SUBSTRUCTURAL DAMAGE IDENTIFICATION USING TIME SERIES OF LOCAL MEASURED RESPONSE

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Abstract

This paper presents a Substructure Isolation method for substructural damage identification using time series of local measured response. Isolated Substructure is a virtual and independent structure which is numerically separated from the global structure by adding virtual supports on the substructure interface. The basic concept of the isolation method is that: first time series of substructural responses are divided into several sub-series with overlap; through the linear combination of all the sub-series, when the boundary response are constrained to zeros, the corresponding inner responses are the constructed responses of the Isolated Substructure; then the substructural damage identification can be performed equivalently by the modes of the Isolated Substructure which are identified from the constructed inner responses. Numerical model of a six-span truss and an experiment of a cantilever beam are used to validate the method. Both the isolation and damage identification are preformed very well using local measured responses.

Introduction

In recent years, Structure Health Monitoring (SHM) has become a widely researched field in civil engineering [Ou, 2005; Kołakowski, 2007]. For important complex structures, the global monitoring of the structure is hard to perform because of two main reasons: (1). its degree of freedoms (Dofs) is quite huge, while the number of sensors placed on the structure is comparatively limited; (2). Structures are complex and may be influenced by many unknown factors, like nonlinear. However in many practical applications, only local structures are crucial, so a local monitoring would be sufficient as well as advantageous with regard to easier implementation and less cost.

Aiming at these problems, the substructuring methods afford good approaches to identify the local damage using only local measured responses. Current substructuring methods usually separate the equation of motion of the concerned substructure from global structure, and estimate the damage of the substructure in time domain, as well as the exposed interface force [Yun and Lee, 1997; Koh and Shankar 2003; Yang and Huang, 2006]. To increase the optimization efficiency and avoid unnecessary estimation of the interface forces, the Substructure Isolation method (SIM) is proposed in [Hou *et al.*, 2010] using the local impulse response. In contrast with other substructure methods, the SIM is efficient and flexible, in which the interface force need not to be estimated, and further the substructural damage can be identified precisely via a virtual, small and independent Isolated Substructure. Hence all existing classical identification methods can be used for local damage identification, such as mode-based method. In the proposed SIM method, two kinds of responses should be measured for isolation, which require zero initial state. However, responses with ideal zero initial are hard to be measured in real application.

This paper employs the time series of local measured response instead of impulse responses for substructure isolation and identification, because the former doesn't need the zero initial state. First, the concerned substructure is separated form the global structure to be a called "Isolated Substructure", which is a small and independent structure, by adding virtual and numerical supports on substructure boundary. Here the isolation process is performed by constructing free responses of the Isolated Substructure only using the local time series of measured responses. Second, natural frequencies of the Isolated Substructure are identified by Eigensystem Realization Algorithms (EAR) method [Juang and Pappa, 1985] from constructed free responses, and then the substructure is identified equally through the Isolated Substructure. In this way, the damage extents can be optimized easily using traditional mode-based identification method, which minimizes the square distance between the Finite Element Modal (FEM) modes and the identified modes of the damaged Isolated Substructure.

A numerical example, a six-span truss, is first introduced to describe the proposed method. Then, an experiment of a cantilever beam, of which the upper part is chosen as the substructure, is used to validate the method. Both the isolation and the identification steps are performed very well using the local measured responses.

Substructure Isolation method using local time series

In the proposed method，two kinds of sensors are placed on the substructure to isolate the substructure from the global structure. Assume *l* degrees of freedom (Dofs) on the substructure interface, then *l* sensors should be placed in these Dofs， denoted by $b_1, b_2, \cdots, b_l$. Furthermore, *n* sensors are placed inside the substructure to obtain the basic information of the substructure, denoted by $1, 2, \cdots, n$.

