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Simultaneous identification of moving mass and structural damage

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1. Abstract
Identification of damage and moving load (or mass) are crucial problems in structural health monitoring (SHM). However, it seems there is not much investigation on simultaneous identification of the two factors, although in practice they usually exist together. This paper proposes a methodology to solve the coupled problem based on the Virtual Distortion Method (VDM): the damaged structure is modeled by an equivalent intact structure (called the distorted structure) subjected to the same moving mass (or in fact to the equivalent response-coupled moving load) and to certain virtual distortions which model the damage. The measured structural response is used to identify the moving mass and the damage; unknown mass and damage extents are used as the optimization variables instead of the usually chosen moving mass-equivalent force. In this way well-conditioning of the identification is ensured and the number of the necessary sensors is decreased. The numerical costs are considerably reduced by using the introduced concept of the moving dynamic influence matrix. The proposed identification method can be used both off-line and online by a repetitive application in a moving time window. A numerical experiment of a beam with 5% measurement error demonstrates that the moving masses can be identified along with the damage extents.
2. Keywords: Structural health monitoring (SHM); Moving mass (load) identification; Damage identification; Virtual distortion method (VDM)

3. Introduction
Identification of structural damage is the primary task of most structural health monitoring (SHM) systems. In many other applications, identification of moving loads (or masses) is also a crucial problem. Especially identification of the moving mass is valuable not only in the assessment of the pavement or bridge but also in traffic studies, in design code calibration, etc. Several techniques have been developed, which address both these identification problems separately. However, in real applications, unknown damage and unknown loads usually coexist and together influence the system response. Though, simultaneous identification of moving masses and structural damage seems to be an unexplored area.
Moving load identification has been studied extensively in the past few decades. An indirect identification through the measured response has received special interest, since in practice it can be performed easier and with lower costs compared with direct measurements. Chan, Law et al. have proposed four methods for indirect identification, which are the time-domain method (TDM) [1], the frequency-time domain method (FTDM) [2], Interpretive Method I (IMI) [3] and Interpretive Method II (IMII) [4]. All of them require that the model parameters of the bridge are known. Each method has its merits and limitations, which are compared in [5]. The numerical ill-conditioning of the problem is the main factor that theoretically worsens the accuracy of the identification results. To improve the accuracy, a pseudo-inverse or singular value decomposition (SVD) techniques have been investigated and adopted for the inverse computation [6]. Some other regularization methods [7] have been proposed, e.g. bounds can be imposed on the identified forces in solving the ill-conditioned problem using different combinations of measured responses. However, finding the optimal value of the regularization parameter in these methods is numerically costly and requires long computational time. Moreover, the regularization parameter turns out to be sensitive to properties of the vehicle and bridge and hard to be precisely assigned [8]. In [8], an iterative regularization method called the updated static component (USC) technique is proposed in order to decrease the sensitivity of the regularization parameter. In general, these and similar methods all require a known and well-defined model of the structure in order to build the load-response relation. Moreover, existing methods usually take the moving force as the unknown variable, which linearizes the identification problem, but at the cost of the increased ill-conditioning and larger number of sensors,

which (in order to obtain a unique solution) are required to be not fewer in number than the moving forces. And the ill-conditioning makes regularization techniques necessary to obtain meaningful solutions. For damage identification, low- frequency SHM is in the scope of this paper. In [9], the existing approaches are categorized as model-based and pattern-recognition. The analysis can be carried out directly in time domain via sampled time signals, utilizing either statistical concepts and time series models [10] or deterministic model-updating approaches, which are often coupled with quick reanalysis techniques [11]. The identification problem is frequently transferred into frequency domain and solved using modal methods, which detect, locate and identify the damages by the respective change of the related modal parameters; see a summary review in [12]. The wavelet analysis becomes a popular tool used often together with pattern-recognition methods [13]. Some of these methods rely on the assumption that the external loads are well-defined and known. Others, like some modal and time series methods, can be used without the exact information of the loads, but they are applicable only under special conditions like ambient excitation or free response of the monitored structure.

Therefore, in the case of coupled moving load (or mass) and structural damage, such as in identification of moving loads that pass a damaged bridge, it is hard to identify the unknown moving load ignoring the unknown damage. In [14], a method based on the virtual distortion method (VDM) [9, 15] is proposed to simultaneously identify a general load and damages of unknown types and extents. This paper presents an efficient VDM-based method, which is tailored to the specific objective of identification of moving loads (or masses) and structural damage (assumed here to be reduced stiffness). The masses and the damage extents are chosen to be the optimization variables, which are identified through minimizing the mean-square distance between the measured and the modeled structural response. The choice of the variables ensures well-conditioning of the identification process. Moreover, given the identified masses, the corresponding moving loads can be easily computed with a high accuracy. In addition, fewer sensors can provide the identification uniqueness and required accuracy level. The numerical costs in each optimization step are considerably reduced by using the proposed here concept of the moving dynamic influence matrix, which is defined as a collection of impulse-responses with respect to the (changing in time) positions of the moving masses. The method can be used both off-line and online, by a repetitive application in a moving time window [14].

4. Virtual Distortion Method for frame structures

The Virtual Distortion Method is a quick reanalysis method applicable in both statics and dynamics [9, 11]. Structural modifications, including damages, are modeled via related response-coupled virtual distortions imposed on the involved elements. The structural response of a *damaged structure* to an external load is expressed in the form of a combination of the responses of the intact structure to the same load and to virtual distortions that occur in the damaged elements (*distorted structure*). Both the *damaged structure* and the *distorted structure* are equivalent in terms of identical element strains and forces. For the sake of notational simplicity, only frame structures, stiffness-related damages and strain sensors are considered here. The methodology can be straightforwardly extended to include other damage patterns as well as types of structures and sensors [9, 11, 14].

