4 mm, parallel to the feed rate direction, in the middle of the zone of melting.

2.3. Study of contact deformations

The aim of the study was to define the impact strain hardening of the burnishing treatment with on the process of deformation of surface layer under the influence of normal stress. After the hybrid treatment, carried out with the following parameters: V_f = 360 mm / min, V_o = 5000 rev/min, X = 7 mm (the central point of the planned experiment), the samples were selected for testing. As the reference surfaces, the sample with a layer of Stellite 6 deposited by plasma spraying, were selected. The study was conducted at the nominal stress to ensure no plastic deformation of the substrate - below 260 MPa. In order to determine the influence of the burnishing process on the course of surface layer deformation, the tests were carried out for samples after two different variants of the alloying process: multipath alloying and the alloying with single path of the laser beam, associated with the oscillations.

The study was conducted on surfaces, in which the surface topography was formed in the laser alloying and the hybrid treatment. The other surfaces in the experiment were finished by the grinding. That allowed eliminating the impact of large differences in surface topography on contact stiffness.

The process of deformation of the samples was carried on the stand using a measuring method of the surface approach proposed by Demkin [9]. The stand enables accurate measurement of the approach *a* as a function of applied nominal pressure *q*. Contact is realized between flat and rough surface of the sample, and smooth and rigid surface of the counter-sample, made of sintered carbide. The counter-samples have three punches, diameter of 5 mm and nominal area of 58.875 mm^2 each. Material of the sample undergoes deformation under the punches. Both samples are placed in a specially constructed device, which is mounted inside the laboratory precision hydraulic press that allows applying the normal pressure up to 1000 MPa. The applied pressure are measured using a compression proving ring, while the approach of samples is measured using the inductive sensor. The results of measurements were recorded in the form of the approach value *a* [um] for the given loading *F* [N].

The study was carried out using the loading from 0 N to 1000 N, which corresponds to nominal pressure 170 MPa, then the unloading to 0 N. Afterwards, the re-loading up to1500N, corresponding the nominal pressure 255 MPa, and the unloading down to 0 N, was applied. The testing enabled to determine the values of total deformation, denoted a_I and $a_{2,}$ and plastic deformation, a_p as well as the elastic one, a_e. It also allowed determining the curves of approach - nominal pressure relation. The values of deformation are the averaged values from three areas of measurement.

Due to the large differences in the geometric structure of surface, which has a significant influence on the contact stiffness, the surfaces of the samples were subjected to grinding to form similar surface topography. Application of the grinding allowed to eliminate the influence of differences in surface roughness on contact stiffness due to the various processes applied previously. It allowed for the analysis of properties of the surface layer, in this case the burnished layer, on the process of contact deformation of SL. After grinding the surface roughness measurement was repeated. The ground samples were re-denoted by adding the letter "g". Table 2 shows the notation of samples, the way of processing and the roughness parameters of filtered profiles.

Table 2. The notation of samples and the results of roughness measurements of surfaces

Sample	**Treatment**	**Surface state**	**Roughness parameters**		
			Ra [μm]	Rt [μm]	RSm [μm]
SMN24	Multipath alloing	After initial treatment	110	421	2017
SMN26	Multipath alloying and burnishing	After initial treatment	55	267	1770
SMN24sz	Multipath alloing	After grinding	2,7	20,3	147
SMN26sz	Multipath alloying and burnishing	After grinding	2,57	21	126
SMN21	Alloing single path	After initial treatment	4,02	34,7	160
SMN23	Alloing and burnishing single path	After initial treatment	2,26	21,1	230
SMN21sz	Alloing single path	After grinding	1,67	12,3	95
SMN23sz	Alloing and burnishing single path	After grinding	1,67	10,2	88
SMN27	Plasma spray layer	After initial treatment	5,47	38,3	116
SMN27sz	Plasma spray layer	After grinding	3.87	28,7	155

3. The results of experiment

3.1. Surface topography after hybrid process

Table 3 shows the selected parameters of 3D and 2D roughness of the surface which underwent only laser alloying while Table 4 presents the parameters due to hybrid treatment. In the case of profile parameters cut-off 0.8 was used for all measured surfaces. Roughness parameters are the average values of 16 measured profiles. In Figure 1a the examples of profiles of surface roughness and the view of the laser alloyed surface are shown. Figure 1b presents geometrical features of hybrid treated surface.