Denote measured responses of *l* boundary sensors and *n* inner sensors respectively as $x(t)$ and $y(t)$, where $x(t) = \{x_1(t), x_2(t), \cdots, x_l(t)\}^T$ and $y(t) = \{y_1(t), y_2(t), \cdots, y_n(t)\}^T$. In real application, the measured responses are discrete. Let sampling frequency be f_s, then sampling time interval is $\Delta t = 1/f_s$, and the discrete responses of $x(t)$ and $y(t)$ are $x(t_i)$ and $y(t_i)$ where $t_i = i\Delta t \quad (i = 1, 2, 3, \cdots)$.

In order to isolate the substructure virtually using time series of measured responses, first define and collect *k* groups of sub-series, which are *series* 1, *series* 2, … , *series* k with overlap, denoted by s_1, s_2, … , s_k. The number of time steps in each sub-series is *w*, i.e. $s_i = \{t_1^i, t_2^i, \cdots, t_w^i\} (i = 1, 2, \cdots, k)$, where t_j^i is the *j*th time step of *i*th sub-series, and $t_1^1 < t_1^2 < \cdots < t_1^k$.

Select necessary data according to the sub-series s_i from measured responses of the boundary sensor and inner sensor. Let $X_{ji} = x_j(s_i) = \{x_j(t_1^i), x_j(t_2^i), \cdots, x_j(t_w^i)\}$, where X_{ji} is the *i*th sub-series of the *j*th boundary sensor responses; $Y_{ji} = y_j(s_i) = \{y_j(t_1^i), y_j(t_2^i), \cdots, y_j(t_w^i)\}$, where Y_{ji} is the *i*th sub-series of the *j*th inner sensor.

Then all boundary responses regard to each sub-series are rearranged into one vector $\tilde{X}_i = \{X_{1i}, X_{2i}, \cdots, X_{li}\}^T$, where $\tilde{X}_i$ is vector with the dimension of *lw*, and *i*=1,2,⋯,*k*. Similarly, all inner responses regard to each sub-series are rearranged into one vector $\tilde{Y}_i = \{Y_{1i}, Y_{2i}, \cdots, Y_{ni}\}^T$, where $\tilde{Y}_i$ is vector with the dimension of *nw*, *i*=1,2,⋯,*k*. Through the linear combination of the above *k* vectors $\tilde{X}_i$, and $\tilde{Y}_i$ respectively, there exists combined responses

$$\begin{cases} U_X = \sum_{i=1}^{k-1} \alpha_i \tilde{X}_i + \tilde{X}_k \\ U_Y = \sum_{i=1}^{k-1} \alpha_i \tilde{Y}_i + \tilde{Y}_k \end{cases} \tag{1}$$

If the combined responses of boundary sensors are zeroed, i.e. $U_X = 0$, the boundary of the substructure can be considered as being constrained, like adding some virtual support on the boundary. In this way, influences on the substructure from its outside are isolated and removed. Then the substructure is numerically isolated from the global structure to be an independent structure, that is Isolated Substructure, which has the same physical parameters as the substructure, and hence can be used for substructure identification. The corresponding combined responses of inner sensors U_Y are the responses of the Isolated Substructure.

Denote $\mathbf{A} = \left[\tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_{k-1} \right]$, $\mathbf{B} = \left[\tilde{X}_k \right]$, $\mathbf{C} = \left[\tilde{Y}_1, \tilde{Y}_2, \cdots, \tilde{Y}_{k-1} \right]$, $\mathbf{D} = \left[\tilde{Y}_k \right]$, $\boldsymbol{\alpha} = \left\{ \alpha_1, \alpha_2, \cdots, \alpha_{k-1} \right\}^T$, and define matrix **A** and **C** as the constraining response, while vector **B** and **D** as the basic response, then we have

$$\begin{cases} 0 = \mathbf{A}\boldsymbol{\alpha} + \mathbf{B} \\ \mathbf{D}_s = \mathbf{C}\boldsymbol{\alpha} + \mathbf{D} \end{cases} \tag{2}$$