The paper is structured as follows:

4.1. Virtual distortion vs. damage

The mutual relations between damage extent, virtual distortion and the final strain of a damaged element can be deduced and expressed in the general terms of the finite element (FE) method. Let the damage extent of the ith element μ_i be the ratio of the modified stiffness $\tilde{\mathbf{K}}_i$ to its original value $\mathbf{K}_i$. The dynamic equation of motion of the damaged structure under external load $\mathbf{f}$ is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \tilde{\mathbf{K}}\mathbf{u} = \mathbf{f}. \tag{1}$$

In the FE analysis, the structural stiffness matrix $\tilde{\mathbf{K}} = \sum_{i=1}^{n_\mathrm{e}} \tilde{\mathbf{K}}_i$, where $\tilde{\mathbf{K}}_i = \mathbf{L}_i^\mathrm{T}\mathbf{T}_i^\mathrm{T}\tilde{\mathbf{K}}_{\mathrm{e},i}\mathbf{T}_i\mathbf{L}_i$ is the stiffness matrix of the ith element expressed in the global degrees of freedom (DOFs); $\tilde{\mathbf{K}}_{\mathrm{e},i}$ is the stiffness matrix of the ith element expressed in its local DOFs; $\mathbf{L}_i$ is the localization matrix linking the global DOFs to the local DOFs of the ith element; $\mathbf{T}_i$ is the transformation matrix from the global coordinates to the local coordinates of the ith element; and n_e is the total number of the elements. Eq.(1) can be rewritten in the following form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} + \sum_{i=1}^{n_\mathrm{e}} (1-\mu_i)\,\mathbf{K}_i\mathbf{u}, \tag{2}$$

which is the equation of motion of the *distorted structure*. Therefore, the response $\mathbf{u}$ is a combination of the responses of the intact structure to the same load $\mathbf{f}$ and to certain virtual forces, which in the global DOFs are defined as $\mathbf{p} = \sum_{i=1}^{n} (1-\mu_i)\,\mathbf{K}_i\mathbf{u}$. The FE formulation yields

$$\mathbf{p} \;=\; \sum_{i=1}^{n} \mathbf{L}_i^{\mathrm{T}}\mathbf{T}_i^{\mathrm{T}} (1-\mu_i)\,\mathbf{K}_{\mathrm{e},i}\mathbf{T}_i\mathbf{L}_i\mathbf{u} \;=\; \sum_{i=1}^{n} \mathbf{L}_i^{\mathrm{T}}\mathbf{T}_i^{\mathrm{T}} (1-\mu_i)\,\mathbf{K}_{\mathrm{e},i}\mathbf{u}_i, \tag{3}$$

where $\mathbf{u}_i = \mathbf{T}_i\mathbf{L}_i\mathbf{u}$ is the vector of the nodal displacements of the ith element in its local coordinate system. Hence the virtual forces that occur locally in the ith element can be expressed as $\mathbf{p}^0_{\mathrm{e},i} = (1-\mu_i)\,\mathbf{K}_{\mathrm{e},i}\mathbf{u}_i$, where $\mathbf{p}_{\mathrm{e},i} = \mathbf{K}_{\mathrm{e},i}\mathbf{u}_i$ are the local nodal forces. Therefore, the local virtual forces and the corresponding local nodal forces are related to each other by

$$\mathbf{p}^0_{\mathrm{e},i} = (1-\mu_i)\,\mathbf{K}_{\mathrm{e},i}\mathbf{u}_i = (1-\mu_i)\,\mathbf{p}_{\mathrm{e},i}.$$

For truss structures with one-dimensional elements, the virtual forces modeling the damage of an element corresponds to a single axial virtual distortion; in other structures more virtual distortions can be necessary. The number and the forms of the distortions can be analyzed using the eigenvalue problem of the local stiffness matrix $\mathbf{K}_{\mathrm{e},i}$ of the element. Its eigenvectors are of two kinds: distortion vectors corresponding to positive eigenvalues and rigid motion vectors corresponding to zero eigenvalues. The matrix $\mathbf{K}_{\mathrm{e},i}$ can be expressed in the terms of its $\bar{n}_i$ positive eigenvalues λ_{ij} and the corresponding eigenvectors $\boldsymbol{\varphi}_{ij}$,

$$\mathbf{K}_{\mathrm{e},i} = \sum_{j=1}^{\bar{n}_i} \lambda_{ij}\boldsymbol{\varphi}_{ij}\boldsymbol{\varphi}_{ij}^{\mathrm{T}}. \tag{4}$$

The vector $\boldsymbol{\varphi}_{ij}$ represents the jth local base distortion of the ith element, and so $\mathbf{K}_{\mathrm{e},i}\boldsymbol{\varphi}_{ij} = \lambda_{ij}\boldsymbol{\varphi}_{ij}$ is the corresponding vector of the forces that realize the base distortion. The damage-modeling virtual forces can be thus expressed in terms of a combination of the base virtual distortions as

$$\mathbf{p}^0_{\mathrm{e},i} \;=\; (1-\mu_i)\sum_{j=1}^{\bar{n}_i} \lambda_{ij}\boldsymbol{\varphi}_{ij}\boldsymbol{\varphi}_{ij}^{\mathrm{T}}\mathbf{u}_i \;=\; \sum_{j=1}^{\bar{n}_i} \mathbf{K}_{\mathrm{e},i}\boldsymbol{\varphi}_{ij}\left[(1-\mu_i)\,\boldsymbol{\varphi}_{ij}^{\mathrm{T}}\mathbf{u}_i\right] \;=\; \mathbf{K}_{\mathrm{e},i}\sum_{j=1}^{\bar{n}_i} \Upsilon^0_{ij}\boldsymbol{\varphi}_{ij}, \tag{5}$$

where $\Upsilon^0_{ij} = (1-\mu_i)\,\boldsymbol{\varphi}_{ij}^{\mathrm{T}}\mathbf{u}_i$. In Eq.(5), $\Upsilon^0_{ij}\boldsymbol{\varphi}_{ij}$ is the jth damage-modeling virtual distortion of the ith element, and Υ^0_{ij} is the combination coefficient of the corresponding jth base distortion $\boldsymbol{\varphi}_{ij}$. The local virtual forces corresponding to the jth virtual distortion can be expressed as $\mathbf{K}_{\mathrm{e},i}\Upsilon^0_{ij}\boldsymbol{\varphi}_{ij}$. Similarly, $\mathbf{p}_{\mathrm{e},i} = \mathbf{K}_{\mathrm{e},i}\mathbf{u}_i = \mathbf{K}_{\mathrm{e},i}\sum_{j=1}^{\bar{n}_i}\Upsilon_{ij}\boldsymbol{\varphi}_{ij}$, where $\Upsilon_{ij} = \boldsymbol{\varphi}_{ij}^{\mathrm{T}}\mathbf{u}_i$, and therefore