A change in shape of melted zone due to burnishing is noticeable. The heights of asperities at the boundary of the alloying zone decreased significantly, while the shape of asperities in the central area of the melting underwent "flattening" in comparison with only alloyed material.

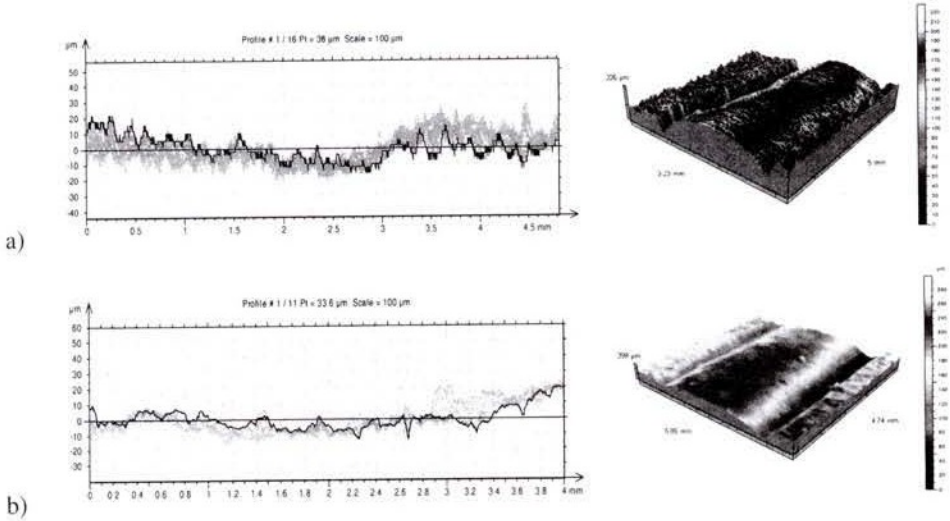

Fig. 1. The profiles and surface topography views: a – alloyed with the laser beam, b – after hybrid treatment

Table 3. Parameters of surface topography after laser alloying of steel with Stellite 6

Process parametrers			3D roughness parameters				2D roughness parameters		
P [kW]	V_f [mm/min]	d [mm]	S_a [µm]	S_z [µm]	S_{dr} [%]	S_{dc} [µm]	R_a [µm]	R_p [µm]	RSm [mm]
2	150	3	33,1	133	32,8	76	6,16	22,9	0,11
2	230	3	22,2	112	31,3	48	7,19	22,3	0,177
2	360	3	23,4	121	31,3	51	7,2	24,1	0,145
2	570	3	25,9	130	32,3	58	7,29	26,9	0,141
2	900	3	35,3	152	33,1	80	9,27	35,8	0,191

