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Scattering model of trabecular bone

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Abstract

In our previous study we have developed the simulation technique that enables determination of the ultrasound signal received at the pulse-echo transducer surface after interrogation of cancellous bone. The simulation can be applied for different scattering models of a trabecular structure. In this study we examined newly developed scattering models of the trabecular bone for their abilities to mimic the frequency dependent backscattering coefficient measured in the cancellous bone. Three types of trabeculae mimicking scatterers were considered. First the bone consisted of cylinders with varying thickness (Gamma distributed) within the population, was assumed. The next two cases accounted for the contribution of thick and thin trabeculae to the total backscattered signal. The second model assumed existence of two populations of the cylindrical scatterers significantly differing in the average value of Gamma distributed diameters. Finally, the mixed model composed of thick and thin trabeculae modeled respectively by cylindrical and spherical scatterers was examined. The last selection resulted from the similarity found between scattering on small sphere and finite cylinder. Calculated echoes demonstrated the usefulness of the mixed model. Frequency dependence of backscattering coefficient agreed well with the experimentally determined dependences..

1. INTRODUCTION

Signals that have been scattered in trabecular bone contain information about the properties of the bone structure. Therefore scattering-based ultrasonic methods potentially enable the assessment of microscopic structure of bone [1].

Theoretical studies of ultrasonic scattering by trabecular bone were performed by Wear [2]. The model of a bone, proposed by Wear, consisted of a random space-distribution of long identical cylinders with a diameter much smaller than the wavelength, aligned perpendicularly to the acoustic beam axis. Therefore the scattering by trabecular bone was modeled as scattering of a plane wave by elastic cylinders.

In our previous study [3] we have developed a simulation technique that enabled determination of the ultrasound signal received at the pulse-echo transducer surface after interrogation of cancellous bone. The simulation can be applied for different scattering models of a trabecular structure. Mimicking the bone structure similarly to Wear's model, by a random distribution of long elastic cylinders but allowing for variations of it's physical parameters we have found the departures from the Rayleigh statistics of the scattered signal instantaneous amplitude (envelope) as the variation of cylinders diameters increased [4]. But we have also found that the frequency dependence of the power backscattered coefficients calculated using modeled signals differed from the published experimental results [5]. Whereas these results show the frequency exponent exceeding the value of 3, in the case of simulated results the dependence was below the cubic one.

The presented investigations were intended to compare the results obtained with the cylindrical model and with the two newly developed models of trabecular bone structure. Both new models accounted for the contribution of thick and thin trabeculae to the total backscattered signal. As in the previous study the frequency dependent backscattering coefficient of the modeled bone structure was determined and analyzed.

2. MODELS OF TRABECULAR BONE

All three models of the trabecular bone structure were simulated as a collection of scatterers (infinitely long, elastic cylinders aligned perpendicularly to the ultrasound beam axis and spherical scatterers), which were randomly (uniformly) distributed in water. Mean values for the mechanical parameters and mean values and standard deviations for the diameters of the trabeculae were determined experimentally using an acoustic microscope [6], or, when needed, were collected from literature. The mechanical properties of scatterers were assumed to be constant and similar to the bone tissue properties.

The first new model assumed the existence of two populations of the cylindrical scatterers significantly differing in the mean diameter. The second one was called the *mixed model* as it was composed of thick and thin trabeculae modeled respectively by cylindrical and spherical elastic scatterers (fig.1.).

Variations in the structural properties of cancellous bone were modeled by changing the mean value and variance of scatterers' diameters. For all bone models the same density value (amount of scatterers per unit volume) of thick trabeculae equal to $\rho = 3/mm^3$ was used. This density value is equivalent to 86% porosity calculated for a constant length (4 mm) and diameter (0.12 mm) of trabeculae. For the thin trabeculae the density $\rho = 9/mm^3$ was assumed.