$$\Upsilon^0_{ij} = (1-\mu_i)\,\Upsilon_{ij}. \tag{6}$$

For a 2D beam element, the local stiffness matrix $\mathbf{K}_{\mathrm{e},i}$ has three positive eigenvalues and three corresponding eigenvectors. Apart from the axial type of distortion $\varepsilon^0_i = \Upsilon^0_{i1}$ (as in a truss element), it also includes pure bending $\kappa^0_i = \Upsilon^0_{i2}$ and bending plus shear terms $\chi^0_i = \Upsilon^0_{i3}$. Hence there are three components of virtual distortions that have to be considered [16]. In case of structures of other types (plates, shells, etc.), the base distortions, the corresponding forces and the relation between a damage and the damage-modeling virtual distortions can be deduced similarly by the eigenvalue problem of stiffness matrices of the elements. In the following, only 2D beam elements are considered.

4.2. Response of damaged frame structure under known force

With the assumption of zero initial conditions, the discretized response $y_\alpha(t)$ of the αth (linear) sensor in an externally loaded damaged structure is modeled by the VDM as the following sum of the linear and the residual parts

$$\begin{aligned} y_\alpha(t) &= y^{\mathrm{L}}_\alpha(t) + y^{\mathrm{R}}_\alpha(t) \\ &= y^{\mathrm{L}}_\alpha(t) + \sum_{\tau=0}^{t}\sum_{i=1}^{n_{\mathrm{e}}} \left[D^{\varepsilon}_{\alpha i}(t-\tau)\varepsilon^0_i(\tau) + D^{\kappa}_{\alpha i}(t-\tau)\kappa^0_i(\tau) + D^{\chi}_{\alpha i}(t-\tau)\chi^0_\beta(\tau)\right] \end{aligned} \tag{7}$$

where $y^{\mathrm{L}}_\alpha(t)$ denotes the response of the intact structure to the same external force; $\varepsilon^0_i(t)$, $\kappa^0_i(t)$ and $\chi_i(t)$ are the damage-related coefficients of the three base distortions of the ith element, and $D^{\varepsilon}_{\alpha i}$, $D^{\kappa}_{\alpha i}$ and

$D^{\chi}_{\alpha i}$ are the discretized impulse-response functions of the intact structure (*dynamic influence matrices* according to the VDM terminology), that is the discretized responses of its αth sensor to impulse base distortions φ_{i1}, φ_{i2} and φ_{i3} of the ith element. The excitations of impulse base distortions are equivalent to local impulse forces $\mathbf{K}_{\mathrm{e},i}\varphi_{i1}$, $\mathbf{K}_{\mathrm{e},i}\varphi_{i2}$ and $\mathbf{K}_{\mathrm{e},i}\varphi_{i3}$ in the βth element. Note that this formulation requires the *assumption of small deformations* in order to allow the responses to be linearly combined.

5. Structural dynamic response under moving mass

5.1. Equation of motion

The moving masses are assumed to attach to the support system. There are d masses on a flat bridge of length L, moving at constant velocities v_1, v_2, …, v_d, see Figure 1. The current coordinates of the mass m_j is $x_j = x_{j,0} + v_j t$, where $x_{j,0}$ is the initial position.

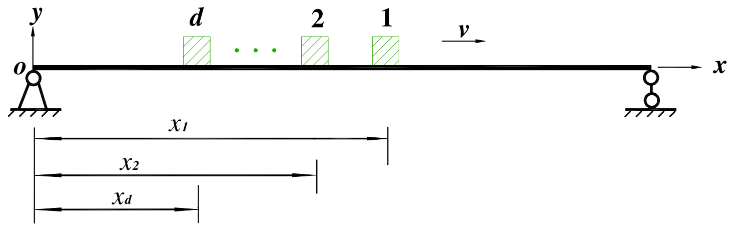

Figure 1: Moving masses and bridge system

The bridge is modeled as a discrete finite element structure. The moving masses and the bridge are collectively considered a single system, which is exposed to constant moving external loads of the gravities of the masses. In each time step, the system mass matrix is assembled with respect to the current positions of the moving masses, and the current force vector is computed using the shape functions of the finite elements currently carrying the masses. The equation of motion of the system can be thus written as

$$[\mathbf{M} + \Delta\mathbf{M}(t)]\,\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) \;=\; \mathbf{B}(\mathbf{x})\bar{\mathbf{m}}\bar{\mathbf{g}}, \tag{8}$$

where $\Delta\mathbf{M}(t) = \sum_{j=1}^{d} \mathbf{L}^{\mathrm{T}}_{i_j}\,\mathbf{n}_{i_j}(\zeta_j) m_j\,\mathbf{n}^{\mathrm{T}}_{i_j}(\zeta_j)\mathbf{L}_{i_j}(x_j)$ is an $n \times n$ time-variant matrix, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the $n \times n$ system matrices of the bridge, i_j is the element number where the mass m_j currently is, ζ_j is the relative coordinate of m_j with respect to the ith element, $\mathbf{L}_i$ is the $n_i \times n$ localization matrix of the ith element, $\mathbf{n}_{i_j}(\zeta_j)$ is the load allocation vector of the i_jth element due to the mass m_j (dependent on the element shape functions), n is the total number of the system DOFs, n_i is the number of the DOFs of the ith element. The matrix $\mathbf{B}(\mathbf{x}) = [\mathbf{b}_1(x_1) \quad \mathbf{b}_2(x_2) \quad \dots \quad \mathbf{b}_d(x_d)]$ denotes the load location matrix, where $\mathbf{b}_j(x_j) = \mathbf{L}^{\mathrm{T}}_{i_j}(x_j)\,\mathbf{n}_{i_j}(\zeta_j)$. Finally, $\bar{\mathbf{m}} = \mathrm{diag}\,[m_1 \quad m_2 \quad \dots \quad m_d]$ is a $d \times d$ diagonal matrix, and $\bar{\mathbf{g}}$ is the d dimensional vector of gravity acceleration constants g. The dynamic nodal displacements, velocities and accelerations of the bridge can be obtained by numeric integration of Eq.(8).