In order to make $\boldsymbol{\alpha}$ satisfy the equation $\mathbf{A}\boldsymbol{\alpha} + \mathbf{B} = 0$, that is to zero the boundary responses of the substructure, let matrix **A** be row full rank matrix, that is $R\left(\mathbf{A}\right) = lw$, where $R\left(\bullet\right)$ expresses the rank of matrix $\left(\bullet\right)$. Because the dimension of **A** is $lw \times \left(k-1\right)$, so the number of its column $k-1$ must be no less than the number of its row lw. Here *k* is the number of the considered sub-series. Therefore, *k* is chosen a bit bigger than $\left(lw+1\right)$ for isolation, and then $\boldsymbol{\alpha}$ can be computed by

$$\boldsymbol{\alpha} = -\mathbf{A}^{+}\mathbf{B} \tag{3}$$

The above analysis shows that generally **A** is a singular matrix, so its generalized inverse matrix $\mathbf{A}^{+}$ is computed by the singular value decomposition (SVD) .Substitute Eq.(3) into Eq.(2), then vector $\mathbf{D}_s$, i.e. the constructed response of the Isolated Substructure is

$$\mathbf{D}_s = \mathbf{D} - \mathbf{C}\mathbf{A}^{+}\mathbf{B} \tag{4}$$

Substructure damage identification

From the above derivation, it tells that the substructure damage identification can be performed equivalently via the identification of Isolated Substructure, since they have the same physical parameters as the substructure. Assume no excitation applied in the inner substructure, the constructed response D_s can be taken as the free response of the Isolated Substructure. Therefore, the modes of the damaged Isolated Substructure, including its natural frequencies $\tilde{\omega}_i$ and mode shapes $\tilde{\varphi}_i$, can be identified by Eigensystem Realization Algorithms (EAR). So the local damage extent μ of the substructure can be identified by minimizing the following objective function:

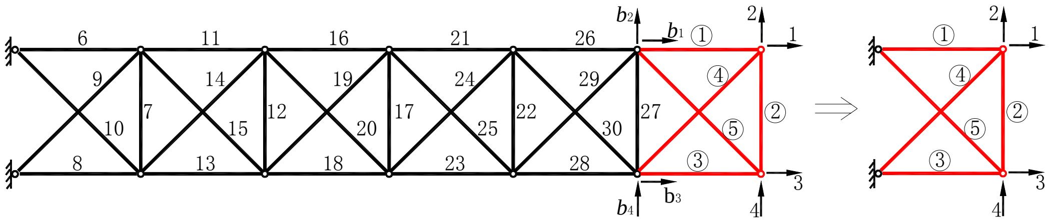
$$\tilde{\eta}(\mu) = \sum_i \left| \frac{\tilde{\omega}_i - \hat{\omega}_i(\mu)}{\tilde{\omega}_i} \right|^2 + \sum_i \gamma_i \left| 1 - MAC\left(\tilde{\varphi}_i - \hat{\varphi}_i(\mu)\right) \right|^2 \tag{5}$$

where $\hat{\omega}_i(\mu)$ and $\hat{\varphi}_i(\mu)$ are respectively the *i*th natural frequency and mode shape of numerical model of the Isolated Substructure to the given damage extent μ; γ_i is the weighting factor of the *i*th mode shape error which is computed using Modal Assurance Criterion (MAC).

Numerical example

A plane truss model

A plane truss structure with 6 spans shown in Figure 1(a) is taken as an example to test the application of the proposed methodology of substructure isolation and local damage identification. The part with 5 elements on the free end is the concerned substructure. Eight acceleration sensors are placed on the substructure, see Figure 1(b), among which four sensors are placed on the substructure boundary, and the rest four sensors are placed inside the substructure. The virtual supports can be constructed by measured time series of the four boundary sensors, so the substructure can be separated from the global structure to be the Isolated Substructure, see Figure 1(c).



a) truss with 6 span b) sensor placement c) isolated substrucutre

Figure 1. A plane truss with six span, sensor placements and Isolated Substructure