Table 4. Parameters of surface topography after the hybrid treatment

Proces parametrers			3D roughness parameters				2D roughness parameters		
V_o [rev/min]	V_f [mm/min]	X [mm]	S_a [μm]	S_z [μm]	S_{dr} [%]	$S_{dc\ 20\text{-}80\%}$ [μm]	R_a [μm]	R_p [μm]	RSm [mm]
4200	230	6	8,16	60,6	12,3	18	2,68	11,9	0,195
5950	230	6	11,1	87,6	16,8	24	4,83	15,3	0,201
4200	570	6	8,64	81,2	13,7	17	4,36	10,7	0,203
5950	570	6	10,1	77,6	15,2	21	3,95	13,9	0,19
4200	230	8,5	8,67	61,6	10,8	19	1,62	5,52	0,251
5950	230	8,5	11,4	73	11,2	24	1,79	6,4	0,242
4200	570	8,5	7,61	57,6	12,2	16	1,67	6,47	0,226
5950	570	8,5	8,4	52,8	11,5	19	1,87	6,06	0,282
3500	360	7	6,16	44,6	11,8	13	1,71	5,82	0,231
7100	360	7	9,01	66,4	13,6	19	2,9	8,73	0,25
5000	150	7	13,3	82,4	14,6	28	3,58	11,7	0,238
5000	900	7	9,93	80,2	17	22	4,44	12	0,207
5000	360	5	10,6	86,4	16,2	23	4,3	14,7	0,213
5000	360	10	7,96	51,2	10,1	17	0,87	3,88	0,28
5000	360	7	9,44	67,8	14,6	20	2,22	7,44	0,221
5000	360	7	12,1	68,6	13,2	26	2,44	8,25	0,219
5000	360	7	14,8	95	13,6	21	2,93	8,54	0,235
5000	360	7	5,97	45,6	10,9	12	2,03	6,65	0,234
5000	360	7	5,81	49,6	19,7	12	1,69	5,25	0,212
5000	360	7	7,84	57,6	12,4	16	2,24	7,73	0,245

Statistical analysis of results of the experimental studies included the choice of regression function, a statistical verification of the adequacy of the approximation function and statistical verification of the significance of coefficients of the approximation function. The approximation tests were performed using the power function and a polynomial of the first degree. Assessment of the correlation and the significance of dependences were carried out on the basis of the I. P. Guilford criteria. The results are provided in Table 5, which include: the approximation function, the value of multiple correlation coefficient *R*, the value of the Fisher's number *F*, *T*-student ratios describing the significance of subsequent independent variables *T1*, *T2*, *T3*. The confidence level $\alpha = 0.1$ was adopted. Credibility of the equations was assessed based on the following criteria:

- critical value of the *F*-statistic, which is $F_{kr} = 2.71$t for the determined equations,
- critical value of the *T*-student coefficients, which is $T_{kr} = 1.41$ for $\alpha = 0.1$ for the determined equations.

Table 5. The regression functions for surface topography parameters: R_a, RS_m, S_{dr}

Topography parameter	Correlation function	R	F	6	T1	T2	T3	T4
R_a	$6+0.00037V_o+0.0017V_f-0.76X$	0.83	12.1	0.62	4.1	1.54	1.82	5.54
RS_m	$0.085-0.000004V_o+0.00002V_f+0.017X$	0.82	11.3	1.86	2.6	1.31	1.1	5.55
S_{dr}	$17.46+0.00061V_o+0.0034V_f-1.22X$	0.79	8.9	1.21	6.17	1.61	1.89	4.53

Multiple correlation coefficients describing the dependence of R_a on the parameters of the hybrid treatment are high, and the dependence between studied properties is significant $F > F_{kr}$. All coefficients are significant at the accepted level of confidence. Figure 6 shows a graphical interpretation of the dependence by a polynomial function that describe the relation between *Ra* and the rotational speed of the burnishing head and the distance between the tool axle and the axis of the laser beam for a fixed feed rate V_f = 360 mm/s. The parameter R_a increases with increasing speed V_o, V_f and it decreases with increasing distance X from the axle of burnishing head to the laser beam axis.

With the increase rotational speed of head V_o the intensity of the burnishing process, and the forces of impact of micro-hammers on machined surface, grow. It causes an increase in surface smoothness. The increase in distance between the axle of burnishing tool and the laser beam caused decreases temperature in the zone of mechanical machining. This is also associated with a decrease of plastic properties of material, smaller plastic deformations of surface asperities and increase of the height parameter R_a.