5.2. Dynamic moving influence matrix

In accordance with the general idea of the VDM, the time-variant matrix $\Delta\mathbf{M}(t)$ in Eq.(8) is moved to the right-hand side. The equation can be stated in the equivalent form of

$$[\mathbf{M} + \Delta\mathbf{M}(t)]\,\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) \;=\; \mathbf{B}(\mathbf{x})\bar{\mathbf{m}}\,[\bar{\mathbf{g}} - \mathbf{a}(t)]\,, \tag{9}$$

which is the equation of motion of the bridge alone subjected to external moving invariant loads and response-coupled loads. In Eq.(9), the vector $\mathbf{a}(t)$ collects the vertical accelerations of the masses, $a_i(t) = \mathbf{b}^{\mathrm{T}}_i(x_i)\ddot{\mathbf{u}}(t)$. Since the acceleration can be represented using the system impulse-response matrix $\ddot{\mathbf{H}}(t)$, which describes accelerations in response to unit impulse excitations and include a distributional term,

$$a_i(t) \;=\; \mathbf{b}^{\mathrm{T}}_i(x_i)\int_0^t \ddot{\mathbf{H}}(t-\tau)\mathbf{B}(\mathbf{x})\bar{\mathbf{m}}\,[\mathbf{g} - \mathbf{a}(\tau)]\;d\tau \;=\; \sum_{j=1}^{d}\int_0^t \mathbf{b}^{\mathrm{T}}_i(x_i)\ddot{\mathbf{H}}(t-\tau)\mathbf{b}_j(x_j)m_j\,[g - a_j(\tau)]\;d\tau. \tag{10}$$

In the discrete-time system, let the vertical acceleration of m_i at the kth time step $a_i(t_k)$ be denoted by a_{ik}, and $\mathbf{b}_i(x_i)$ by $\mathbf{b}_{ik}$. Then

$$a_{ik} = \sum_{j=1}^{d}\sum_{l=1}^{k} \mathbf{b}^{\mathrm{T}}_{ik}\ddot{\mathbf{H}}(k-l)\mathbf{b}_{jl}\,m_j\,(g - a_{jl})\,, \tag{11}$$

where $\ddot{\mathbf{H}}(k-l)$ is the discretized counterpart of the continuous impulse-response from Eq.(10), that is its element $\ddot{H}_{ij}(k-l)$ is the acceleration response of the ith DOF at time t_k to the unit excitation applied in the jth DOF at time t_l. With proper ordering of the data, Eq.(11) can be stated in the matrix form,

$$\mathbf{a} = \mathbf{D}^{\mathrm{mm}}\mathbf{m}\left(\mathbf{g} - \mathbf{a}\right), \tag{12}$$

where $\mathbf{a} = \begin{bmatrix}\mathbf{a}_1^{\mathrm{T}} & \mathbf{a}_2^{\mathrm{T}} & \ldots \mathbf{a}_d^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, and the column vector $\mathbf{a}_i$ collects all discrete accelerations of the mass m_i in successive time steps. $\mathbf{D}^{\mathrm{mm}}$ is the proposed in this paper dynamic moving influence matrix; it is an $n_t d \times n_t d$ block matrix, composed of d^2 lower triangular matrices $\mathbf{D}_{i,j}^{\mathrm{mm}}$, each of dimension $n_t \times n_t$, where n_t is the total number of the time steps. An element $D_{ij}^{\mathrm{mm}}(k,l)$ is the vertical acceleration of the mass m_i at the time t_k caused by a unit impulse applied at time t_l on the location of the mass m_j (that is, at $x_{j,0} + v_j t_l$),

$$D_{ij}^{\mathrm{mm}}(k,l) = \begin{cases} \mathbf{b}_{ik}^{\mathrm{T}}\ddot{\mathbf{H}}(k-l)\mathbf{b}_{jl} & \text{if } l \le k, 0 \le x_{i,0} + v_i t_k \le L, 0 \le x_{j,0} + v_j t_l \le L, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

In Eq.(12), $\mathbf{m}$ is a $n_t d \times n_t d$ diagonal block matrix, in which the ith diagonal matrix is $m_i \mathbf{I}_{n_t \times n_t}$, and $\mathbf{g}$ is a column vector composed of $n_t d$ elements: either gravity acceleration constants g (when the corresponding mass is on the bridge) or of zero elements at the time steps when the corresponding mass is off the bridge.

The dynamic moving influence matrix needs to be computed only once for a certain bridge and velocities of the masses. Therefore, for the coupled bridge-moving mass analysis, the accelerations $\mathbf{a}$ of the masses can be solved quickly by Eq.(12) for different moving masses. In this way, the repeated assembling of the system mass matrix in each time step is avoided. This affords an important advantage in moving mass identification, as described in the following section.