Table 1. The first 16 natural frequencies of the intact plane truss (Hz)

order	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
frequency	10.9404	56.6306	69.1922	133.4951	209.7619	221.8867	313.3409	352.285
order	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
frequency	387.8706	410.5971	429.4148	440.6975	453.7782	501.3869	505.8725	526.8817

Table 2. Natural frequencies of the intact Isolated Substructure(Hz)

Order	*1*	*2*	*3*	*4*
Frequency	179.548	406.572	501.524	651.648

The entire truss is 12 m long and 2m high, originally made of steel with density 7800kg/m^3 and Young's modulus 210GPa. The cross-sections of the truss bars are 210cm^2. The Rayleigh damping model is assumed with both damping ratios equal to 1%. The first 16 natural frequencies of the intact truss are shown in Table 1, and the natural frequencies of intact Isolated Substructure are shown in Table 2.

All the element damage extents (ratio of the modified element stiffness to the original value) of the entire truss (Figure 1(a)) are shown in Figure 2. The first 5 values are regard to the concerned substructure and are to be identified using the proposed method based on the local time series of measured responses.

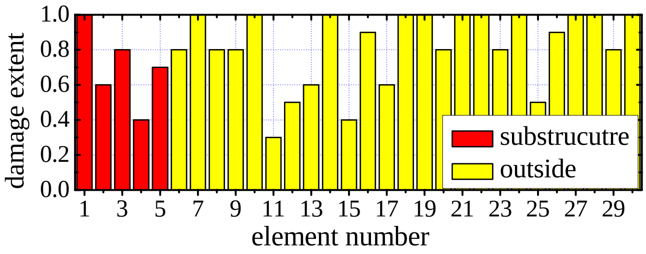


Figure 2. Actual damage extent of the plane truss

Substructure isolation and identification

Apply random white noise on each Dofs of all the nodes except the Dofs inside the substructure. The corresponding response of boundary and inner sensors are shown respectively in Figure 3 and Figure 4. The sampling frequency is 10000 Hz.

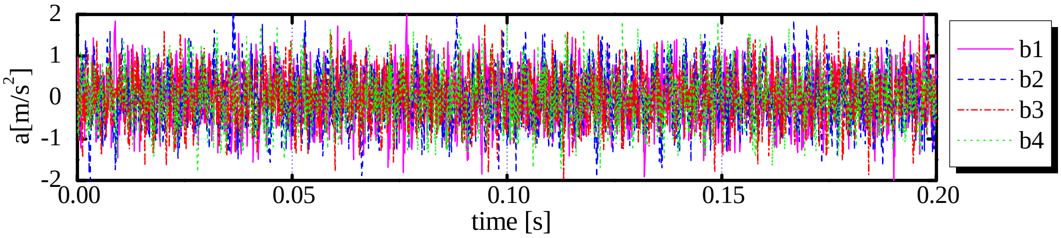


Figure 3. time series of substructure boundary sensors

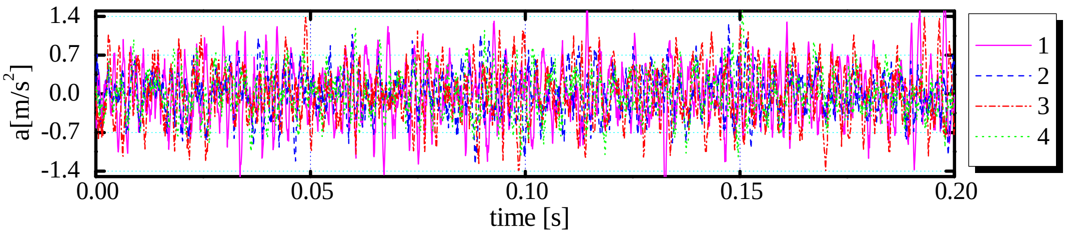


Figure 4. time series of substructure inner sensors

500 groups of sub-series are collected form the response of Figure 3 and Figure 4, and each sub-series contains 120 (w=120) time steps. The overlap of the two next sub-series is 118 time steps. According to Eq.(2), when the responses of boundary sensor (b_1,b_2,…,b_4) are constrained to zero by linear combination of all the 500 sub-series, the same linear combination of responses of inner sensors (1,2,…,4) belong to the free response of Isolated Substructure, see Figure 5.