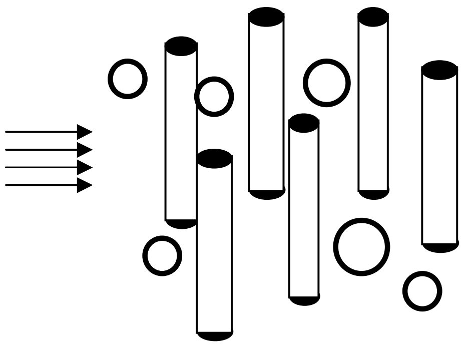


Fig. 1. Trabecular bone model consisting of two populations of cylindrical and spherical scatterers

In the *one population model* three values of mean diameters <d> of the trabecula mimicking cylinders equaled 0.05 mm, 0.12 mm and 0.2 mm were considered. In the *two populations model* (fig. 1) three pairs of the cylinder's mean thickness of 0.05 mm and 0.033 mm, 0.12 mm and 0.08 mm and 0.2 mm and 0.13 mm for modeling respectively, thick <d> and thin <d1> trabeculae diameters were used. In the *mixed model* (fig. 2) the thick trabeculae properties remained unaltered while the spheres diameters <d1> were equal to 0.05 mm, 0.1 mm and 0. 14 mm, respectively.

The chosen values of the mean diameters of the sphere assured that the amplitudes of the backscattered signal (at 1 MHz) from the sphere and from the finite, 0.65 mm long thin cylinder [7] of the diameter corresponding respectively to the mean diameters of the thin trabeculae from *two population model* were the same.

The variation of trabeculae thickness was described by the thickness SNR coefficient (tSNR) defined as a ratio of the mean thickness to the standard deviation of thickness (tSNR = $\langle d\rangle/\sigma(d)$) and was equal to 2.7 and 3.2 for simulating thick and thin trabeculae, respectively [8].

3. SPHERICAL MODEL OF THIN TRABECULA

The choice of a spherical scatterer for the simulation of a thin trabecula resulted from an exact calculation of the pressure field for the scattering on the finite thin cylinder. The calculations were performed by solving numerically the integral form of the Sturm-Liouville equation that describes scalar wave in inhomogeneous media [9]. The results indicated similarity to the field scattered on the sphere. Also, the frequency (f) dependence of the backscattering coefficient calculated for the scattering on thin cylinders used in the *two population model* was close to the power of 4 that is characteristic to the scattering on the sphere (fig. 2).

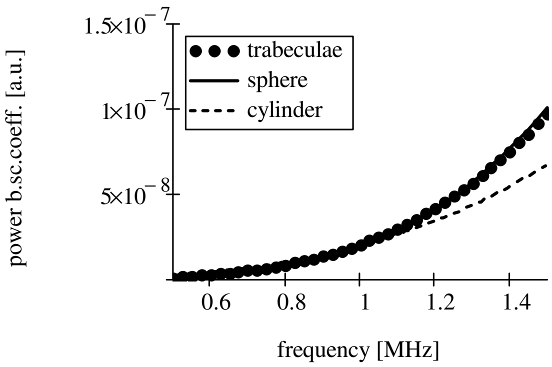


Fig. 2. Frequency dependence of the power backscattered coefficient calculated for the finite length cylinder (0.65 mm long, 0.05 mm diameter), infinite thin cylinder and sphere

4. MODELING THE BACKSCATTERED SIGNAL

In order to simulate the signal received at the transducer we have applied the 1D geometry model as a direct consequence of the 180° angular scattering (backscattering) assumption. All simulations were carried out with a transmitted pulse waveform identical to the pulse emitted by a pulse-echo transducer used to collect the empirical densitometric data.

The backscattered signal that was received at the pulse-echo transducer surface after scattering in trabecular bone structure was simulated in the following way. First, each scatterer (trabecula) was considered as a secondary source of an ultrasound wave. Then, each scatterer's backscattering coefficient was modified by a field coefficient that depended on the position of the scatterer considered and the distribution of the field generated by the pulse-echo transducer. Next, the spectrum of each individual pulse scattered from each cylinder was obtained as a product of the emitted pulse spectrum and the modified, complex backscattering coefficient of the scatterer. The received signal was simulated by superposition of all of the elementary, individually scattered pulses, taking into account the phase differences caused by each scatterer spatial location associated with the individual cylinder or sphere. The spectrum of the simulated signal was band limited according to the experimentally determined transfer function of the transducer.