6. Simultaneous identification of moving masses and structural damage

6.1. Response of a damaged frame structure to moving masses

For a damaged bridge excited by moving masses, let the moving masses and the damage extents be both unknown. Sensor responses can be computed by Eq.(7), where the damage is modeled by the distortions and the linear part $\mathbf{y}^{\mathrm{L}}$ is the response of the undamaged bridge to the equivalent moving force vector $\mathbf{p} = \mathbf{m}(\mathbf{g} - \mathbf{a})$. Eq.(7), similarly as Eq.(11), can rewritten for all considered time steps n_t and for all sensor locations α and stated in the form of a single large linear equation as

$$\mathbf{y} = \mathbf{D}^{\mathrm{m}}\mathbf{p} + \mathbf{D}^{\varepsilon}\boldsymbol{\varepsilon}^0 + \mathbf{D}^{\kappa}\boldsymbol{\kappa}^0 + \mathbf{D}^{\chi}\boldsymbol{\chi}^0, \tag{14}$$

where $\mathbf{y}$ is the discrete response (an $n_t s$ column vector, where s is the number of the sensors), $\mathbf{D}^{\mathrm{m}}$ is an $n_t s \times n_t d$ block matrix, composed of $n_t \times n_t$ lower triangular matrices $\mathbf{D}_{\alpha j}^{\mathrm{m}}$ such that $D_{\alpha j}^{\mathrm{m}}(k,l)$ is the response of the αth sensor at time t_k to the unit impulse applied at time t_l on the location $x_{j,0} + \nu_j t_l$ (position of the mass m_j),

$$D_{\alpha j}^{\mathrm{m}}(k,l) = \begin{cases} \mathbf{g}_{\alpha}\mathbf{H}(k-l)\mathbf{b}_{jl} & \text{if } l \le k, 0 \le x_{j,0} + v_j t_l \le L, \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

where $\mathbf{H}(k-l)$ is the displacement impulse-response of the undamaged structure in time t_k to the unit impulses applied in all DOFs in time t_l and $\mathbf{g}_{\alpha}$ is the observation vector with respect to the sensor α.

6.2. Objective function

In general, there are two ways to identify the unknown moving masses and damage extents (that is, the stiffness reduction ratios μ_i). The first way is to solve directly the linear system Eq.(14) in order to obtain the equivalent moving force vector $\mathbf{p}$ and the virtual distortions $\boldsymbol{\varepsilon}^0$, $\boldsymbol{\kappa}^0$, $\boldsymbol{\chi}^0$. They can be then treated as excitations of the undamaged structure and used to compute the corresponding accelerations $\mathbf{a}$ and the responses $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$, $\boldsymbol{\chi}$ (i.e. Υ_{ij} of Eq.(5)) of the damaged elements. The unknown masses and the damage extents can be then estimated using the expression $\mathbf{p} = \mathbf{m}(\mathbf{g} - \mathbf{a})$ and Eq.(6), respectively. However, the problem of computing the direct solution of Eq.(14) is a well-known ill-conditioned problem, and thus extremely sensitive to measurement errors. Moreover, since the elements of the vectors $\mathbf{p}$, $\boldsymbol{\varepsilon}^0$, $\boldsymbol{\kappa}^0$ and $\boldsymbol{\chi}^0$ are treated as independent, the number of sensors has to be equal or greater than the number of the unknown loads plus the threefold number of the potentially damaged elements.

Therefore, this paper proposes a more practical way of identification of the unknown m_i and μ_j by minimization of the normalized mean-square distance between the measured structural response $\mathbf{y}^{\mathrm{M}}$ and the computed response $\mathbf{y}$ of the distorted structure, where the unknown variables are m_i and μ_j. In this way the number of the unknown variables is significantly reduced, and the ill-conditioning is avoided. The proposed objective function is thus

$$f(m_1, \ldots, m_d, \mu_1, \ldots, \mu_{n_d}) = \frac{\|\mathbf{y}^{\mathrm{M}} - \mathbf{D}^{\mathrm{m}}\mathbf{p} - \mathbf{D}^{\varepsilon}\boldsymbol{\varepsilon}^0 - \mathbf{D}^{\kappa}\boldsymbol{\kappa}^0 - \mathbf{D}^{\chi}\boldsymbol{\chi}^0\|}{\|\mathbf{y}^{\mathrm{M}}\|}, \tag{16}$$

where n_d is the number of the potentially damaged elements.

6.3.Estimation of the equivalent forces and virtual distortions

In the objective function Eq.(16), the equivalent forces $\mathbf{p}$ and the virtual distortions $\boldsymbol{\varepsilon}^0$, $\boldsymbol{\kappa}^0$, $\boldsymbol{\chi}^0$ depend on the unknown masses m_i and the damage extents μ_j. In each optimization step they can be quickly constructed using precomputed corresponding influence matrices of the distorted structure, based on the following formulas, which are similar to Eq.(14):

$$\begin{aligned}
\boldsymbol{\varepsilon} &= \mathbf{D}^{\varepsilon\varepsilon}\boldsymbol{\varepsilon}^0 + \mathbf{D}^{\varepsilon\kappa}\boldsymbol{\kappa}^0 + \mathbf{D}^{\varepsilon\chi}\boldsymbol{\chi}^0 + \mathbf{D}^{\varepsilon\mathrm{m}}\mathbf{p},\\
\boldsymbol{\kappa} &= \mathbf{D}^{\kappa\varepsilon}\boldsymbol{\varepsilon}^0 + \mathbf{D}^{\kappa\kappa}\boldsymbol{\kappa}^0 + \mathbf{D}^{\kappa\chi}\boldsymbol{\chi}^0 + \mathbf{D}^{\kappa\mathrm{m}}\mathbf{p},\\
\boldsymbol{\chi} &= \mathbf{D}^{\chi\varepsilon}\boldsymbol{\varepsilon}^0 + \mathbf{D}^{\chi\kappa}\boldsymbol{\kappa}^0 + \mathbf{D}^{\chi\chi}\boldsymbol{\chi}^0 + \mathbf{D}^{\chi\mathrm{m}}\mathbf{p},\\
\mathbf{a} &= \mathbf{D}^{\mathrm{m}\varepsilon}\boldsymbol{\varepsilon}^0 + \mathbf{D}^{\mathrm{m}\kappa}\boldsymbol{\kappa}^0 + \mathbf{D}^{\mathrm{m}\chi}\boldsymbol{\chi}^0 + \mathbf{D}^{\mathrm{mm}}\mathbf{p},
\end{aligned} \tag{17}$$

where all $\mathbf{D}^{(\cdot)(\cdot)}$ are influence matrices that contain responses to impulse excitations, similar to the matrices in Eq.(14), but with respect to distortions imposed on the potentially damaged element and forces or accelerations at the locations of the masses, as defined by the superscripts. The responses $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$, $\boldsymbol{\chi}$ and the equivalent forces $\mathbf{p}$ are related to the virtual distortions $\boldsymbol{\varepsilon}^0$, $\boldsymbol{\kappa}^0$, $\boldsymbol{\chi}^0$ and the accelerations $\mathbf{a}$ via the unknown masses and damage extents by (see Eq.(6))