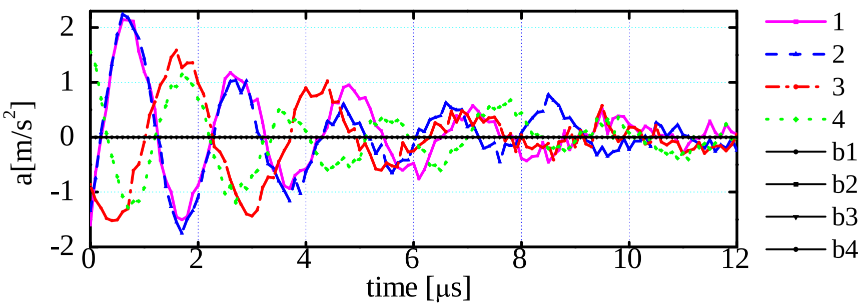


Figure 5. Constructed free responses of the Isolated Substructure

Natural frequencies and mode shapes of the damaged Isolated Substructure can be identified using ERA method by the constructed free response, see Table 3 and Table 4 respectively, where natural frequencies and mode shapes computed from the numerical model of the intact and damaged Isolated Substructure are also listed. It can be seen that the identified values are very close to the actual values, which proves that responses of the Isolated Substructure can be constructed accurately from the local measured responses of the global structure.

Table 3. Natural frequencies of the Isolated Substructure (Hz)

order	intact	actual	Identified	error
1	179.5475	141.9209	131.8341	-7.11%
2	406.5722	358.8842	357.4369	-0.40%
3	501.5236	445.8067	445.6464	-0.04%
4	651.6481	507.5158	502.8965	-0.91%

Table 4. The mode shapes of Isolated Substructure

order	1		2		3		4	
Dof.	Actual	Identified	Actual	Identified	Actual	Identified	Actual	Identified
1	0.1474	0.2543	0.4099	0.4135	0.7745	0.7743	-0.3727	-0.3608
2	-0.6725	-0.6448	-0.1947	-0.1789	-0.1554	-0.237	-0.7004	-0.7096
3	-0.1217	-0.1683	0.8796	0.879	-0.5136	-0.4359	0.010	-0.0237
4	-0.715	-0.7009	0.143	0.1565	0.3351	0.3928	0.6086	0.6047
1-MAC	1.45E-02		4.44E-04		1.60E-02		1.37E-03	

Then the damage extents of the Isolated Substructure (Figure 1(c)) can be optimized by minimizing the objective function Eq.(5) via the identified modes in Table 3 and Table 4. The identified substructural damages are shown in Figure 6, which is accurate.

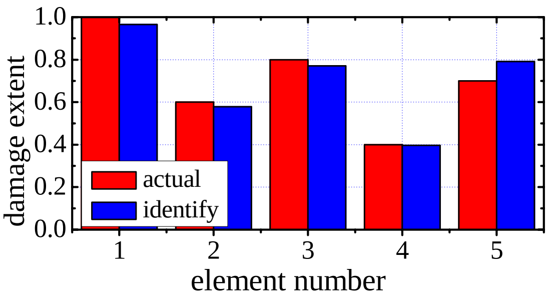


Figure 6. Identified damage extent

Experiment

An aluminum cantilever beam(136.15 cm long, cross-section 2.7 cm×0.31 cm), of which the upper part is the concerned substructure(79.4 cm long), is used for experimental verification, see Figure 7. Three strain sensors are placed on the substructure, of which the sensor 3 is placed on the boundary. The boundary velocity is measured by laser vibrometer.