When analyzing the influence of process parameters on roughness sampling *RSm a* good correlation was also found. Polynomial function, for which all the coefficients of the equation are significant at a confidence level of 0.1, show good fit. With the increase in the rotational speed of head and in the feed rate, the roughness spacing increases, while the increasing distance between the head and the laser beam causes a decrease of RS_m. Analysis of the equation shows that the temperature rise of the burnishing process and the higher impact of the tool results in surface geometrical structure with increased micro-asperity spatial property. This is also confirmed by the results of analysis of relation between the developed interfacial area ratio S_{dr} and the process parameters. Small value of parameter S_{dr} is obtained for treatment at low temperatures (high values of feed rate and tool distance X), and low rotational speed of the head. The parameter $S_{dc\ 20\text{-}80\%}$ characterized surface bearing curve also show good correlation with treatment parameters.

The influence of rotational speed of the head and distance between the tool axle and the laser beam axis on surface topography parameters R_a and S_{dr} for constant value of feed speed V_f=360 mm/min are shown on Figure 2.

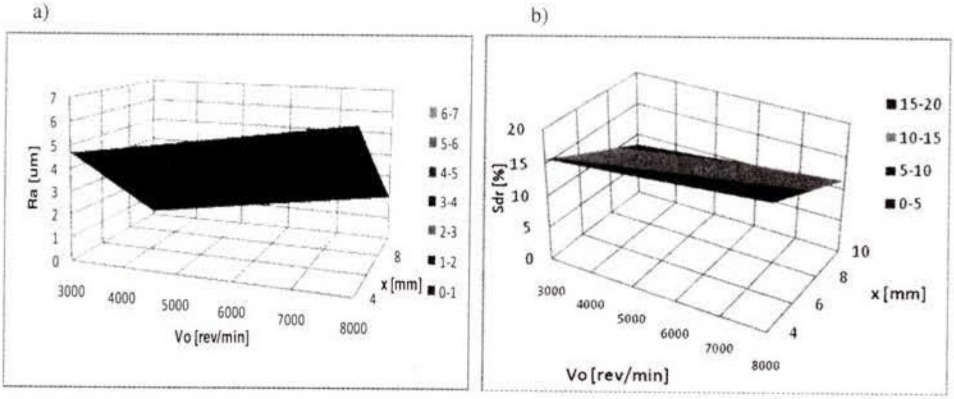

Fig. 2. The influence of rotational speed of the head and distance between the tool axle and the laser beam axis on surface roughness parameters: a - R_a, b - S_{dr}

3.2. The results of contact stiffness

The results of measurements of all samples are presented in the form of graphs of the approach a [μm] as a function of nominal pressure q [MPa]. In addition, the table shows the numerical values of the total deformation (a_1, a_2), elastic (a_{1e}, a_{2e}) and plastic deformation ($a1_{pl}$, $a2_{pl}$) for the maximum pressure in the experiment: 155 MPa and 270 MPa. The data allowed the determination of contact stiffness for the tested surfaces.

The value of the contact stiffness was calculated on the basis of the formula:

$$j = F / A_n y \quad [\mathrm{N/mm^2 \mu m}]$$

Where: F- normal force, A_n - nominal area, y - contact deformations (contact surface displacement due to deformation of surface asperities), j - contact stiffness - the pressure that cause local deformation of the contact area of 1 μm.

The difference between the values of maximal approach for the pressure 270 MPa and 155 MPa was calculated. This allowed for the determination of incremental contact stiffness in range of 155-270 [MPa] according to the formula: $j=\Delta F/A_n\,\Delta y$ [N/mm^{2}μm].

The results of contact strain and contact stiffness are shown in Table 6 and on Figure 3, 4.

