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MODEL-FREE MONITORING OF STRUCTURES

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1 INTRODUCTION

This contribution presents the model-free approach to structural identification and monitoring, which has recently been developed in IPPT PAN [1–3]. The approach adapts the essentially nonparametric methodology of the virtual distortion method (VDM, [4]). Monitored structure is characterized in a purely experimental way, so that no parametric numerical modelling is required: the monitoring process is based directly on experimentally measured local impulse response functions. Even though, the approach can be used for identification of parametrized modifications of mass and stiffness or inelastic impacts. In comparison to other monitoring methods, it is characteristic enough to warrant the name of a *model-free* approach.

Most of the low-frequency methods used for global structural health monitoring (SHM, see the references in [1]) can be classified into two general groups:

1. *Model-based methods*, which rely on a parametric numerical model of the monitored structure. An appealing feature of these methods is the physicality of the model and identified damages; however, an accurate parametric model is often not easy to obtain.
2. *Pattern recognition methods* rely on a database of numerical fingerprints extracted from the experimentally measured responses. No parametric modeling is required, but at the cost of the physicality of the model. The identification rarely goes beyond damage detection or approximate localization.

The developed approach is aimed at exploiting the advantages of both groups of methods: it makes use of a nonparametric model of the monitored structure composed of experimentally measured data, but it enables full identification of parametrically expressed modifications and inelastic impacts.

2 FORMULATION

Consider modifications $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$ to the structural mass and stiffness matrices, and denote by $\mathbf{f}(t)$ an external testing excitation. In case of an inelastic impact, $\mathbf{f}(t) := mv\mathbf{e}\delta(t)$, which involves the impacting mass m, impact velocity v, versor $\mathbf{e}$ of direction of the impact and the Dirac delta. In time domain, the response of the modified/impacted structure satisfies:

$$(\mathbf{M} + \Delta\mathbf{M})\,\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + (\mathbf{K} + \Delta\mathbf{K})\,\mathbf{u}(t) = \mathbf{f}(t), \tag{1}$$

which yields the equation of motion of the unmodified structure,

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{p}^0(t), \tag{2}$$

in which the modification or impact is modeled with the equivalent pseudo load vector $\mathbf{p}^0(t)$,

$$\mathbf{p}^0(t) = -\Delta\mathbf{M}\ddot{\mathbf{u}}(t) - \Delta\mathbf{K}\mathbf{u}(t). \tag{3}$$

As seen from (2), the response of the modified structure to $\mathbf{f}(t)$ is the sum of the known response $\mathbf{u}^{\mathrm{L}}(t)$ of the unmodified structure and the cumulative effects of the pseudo loads,

$$\mathbf{u}(t) = \mathbf{u}^{\mathrm{L}}(t) + \left(\boldsymbol{\mathcal{B}}\mathbf{p}^0\right)(t), \qquad \ddot{\mathbf{u}}(t) = \ddot{\mathbf{u}}^{\mathrm{L}}(t) + \left(\ddot{\boldsymbol{\mathcal{B}}}\mathbf{p}^0\right)(t), \tag{4}$$

where $\boldsymbol{\mathcal{B}}$ and $\ddot{\boldsymbol{\mathcal{B}}}$ are the operators of convolution with measured impulse response functions. Equations (2) to (4) yield together the the following system of Volterra integral equations of the second kind with the pseudo load vector $\mathbf{p}^0(t)$ as the unknowns:

$$-\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t) - \Delta\mathbf{K}\mathbf{u}^{\mathrm{L}}(t) = \left[\mathbf{I} + \Delta\mathbf{M}\mathbf{M}^{-1}\right]\mathbf{p}^0(t) + \left(\left(\Delta\mathbf{M}\ddot{\boldsymbol{\mathcal{B}}} + \boldsymbol{\Delta}\mathbf{K}\boldsymbol{\mathcal{B}}\right)\mathbf{p}^0\right)(t), \tag{5}$$

which confirms the inherent ill-conditioning of the considered inverse problem. Solved equation (5), then the response of the modified/impacted structure can be computed by (4).

The frequency-domain counterparts of (4) and (5) are

$$\mathbf{u}(\omega) = \mathbf{u}^{\mathrm{L}}(\omega) + \mathbf{H}(\omega)\mathbf{p}^0(\omega), \tag{6}$$

$$\left(\mathbf{I} - \omega^2\Delta\mathbf{M}\mathbf{H}(\omega) + \Delta\mathbf{K}\mathbf{H}(\omega)\right)\mathbf{p}^0(\omega) = -\left(-\omega^2\Delta\mathbf{M} + \Delta\mathbf{K}\right)\mathbf{u}^{\mathrm{L}}(\omega). \tag{7}$$

The frequency-domain formulation is computationally more effective, but allows for much less control over the numerical regularization of the solution, which is performed indirectly through the decay coefficient of the exponential FFT window and needs further investigation.

The inverse problem of damage identification is stated in the form of minimization of the discrepancy between the measured and the modeled responses of the modified structure, which can involve response time histories, windowed time histories, natural frequencies, etc.

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