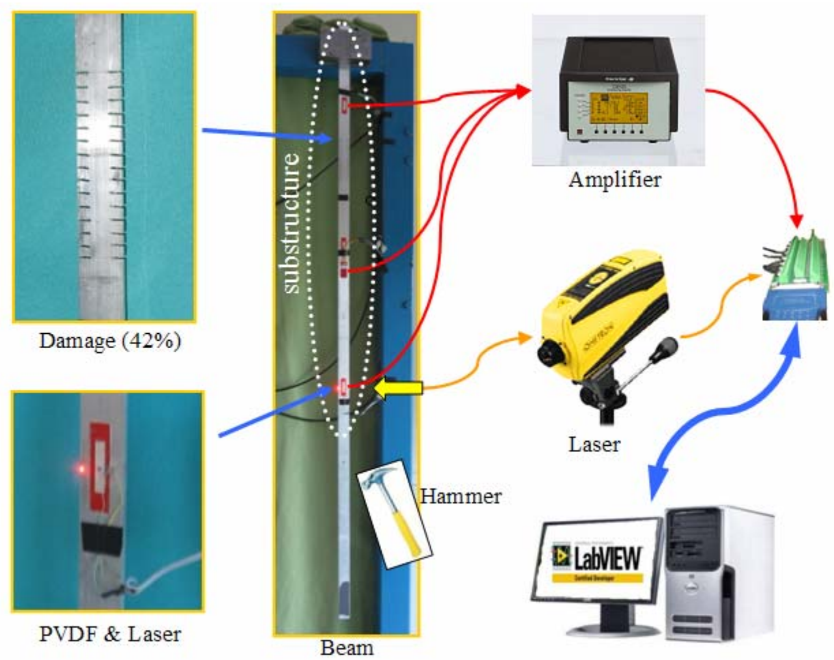


Figure 7. Experimental setup

The single virtual pinned support placed on the substructure boundary is constructed by constraining the responses of the velocity sensor and the strain sensor 3 to zero, then the substructure can be isolated from the global system, see Figure 8. The other two strain sensors ‘strain 1’ and ‘strain 2’ were placed inside the substructure to identify its damage. In order to identify the location and extent of the damage, the Isolated Substructure is divided into five parts (Figure 9), of which the real damage extents are [1 0.42 1 1 1].