$$\boldsymbol{\varepsilon}^0 = (\mathbf{I} - \boldsymbol{\mu})\boldsymbol{\varepsilon}, \qquad \boldsymbol{\kappa}^0 = (\mathbf{I} - \boldsymbol{\mu})\boldsymbol{\kappa}, \qquad \boldsymbol{\chi}^0 = (\mathbf{I} - \boldsymbol{\mu})\boldsymbol{\chi}, \qquad \mathbf{p} = \mathbf{m}(\mathbf{g} - \mathbf{a}), \tag{18}$$

where $\boldsymbol{\mu}$ is an $n_d n_t \times n_t n_d$ block diagonal matrix, in which the ith diagonal matrix equals $\mu_i \mathbf{I}_{n_t \times n_t}$. Eq.(17) and Eq.(18) yield together the following linear system:

$$\left(\mathbf{I} - \begin{bmatrix} (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\varepsilon\varepsilon} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\varepsilon\kappa} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\varepsilon\chi} & (\boldsymbol{\mu}-\mathbf{I})\mathbf{D}^{\varepsilon\mathrm{m}}\mathbf{m} \\ (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\kappa\varepsilon} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\kappa\kappa} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\kappa\chi} & (\boldsymbol{\mu}-\mathbf{I})\mathbf{D}^{\kappa\mathrm{m}}\mathbf{m} \\ (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\chi\varepsilon} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\chi\kappa} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\chi\chi} & (\boldsymbol{\mu}-\mathbf{I})\mathbf{D}^{\chi\mathrm{m}}\mathbf{m} \\ \mathbf{D}^{\mathrm{m}\varepsilon} & \mathbf{D}^{\mathrm{m}\kappa} & \mathbf{D}^{\mathrm{m}\chi} & -\mathbf{D}^{\mathrm{mm}}\mathbf{m} \end{bmatrix}\right) \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa}^0 \\ \boldsymbol{\chi}^0 \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\varepsilon\mathrm{m}} \\ (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\kappa\mathrm{m}} \\ (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\chi\mathrm{m}} \\ \mathbf{D}^{\mathrm{mm}} \end{bmatrix} \mathbf{mg}, \tag{19}$$

where the unknowns are the virtual distortions and the accelerations of the moving masses. Thanks to the dynamic moving influence matrices, these key vectors can be obtained in each optimization step fast and with high accuracy, because (1) in practical cases (relatively small damages and masses) the matrix of Eq.(19) is well-conditioned square full rank; (2) all the data necessary to form Eq.(19) are computed based on the FE model, so without any measurement errors. That two factors make the proposed method of moving mass and damage identification robust to noise, in comparison to the methods based on the direct solution of Eq.(14).

6.4. Remarks and generalizations

For relatively small damages and masses, the coefficient matrix of Eq.(19) is full rank and well-conditioned; hence its inverse matrix can be in principle used directly. However, in off-line identification, in the case of a dense time discretization or a longer sampling time, Eq.Eq.(19) can become prohibitively large and computationally hardly manageable. To reduce the numerical costs, one can exploit the fact that the coefficient matrix is a block matrix composed of lower triangular matrices and that all the data in the equation are computed based on the ideal FEM model without any measurement errors. Thus, the unknown vectors can be computed stepwise, without a significant loss of accuracy. The equations and the unknowns in Eq.(19) can be rearranged into a block lower triangular form that can be quickly solved by block forward-substitution.

In addition, the optimization can be made quicker by specifying proper initial trial values $\tilde{m}_i$ of the moving masses. They can be estimated by approximating the masses by moving forces with constant values of $m_i g$ and by assuming that the bridge is undamaged, that is by solving

$$\mathbf{y}^{\mathrm{M}} = \mathbf{D}^{\mathrm{m}}\tilde{\mathbf{m}}\mathbf{g}. \tag{20}$$

Given the trial mass values, the mass unknowns in Eq.(16) can be rescaled into mass modification coefficients $\mu_i^{\mathrm{m}} = m_i/\tilde{m}_i$, so that all optimization variables are of comparable magnitudes, which usually makes the optimization quicker.

Furthermore, the proposed method of moving mass and damage identification can be straightforwardly extended to online identification by repetitive applications in a moving time window [14], that is by replacing the measured structural response $\mathbf{y}^{\mathrm{M}}$ in Eq.(16) by

$$\mathbf{y}^{\mathrm{M}} \leftarrow \mathbf{y}^{\mathrm{M}(n)} - \bar{\mathbf{y}}^{(n)}, \tag{21}$$

where $\mathbf{y}^{\mathrm{M}(n)}$ is the measured data in time section n and $\bar{\mathbf{y}}^{(n)}$ is the free vibration response of the undamaged structure caused by nonzero initial conditions at the beginning of the time section. The initial conditions of each time section and the corresponding free vibrations can be computed straightforwardly, provided the moving masses and the virtual distortions in the previous sections are already identified.

Certainly, if the system parameters are known, then the model of the damaged structure can be used directly, the virtual distortions in Eq.(16) vanish, and the method can be also used for robust identification of moving mass alone.

7.Numerical examples

A simply supported beam (Figure 2) is employed to validate the proposed method, with a uniform mass of 15.3×10^3 kg/m, length 100 m, Young's modulus 2.15×10^{11} N/m^2 and the inertia moment of 0.8 m^4. The beam is divided into 20 elements of equal lengths. Strain sensors are used, and placed on the upper face of the beam (that is, off the neutral axis). The two following cases are discussed:

1. One moving mass $m = 61.2 \times 10^3$ kg passes over the beam with a constant velocity 40 m/s. Element no 10 is damaged with the stiffness reduction ratio $\mu = \tilde{E}/E = 0.6$. A single sensor is located at 22.7 m, see Figure 2 (left).
2. Two masses $m_1 = 61.2 \times 10^3$ kg and $m_2 = 53 \times 10^3$ kg moving with constant velocities of $v_1 = 50$ m/s and $v_2 = -40$ m/s. The initial positions of the masses are $x_{1,0} = -10$ m and $x_{2,0} = 120$ m. Elements no 9 and 15 are damaged with the stiffness reduction ratios of $\mu_9 = 0.60$ and $\mu_{15} = 0.35$. Two sensors are employed: s1 at location 32.7 m and s2 at 57.7 m. The case is illustrated in Figure 2 (right).

