

## Efficient reliability analysis of sheet metal forming processes accounting for forming limit curve uncertainty

Jerzy Rojek<sup>1</sup>, Rafał Stocki<sup>2</sup>, Rafał Lasota<sup>3</sup> and Piotr Tazowski<sup>4</sup> \*

*Institute of Fundamental Technological Research, Polish Academy of Sciences*

*ul. Pawińskiego 5B, 02-106 Warsaw, Poland*

*e-mail: <sup>1</sup>jrojek@ippt.pan.pl, <sup>2</sup>rstocki@ippt.pan.pl, <sup>3</sup>rlasota@ippt.pan.pl, <sup>4</sup>ptazow@ippt.pan.pl*

### Abstract

The failure probability estimation of FEM simulated sheet metal forming process is a computationally challenging task. The application of efficient gradient-based reliability techniques is very much limited due to the numerical noise introduced by the explicit dynamic algorithm used to perform the sheet stamping analysis and by the nonlinearity of the failure function. To cope with this difficulty, in the current study a two stage metamodel-based adaptive importance sampling method is employed. In order to assess the reliability of sheet metal forming operations the stochastic character of such parameters as friction, blankholding force, blank thickness, strain hardening parameters of the constitutive law as well as parameters defining the forming limit curve (FLC) are considered. Using the numerical example of a square cup deep drawing, the benchmark problem of the Numisheet'93 conference, it is investigated how the assumptions concerning the probabilistic distribution of the FLC location parameter affect the probability of sheet metal fracture .

*Keywords: sheet metal forming simulation, forming limit diagrams, reliability analysis, adaptive importance sampling*

### 1. Introduction

The so-called forming limit diagrams (FLD) are commonly used to define criteria of material breakage or sheet metal wrinkling – defects that are encountered in the industrial practice of sheet metal forming operations, see e.g. Ref. [1]. Points on FLD representing strain states all over the deformed sheet are confronted with the forming limit curve (FLC). The curve is supposed to represent the boundary between the strain combinations which produce local instability and/or fracture and those that are permissible in forming operations. A zone below FLC, where good results are guaranteed with sufficient probability, is considered as safe zone. Unfortunately, the use of FLD has definite limitations. One of them is the effect of inherent uncertainties of the process and material parameters that lead to uncertainties in the FLD evaluation for a given forming process. Moreover, FLC usually only approximates experimental results which exhibit unavoidable scatter. Therefore any FLC can be regarded as bounding the safe zone with some probability only. In design practise a marginal zone is usually introduced between safe and failure zones to guarantee that a failure due to an inaccurate assessment of the FLC location is unlikely, see Fig. 1. This approach, however, does not allow to evaluate the failure probability of the forming process, which requires a proper stochastic description of frictional contact, blankholding force, parameters of the constitutive law as well as those defining FLC position.

As it was pointed out in one of the first papers dealing with this topic, see Ref. [2], a numerical realization of the sheet metal forming reliability analysis is computationally challenging. Any direct application of efficient gradient-based reliability techniques is very much restricted due to the noisy character of the failure function. The numerical noise is mainly due to the explicit dynamic algorithm used to perform sheet stamping simulation and due to the failure function definition that is based on the distance of the points on FLD from the bounding FLC composed of line segments. Contrary to the expensive Monte Carlo sampling approach employed in [2], in the current study the

above-mentioned computational difficulties are overcome by using a two stage metamodel-based adaptive importance sampling method, previously used by the authors to solve crashworthiness reliability problems [3]. Using the benchmark problem of the Numisheet'93 conference [4] it is showed how the use of different probability distributions to model the uncertainty of FLC position influence the computed probability of sheet metal forming failure.

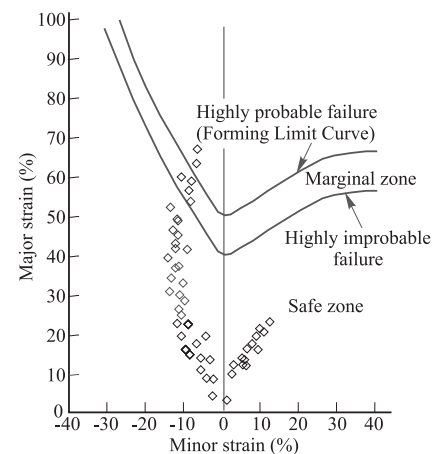


Figure 1: FLD with marginal zone

### 2. Definition of the failure function

The failure function (limit state function) used in reliability analysis takes advantage of the concept of FLD. It is defined as the signed minimal distance from FLC of the point corresponding to principal strains in the given finite element, see Fig. 2. If all

\*This research was financially supported by European Regional Development Fund within the framework of the Innovative Economy Programme, project no. POIG.01.03.01-14-209/09, acronym - NUMPRESS.

the points are below FLC the limit state function is equal to the distance of the point closest to FLC. On the other hand, if some of the points are above FLC the value of the function equals the maximal distance of the point from those above the curve, taken with the minus sign. Depending on the realization of the vector of random variables the cloud of points on FLD can take different shapes which implies that different points in the sheet metal may be located close to FLC. Moreover, the piecewise linear FLC and some numerical noise introduced by the explicit dynamic approach in the FEM analysis renders the limit state function non-differentiable.