Table 6. The values of strain for the applied nominal pressure and the contact stiffness for nominal pressure 155 MPa and 270 MPa

Sample No.	Contact strain at nominal stress 155 MPa			Contact strain at nominal stress 270 MPa			Contact stiffness		
	a_1 [μm]	a_{1e} [μm]	a_{1pl} [μm]	a_2 [μm]	a_{2e} [μm]	a_{2pl} [μm]	q=155 MPa	q=270 MPa	q=155-270 MPa
smn24	202	22	180	234	20	212	0,77	1,15	3,8
smn26	68	24	44	90	31	59	2,28	3,00	5,2
smn24sz	29	18	10	39	29	14	5,34	6,92	11,5
smn26sz	20	18	2	25	22	3	7,75	10,80	23
smn21	88	40	48	138	90	88	2,77	3,38	4,8
smn23	56	20	36	80	28	52	1,76	1,96	2,3
smn21sz	24	4	8	40	28	12	6,46	6,75	7,2
smn23sz	14	9	5	24	14	10	11,07	11,25	11,5
smn27	47	23	22	57	36	27	3,30	4,74	6,8
smn27sz	31	24	7	40	28	12	5,00	7,11	12,8

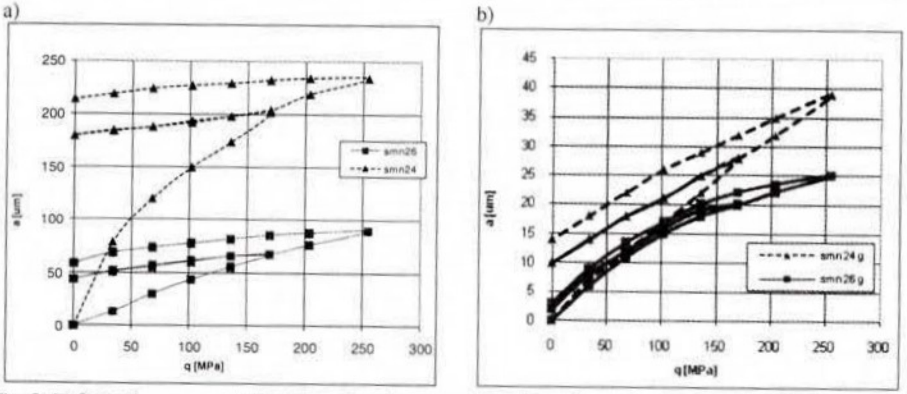

Fig. 3. Deformation process in function of nominal pressure of the alloyed surface (sample SMN24) and the alloyed and simultaneously burnished surface (sample SMN26);
a – surfaces after laser and hybrid treatment, b – surfaces after grinding

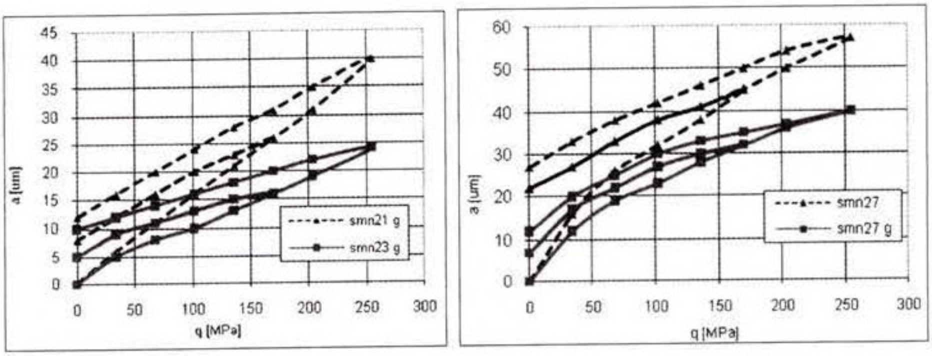

Fig. 4. Deformation process in function of nominal pressure after: a-laser alloing and hybrid treatment for the ground surface; b-plasma spraying (SMN27) the ground surface (SMN27g)