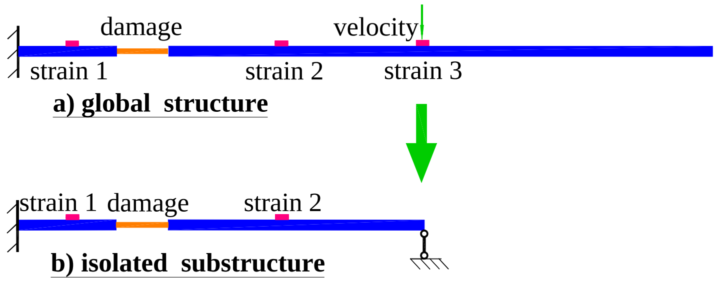


Figure 8. The isolation of the substructure

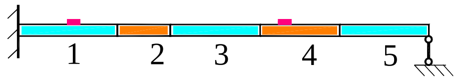


Figure 9. Division of the Isolated Substructure into five parts

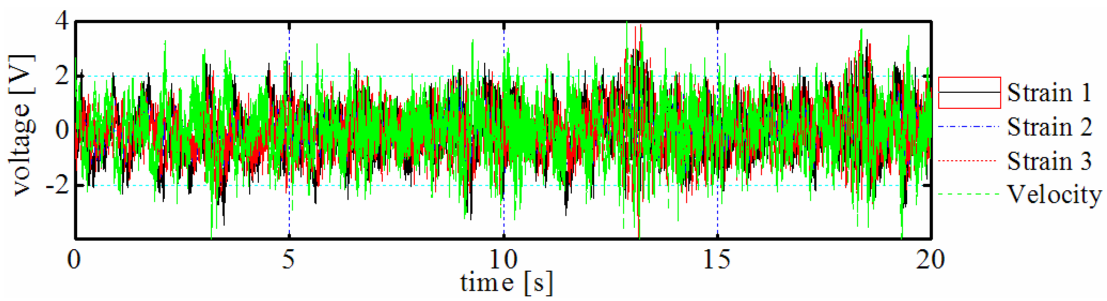


Figure 10. The time series of substructure excited by random hammer impact

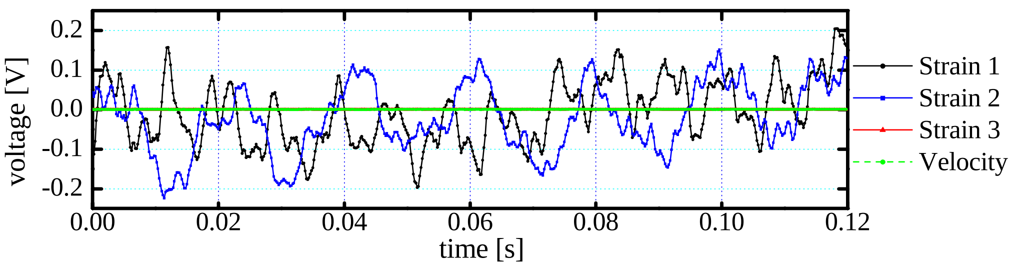


Figure 11. Constructed free response of Isolated Substructure

Responses of the four sensors (Figure 10) are excited by hammer randomly on the outside of the substructure. The sampling frequency is 10000Hz, and the measured time is 20s. The free responses of the Isolated Substructure(Figure 9) can be constructed from the measured responses by Eq. (2) which are shown in Figure 11. In the experiment, only natural frequencies are used to identify the damage, so the measured voltage signals needn't to be calibrated to stains or velocity.

Then natural frequencies of the Isolated Substructure are identified using ERA method by the constructed free responses, see Table 5. Damages of the substructure are then identified by minimizing the square distance between the constructed natural frequencies of the Isolated Substructure and the natural frequencies computed using its Finite Element model under given damage extents, see Figure 12. It shows that both damage extents and locations can be identified very well using the proposed method.

Table 5. Natural frequencies of the Isolated Substructure (Hz)

order		1	2	3	4	5	6	7
FEM	intact	17.6848	57.3318	119.1544	203.2967	310.4712	439.9466	592.4764
	damaged	17.5192	52.0069	112.9491	195.6606	290.0367	413.9329	551.0665
Identified		17.5297	51.6593	112.5637	193.58	290.3337	415.0462	547.618

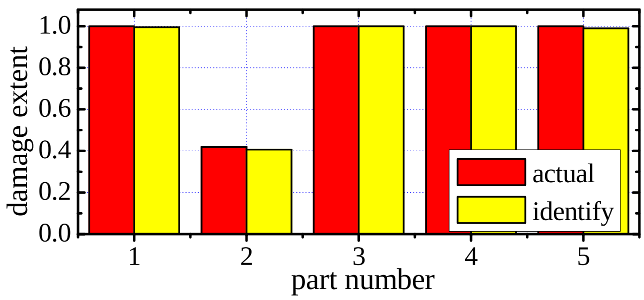


Figure 12. Identified Damage extents

Conclusions

A Substructure Isolation Method is presented for substructural damage identification using time series of local measured response. A numerical example of a truss model and an experiment of a cantilever beam have verified that the proposed method is efficient and flexible for local substructure monitoring. Conclusions are summarized as following:

(1). in the procedure of the substructure isolation, the basic response and constraining response can be selected with overlap from only one local time series. In this way, the measured data are used efficiently which is very practical in real application.

(2). after isolation, substructural damages can be identified by the existing classical identification methods, which is easy to perform.

(3). the requirement of measuring all the boundary responses is the limitation of this method, which needs to be overcome in the future study.

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