MODEL-FREE IDENTIFICATION OF STRUCTURAL DAMAGES

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1. Introduction

This work presents and verifies experimentally a model-free methodology for off-line identification of structural damages. The Virtual Distortion Method (VDM) [1] is used, which allows the structure to be modeled locally in an essentially non-parametric way, so that no error-prone parametric modeling is necessary. The damage is modeled using damage-equivalent pseudo-loads, which are convolved with the experimentally obtained responses of the original structure to compute the response of the damaged structure. A related approach has been earlier used for model-free impact and mass identification [2, 3]. An effective sensitivity analysis is possible via the adjoint variable method.

2. Response of the damaged structure

Let the original undamaged structure obey the following equation of motion:

$$\text{(1)} \quad \mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{C}\dot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{K}\mathbf{u}^{\mathrm{L}}(t) = \mathbf{f}(t),$$

where $\mathbf{f}(t)$ is a given excitation. Let the damage be described by modifications $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$ to its mass and stiffness matrices and use $\mathbf{u}(t)$ to denote the response of the damaged structure to $\mathbf{f}(t)$,

$$\text{(2)} \quad [\mathbf{M} + \Delta\mathbf{M}]\,\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + [\mathbf{K} + \Delta\mathbf{K}]\,\mathbf{u}(t) = \mathbf{f}(t).$$

The VDM [1] can be used to model a damage defined this way with a response-coupled pseudo-load $\mathbf{p}^0(t)$, which acts in the unmodified structure to imitate the effects of the damage:

$$\text{(3)} \quad \mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{p}^0(t), \qquad \text{where} \qquad \mathbf{p}^0(t) = -\Delta\mathbf{M}\ddot{\mathbf{u}}(t) - \Delta\mathbf{K}\mathbf{u}(t).$$

Therefore, the response of the modified structure to the load $\mathbf{f}(t)$ can be expressed in terms of the convolution with the system impulse-responses:

$$\text{(4)} \quad \mathbf{u}(t) = \mathbf{u}^{\mathrm{L}}(t) + \int_0^t \mathbf{B}^0(t-\tau)\mathbf{p}^0(\tau)\,d\tau \qquad \text{and} \qquad \ddot{\mathbf{u}}(t) = \ddot{\mathbf{u}}^{\mathrm{L}}(t) + \int_0^t \ddot{\mathbf{B}}^0(t-\tau)\mathbf{p}^0(\tau)\,d\tau,$$

where $\mathbf{B}^0$ and $\ddot{\mathbf{B}}^0$ contain the impulse-responses. Equations (4) can be stated in the operator form as $\mathbf{u} = \mathbf{u}^{\mathrm{L}} + \boldsymbol{\mathcal{B}}^0\mathbf{p}^0$ and $\ddot{\mathbf{u}} = \ddot{\mathbf{u}}^{\mathrm{L}} + \ddot{\boldsymbol{\mathcal{B}}}^0\mathbf{p}^0$. A substitution of (4) into the second equation of (3) yields the following system of Volterra linear integral equations with the unknown pseudo-load vector $\mathbf{p}^0$:

$$\text{(5)} \quad \mathbf{p}^0 + \left[\Delta\mathbf{K}\boldsymbol{\mathcal{B}}^0 + \Delta\mathbf{M}\ddot{\boldsymbol{\mathcal{B}}}^0\right]\mathbf{p}^0 = -\Delta\mathbf{K}\mathbf{u}^{\mathrm{L}} - \Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}},$$

which can be proved to be uniquely solvable, if the matrix $\mathbf{M} + \Delta\mathbf{M}$ is non-singular. Given $\mathbf{p}^0$, the response of the damaged structure can be computed by (4). The impulse-responses need to be measured only in the degrees of freedom (DOFs) related to the potential modifications, as in other DOFs the pseudo-loads vanish. The measurements are especially straightforward in skeletal structures.

In reality, only responses to excitations that last several steps can be measured. These responses can be also used, provided the pseudo-loads are expressed in the form of the following convolution:

$$\text{(6)} \quad p_i^0(t) = (q_i \star p_i)(t) = \int_0^t q_i(t-\tau)p_i(\tau)\,d\tau,$$

where $q_i(t)$ is the actually applied non-impulsive excitation in the ith DOF that has to satisfy $q_i(t) < 0$ for $t < 0$ and $p_i(t)$ is a certain unknown function. Equation (6), collected for all involved DOFs and stated in the operator notation, takes the following form:

$$(7) \quad \mathbf{p}^0 = \boldsymbol{\mathcal{Q}}\mathbf{p},$$

where $\boldsymbol{\mathcal{Q}}$ is the corresponding diagonal matrix convolution operator. Substitute (7) into (4) and (5),

$$(8) \quad \mathbf{u} = \mathbf{u}^{\mathrm{L}} + \boldsymbol{\mathcal{B}}\mathbf{p}, \qquad \ddot{\mathbf{u}} = \ddot{\mathbf{u}}^{\mathrm{L}} + \ddot{\boldsymbol{\mathcal{B}}}\mathbf{p}, \qquad \left[\boldsymbol{\mathcal{Q}} + \Delta\mathbf{K}\boldsymbol{\mathcal{B}} + \Delta\mathbf{M}\ddot{\boldsymbol{\mathcal{B}}}\right]\mathbf{p} = -\Delta\mathbf{K}\mathbf{u}^{\mathrm{L}} - \Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}},$$

where $\boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{B}}^0\boldsymbol{\mathcal{Q}}$ and $\ddot{\boldsymbol{\mathcal{B}}} = \ddot{\boldsymbol{\mathcal{B}}}^0\boldsymbol{\mathcal{Q}}$, so that, contrary to (4) and (5), all data necessary to form (8) can be directly measured.

3. Damage identification

The inverse problem amounts to a minimization of the following objective function:

$$(9) \quad F(\Delta\mathbf{M}, \Delta\mathbf{K}) = \frac{1}{2}\int_0^T \|\mathbf{d}(t)\|^2\, dt \qquad \text{where} \qquad \mathbf{d}(t) = \mathbf{u}^{\mathrm{M}}(t) - \mathbf{u}(t),$$

which is the mean-square discrepancy between the measured and the computed responses of the damaged structure. It is minimized with respect to a chosen set of parameters that define the damage via the modifications $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$. The adjoint variable method can be used for fast sensitivity analysis: for example, the derivative of F with respect to the αth parameter can be computed as

$$(10) \quad F_\alpha(\Delta\mathbf{M}, \Delta\mathbf{K}) = \int_0^T \boldsymbol{\lambda}^{\mathrm{T}}(t)\left[\Delta\mathbf{K}_\alpha\mathbf{u}(t) + \Delta\mathbf{M}_\alpha\ddot{\mathbf{u}}(t)\right] dt,$$

where $\boldsymbol{\lambda}(t)$ is the vector of the adjoint variables, defined as the solution to

$$(11) \quad \left[\boldsymbol{\mathcal{Q}}^{\mathrm{T}} + \boldsymbol{\mathcal{B}}^{\mathrm{T}}\Delta\mathbf{K} + \ddot{\boldsymbol{\mathcal{B}}}^{\mathrm{T}}\Delta\mathbf{M}\right]\boldsymbol{\lambda} = \boldsymbol{\mathcal{B}}^{\mathrm{T}}\mathbf{d}.$$

In a similar way, a fast second-order sensitivity analysis is also possible.

4. Experimental verification

The approach has been verified experimentally using a 4-meter long 3D truss structure with 70 elements. Due the space constraints of this abstract, the results will be presented in the conference.

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6. References

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