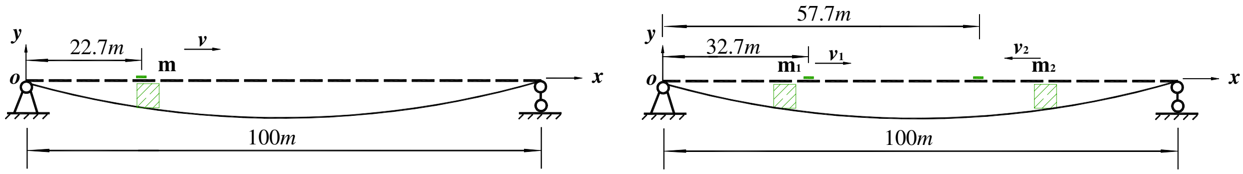

Figure 2: Moving mass-bridge coupled system: (left) case 1; (right) case 2

The respective dynamic responses of the sensors are calculated using the discrete FE model and the Newmark integration method with the parameters $\alpha = 0.25$ and $\beta = 0.5$. The integration step equals 0.01 s, thus the sampling frequency is 100 Hz. A total of 200 time steps is used, which corresponds to the sampling time interval of 2 s. Measurement errors of the simulated measurement data are modeled by an independent Gaussian noise at 5% rms level. The simulated sensor responses are shown in Figure 3.

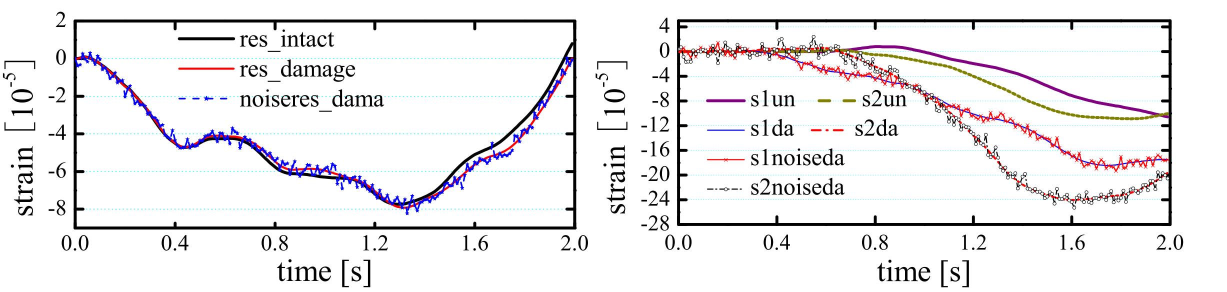

Figure 3: Strain responses of the damaged and the intact systems: (left) case 1, (right) case 2. Noise-free measurement, intact beam („res_damaged", „s1un", „s2un"); noise-free, damaged beam („res_damage", „s1da", „s2da"); noisy measurement, damaged beam („noiseres_dama", „s1noiseda", „s2noiseda")

In the following subsections, first the moving mass in case 1 is identified under the assumption that the damage is known and used directly in the FE model, then the moving masses and damage extents are identified simultaneously in the two considered cases. The identification results are assessed by their relative accuracy:

$$\delta = 100\% \times \frac{\|\text{estimated}_{\text{value}} - \text{actual}_{\text{value}}\|}{\|\text{actual}_{\text{value}}\|}.$$

7.1 Moving mass identification under known damage

Assume that the damage is known. The moving mass m is identified first by a direct solution of Eq.(14). The results are shown in Figure 4. They are computed using the truncated singular value decomposition (TSVD). The regularization level has been defined by the number k of the truncated singular values, which in each case has been determined using the L-curve technique [17]. The L-curves computed for the noise-free and the noise-polluted measurements are depicted in Figure 4 (top right and top left) and attest that the equation Eq.(14) is seriously ill-conditioned. Moreover, consistently high values of the regularizing parameter $\log_{10} \|\mathbf{Lp}\|$ suggest that it is impossible to get accurate results even at the optimal regularization level.

In the noise-free case, the optimum regularization level is $k = 7$. The corresponding computed moving force is shown in Figure 4 (bottom left); the end part diverges suddenly from the actual mass-equivalent moving force. With noise pollution, the force is computed at the optimal value $k = 40$ and shown in Figure 4 (bottom right); both the front and the end parts diverge largely from the actual values. Table 1 lists the values of the mass identified via the constructed moving force by the formula $\mathbf{p} = \mathbf{m}(\mathbf{g} - \mathbf{a})$. The identification errors confirm that the result can be very sensitive to the disturbances of the measured response.

In comparison, the method of optimization of Eq.(16) turns out to be robust to noise (Table 1). The trial value of the mass via Eq.(20) is $\tilde{m} = 61.778 \times 10^3$ kg; in each optimization step, the acceleration $\mathbf{a}$ is calculated fast using the moving influence matrix $\mathbf{D}^{\text{mm}}$ by Eq.(17), which reduces to $[\mathbf{I} + \mathbf{D}^{\text{mm}}\mathbf{m}]\mathbf{a} = \mathbf{D}^{\text{mm}}\mathbf{mg}$. The optimal value of the mass-related coefficient is $\mu_m = 0.995$. Figure 5 (left) compares the constructed moving load to the actual equivalent moving load; the result is very satisfactory under 5% rms noise pollution.