4. DISCUSSION AND CONCLUSIONS

Studies of the influence of hybrid treatment parameters showed that, as a result of micro-hammering the reconstruction of surface topography occurred compared to laser alloying. The increase of surface smoothness was recognized in the whole range of tested parameters of the treatment. The laser alloying process combined with the burnishing can provide reduction of surface roughness heights in relation to the values of asperity heights obtained in the laser alloying process. The shape of the melted zone has improved as well. It was found that the micro-hammering of surface reduces more than twice the maximum height of asperity, S_z, and three times the average value of asperity height, S_a, in relation to the laser alloyed samples. The surface topography due to micro-hammering has a smaller developed area S_{dr}, which is connected with the elimination of small summits of roughness and the flattening of large summits in burnishing process. The analysis of results showed that after hybrid treatment the surface topography is characterized by larger roughness spacing S_m and improved surface bearing area ratio ($S_{dc20\text{-}80\%}$) compared to only laser alloyed samples.

A good correlation between process parameters and the roughness parameters was found. The increase in rotational speed of head and temperature of burnishing treatment provides the smoothest surfaces. The result analysis revealed that the effect of feed rate on the parameters of the roughness is much smaller in the investigated range of parameters comparing to other factors.

The analysis of the deforming as a function of nominal pressure shows that for all examined cases, the surfaces after the burnishing show significantly lower total and plastic deformation in contact with counter sample. In case of surfaces without grinding this effect is associated primarily with surfaces topography. In the process of burnishing the rebuilding of geometrical structure occurs, surface irregularities have smaller heights while their shapes become preferable. Therefore, the real contact area is increased and the unit pressure on individual asperities is reduced. This effect is particularly evident in case of the samples after multipath alloying (Fig.3). At nominal pressure of 155 MPa the plastic deformation is 180 μm for samples that were only alloyed, whereas the burnished surface deformed 44 μm. For higher nominal loading the similar difference between plastic deformation values is observed: 212 μm for the alloyed sample while 68 μm for the burnished one. It is the result of more profitable geometric structure and better physical characteristic of surface layer.

Grinding process has provided similar geometric structure - the surface heights and the shape of asperities are similar for all samples. In this case, examining the process of deformation of the surface under the influence of contact pressure, the differences in elastic and plastic properties of the alloyed layer material and the layer material which underwent alloying with simultaneous burnishing, can be established. For the samples that were multipath alloyed can be concluded that after hybrid treatment there is nearly five times less plastic deformation than after only the multipath alloying, similar to the case of not ground samples. Minor differences exist for samples with single laser tracks (Fig. 4), although in this case less plastic and total deformation for the hybrid treated samples is observed.

Differences in the behavior of the samples after multipath and single path alloying can be explained by different degree of plastic work, different surface layer stresses and the differences in microstructure. In the case of laser-mechanical treatment by scanning the surface the degree of cold work is larger due to multiple burnishing of surface during processing of subsequent paths. The repeated burnishing is carried out at lower temperatures than the burnishing of individual paths, resulting in higher strain hardening of surface layer.

The sample after alloying and multipath burnishing shows the smallest plastic deformation and the highest contact stiffness in case of ground samples, at the both applied nominal pressure. The sample after hybrid treatment with single paths and the laser alloyed samples were more deformable and their value of contact stiffness were lower. The largest deformation shows the sample with the Stellite 6 layer formed by plasma spraying. The calculated values of contact stiffness for contact pressure 155 and 270 MPa, presented in Table 9, confirm these observations. The highest value of contact stiffness, equal to 23 MPa/1μm, from the range of loading pressure 150-270 MPa, was observed after the alloying and the multipath burnishing of the ground surface.

CONCLUSIONS

- The laser-mechanical treatment reconstructs the surface topography and causes the increase in surface smoothness compared to laser alloying. More than threefold decrease in the average height of roughness, *Sa*, after the burnishing process was stated.
- Good correlation between the parameters of the hybrid treatment and roughness parameters was demonstrated. The obtained dependences enable to control the process.
- The laser-mechanical treatment increases the contact stiffness of surface in relation to the laser alloying and the plasma sprayed layers, the highest values of contact stiffness were found for the surfaces after multipath burnishing.

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