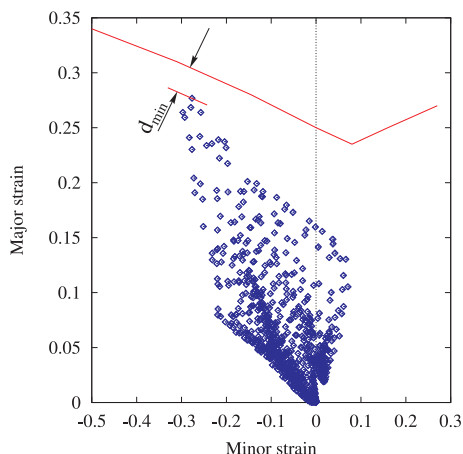


Figure 2: Definition of the limit state function

### 3. Reliability analysis method

The employed approach is described in detail in Ref. [3]. It is composed of two major parts. In the first part, the most probable point search is performed by means of adaptive metamodel based on the optimal Latin hypercube design of experiments. In the second part a multi-modal adaptive importance sampling method is applied to improve the estimate of the first part. The sampling phase, utilizing the concept of the so-called representative points is schematically shown in Fig. 3. Multi-modal sampling density concentrated over selected representative points ( $\hat{v}^{(1)}, \dots, \hat{v}^{(4)}$  in Fig. 3) allows for better adjustment to the nonlinear failure domain  $\Delta_f$ , leading to more accurate failure probability estimation.

### 4. Numerical example

Deep drawing of an aluminium square cup (Fig. 3), the benchmark problem at the Numisheet'93 conference [4] has been analyzed. To formulate the reliability analysis problem eight random variables are identified. They are as follows: the initial thickness of the blank, two parameters ( $K$  and  $n$ ) of the uniaxial true-stress true-strain curve  $\bar{\sigma} = K(a + \bar{\epsilon}^p)^n$ , blankholder force, three friction coefficients—between sheet metal and punch, die and blankholder, respectively, and the vertical shift parameter to describe the uncertainty of the FLC location. It was assumed that the variables are uniformly or normally distributed, the friction coefficients are strongly positively correlated and the shift parameter takes either exponential or uniform distribution. It is the influence of the probabilistic modeling of the FLC uncertainty that seems to be the most important for design practise. In the presentation it will be shown that uniform distribution of the shift parameter, consistent with the concept of marginal zone shown in Fig. 1, leads to more than 10 times higher failure probability with

respect to the exponential model, which is more realistic when the qualitative character of the scatter of experimentally observed fracture failures is considered.

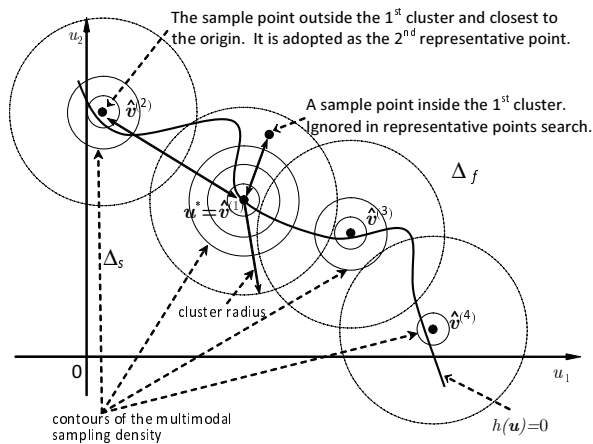


Figure 3: General idea of multi-modal adaptive importance sampling strategy.  $u^*$  is the most probable failure point.

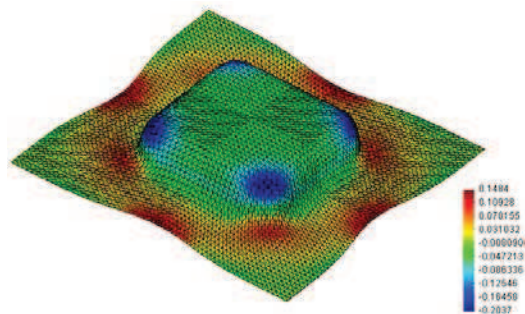


Figure 4: Deep drawing of a square cup – distribution of logarithmic thickness strain at 20mm depth of drawing

### References

- [1] Marciniak, Z., Duncan, J.L. and Hu, S.J., *Mechanics of sheet metal forming*, Butterworth-Heinemann, Oxford, 2002.
- [2] Kleiber, M., Rojek, J. and Stocki, R. Reliability assessment of sheet metal forming operations, *Computer Methods in Applied Mechanics and Engineering*, 191, pp. 4511–4532, 2002.
- [3] Stocki, R., Tazowski, P. and Knabel, J. Reliability analysis of a crashed thin-walled s-rail accounting for random spot weld failures, *International Journal of Crashworthiness*, 13, pp. 693–706, 2008.
- [4] Makinouchi, A., Nakamachi, E., Oñate, E. and Wagoner, R.H. (Eds.), *Proc. 2nd International Conference NUMISHEET'93: Numerical Simulation of 3-D Sheet Metal Forming Processes—Verification of Simulation with Experiment*, Isehara, Japan, 1993.