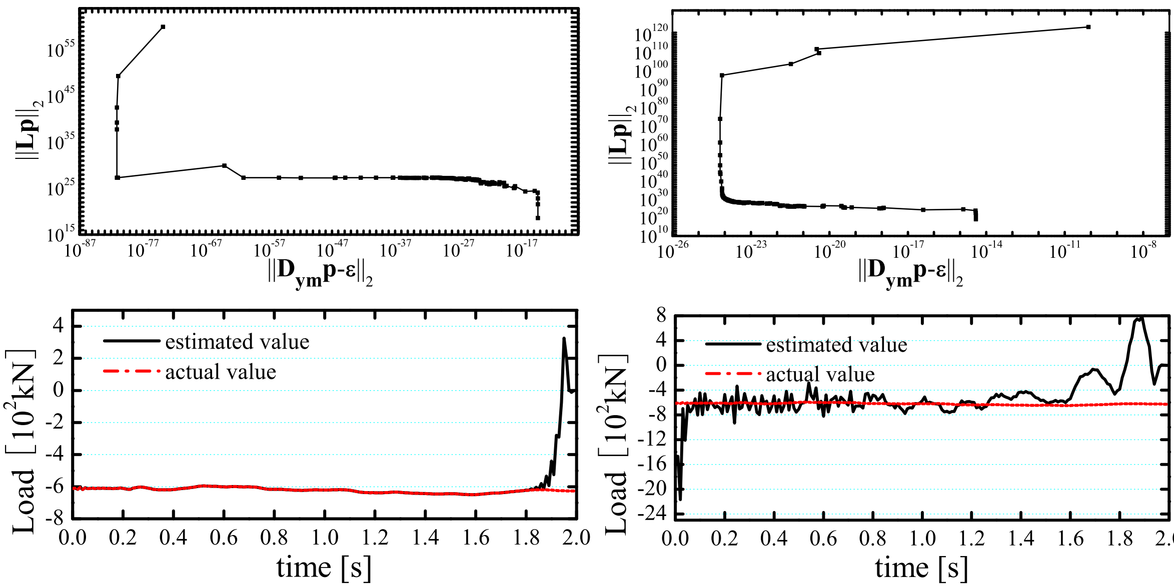

Figure 4: Case I, moving load computed by a direct solution of Eq.(14): (top left) noise-free L-curve; (top right) L-curve with 5% rms noise; (bottom left) computed moving load, noise-free case; (bottom right) computed moving load, 5% rms noise

Table 1: Case I, identified mass and relative error

	via solving Eq.(14)		via optimizing Eq.(16)
	noise free	5% noise	5% noise
mass [10^3 kg]	61.017	52.344	61.469
error δ [%]	0.30	14.47	0.44

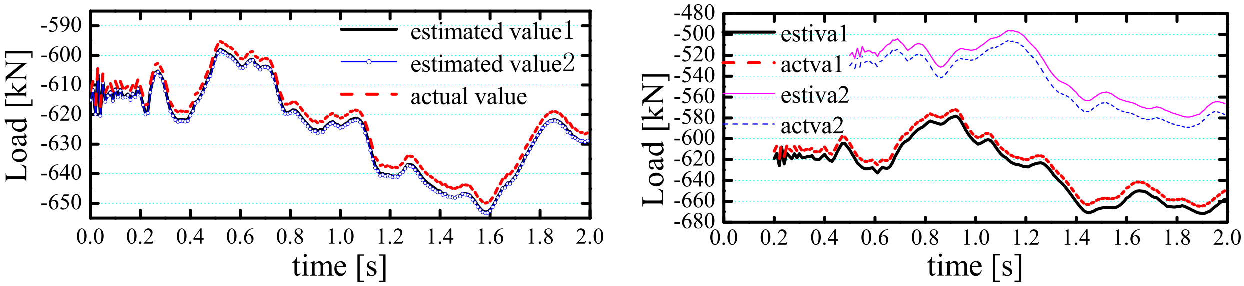

Figure 5: Comparison of the constructed equivalent moving force to the actual value: (left) case 1, ,,value1" is estimated assuming known damage extent, ,,value2" is estimated along with the unknown damage extent; (right) case 2, ,,actva1" (actual) and ,,estiva1" (estimated) correspond to m_1, while ,,actva2" and ,,estiva2" correspond to m_2

7.2 Simultaneous identification of moving masses and damage

Assume that the damage location is known, but the extents are unknown. The moving mass and the damage extent are identified by minimizing the objective function Eq.(16).

In case 1, the trial mass value of $m_0 = 62.561 \times 10^3$ kg is obtained by Eq.(20), and the identified mass modification coefficient is $\mu_m = 0.9833$. The relative identification errors are much less than 1%, as seen in Table 2. In case 2, the trial mass values are $m_1 = 43.461 \times 10^3$ kg and $m_2 = 76.509 \times 10^3$ kg. The corresponding optimal mass modification coefficients are $\mu_{m,1} = 1.4243$ and $\mu_{m,2} = 0.6798$. The relative identification error is less than 2% for masses and less than 7% for the damage extents. In addition, the equivalent moving forces corresponding to the identified masses and damage extents can be computed by $\mathbf{p} = \mathbf{m}(\mathbf{g} - \mathbf{a})$. The reconstructed moving forces are close to the actual, see Figure 5.

Table 2: Case II, identified masses, damage extents and relative errors

	case 1		case2			
	μ_{10}	$m[10^3$ kg$]$	μ_9	μ_{15}	$m_1[10^3$ kg$]$	$m_2[10^3$ kg$]$
identified	0.6188	61.516	0.5727	0.3261	61.902	52.011
actual	0.60	61.2	0.60	0.35	61.2	53
error δ [%]	3.13	0.52	4.55	6.82	1.15	1.87

8. Conclusion

Based on the virtual distortion method (VDM), this paper presents an effective method to simultaneously identify moving masses and structural damage. In comparison to other approaches, fewer sensors are necessary by taking the moving masses and damage extents as optimization variables, and the ill-conditioning of direct force identification is avoided. Furthermore, the proposed dynamic moving influence matrix makes the computation of the system response fast and precise without the need for repeated assembly of the time-variant mass matrix in every time step. Since the dynamic moving influence matrix needs to be computed only once for a certain beam and mass velocities, it affords an effective way to estimate the responses with different moving masses and damages. The proposed method is robust to noise and can be used both off-line and online by repetitive applications in a moving time window.

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