Self-consistent mean-field approximation in a coordinate representation

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The rich structure of microscopic quantum many-body systems (metalic, clusters, atoms, and nuclei) can be well described within a simple model; namely, *the deformed oscillator* and *the mean-field approximation*. The key observation is the *shell structure* exhibited by the energy eigenspectrum. The filling of eigenstates by spin-$^1/_2$ particles follow the rules of atomic physics; including shell-filling and Hund's rule. The number of electrons that can occupy any given state is limited by Pauli's exclusion principle. An anisotropic oscillator confinement potential in two-dimensions ($q_1 \perp q_2$) is

$$V(q) = m^2\omega^2\left(\delta q_1^2 + \frac{1}{\delta}q_2^2\right), \tag{1}$$

as an empirical mean-field potential in which N spin-$^1/_2$ particles with mass m are assumed to move independently. Imposing the constraint $\omega^2 = \omega_1\omega_2$ conserves the area of the ellipsoidal with respect to changes in the deformation ratio $\delta = {}^{\omega_1}/_{\omega_2}$. The following quantum conditions hold:

$$[H, H_1] = 0\,, \quad [H, H_2] = 0\,, \quad [H_1, H_2] = 0\,, \tag{2}$$

and the corresponding single-particle eigenspectrum is

$$\epsilon(n_1, n_2) = \hbar\omega\left[\sqrt{\delta}\left(n_1 + \frac{1}{2}\right) + \frac{1}{\sqrt{\delta}}\left(n_2 + \frac{1}{2}\right)\right]. \tag{3}$$

The isotropic case, $\delta = 1$, corresponds to an $(n_1 + n_2 + 1)$-fold degeneracy of the quantum numbers $n_1, n_2 \in \{0, 1, 2, \ldots\}$. It is shown how the *self-consistent solution* to the many-body problem (*cf.* Eqn. (1)) is obtained in a coordinate representation within the framework `deal.II`.