5. RESULTS

Analysis of the simulated transducer responses to bone backscattered waves was performed in order to calculate the

Tab. 1. The exponents (n) of the power-law function and the coefficients (in brackets) describing the quality of fit of the frequency-dependent backscattered coefficient determined for three bone models

thick scatterers diameters	Cylindrical model	Two population cylindrical model – thin scatterers diameter and (n)	Two population mixed model - thin scatterers diameter and (n)
<d>=0.05mm	**2.587** (0.998)	<d1>=0.033mm, **2.978** (0.988)	<d1>=0.05mm, **3.853** (0.978)
<d>= 0.12 mm	**1.789** (0.991)	<d1>=0.08mm**, 2.306** (0.976)	<d1>=0.1mm, **3.693** (0.961)
<d>= 0.20 mm	**1.603** (0.996)	<d1>=0.13mm, **1.416** (0.958)	<d1>=0.14mm, **2.961** (0.946)

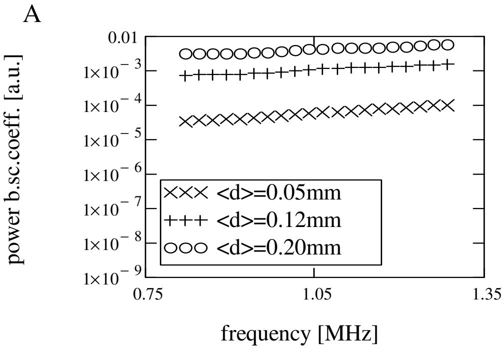
frequency dependence of the power backscatter coefficient (PBSC). This coefficient was determined following the substitution method [10].

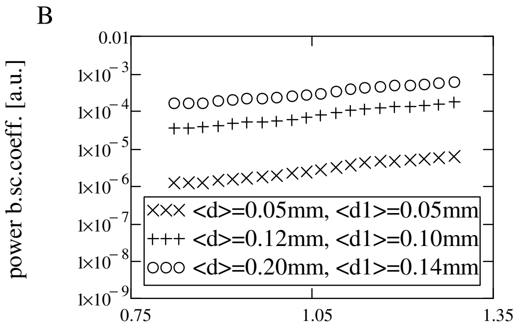
Using this method the PBSC (formula 1) was calculated by averaging of the 48 simulated spectra $\langle S_i(f)\rangle$ of the echoes which were scattered by a trabecular bone and next by comparing them to those obtained from the plane reflector $S_p(f)$ (calibration spectrum). Each echo corresponded to another random location of the scatterers but was assigned to have the same statistical properties of the bone model. The frequency-dependent coefficient was next computed (fig.3.), applying volume compensation correction factor C(f) described by the formula given in [10] and under the assumption of non-attenuating medium.

$$PBSC(f) = \frac{\langle S_i(f)\rangle \cdot C(f)}{S_p(f)} \qquad (1)$$

Next, the values of backscattering coefficient were least-squared fit to frequency f power-law function (Af^n) over the bandwidth from 0.8 MHz to 1.3 MHz (Tab. 1.).

When the spherical scatterers were used to describe thin trabeculae (mixed model of bone) the exponents (n) of the power-law function fitting the frequency dependence of backscattering coefficient increased to the value exceeding 3 (see Table 1). For the smallest of the concerned spherical scatterers, outnumbering three times an amount of thick cylindrical trabeculae the value of n equal to 3.853 was achieved.





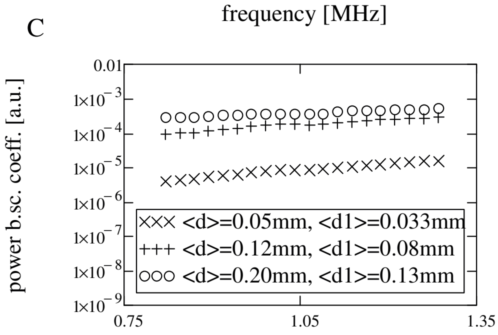


Fig. 3. Power backscattered coefficients calculated from simulated RF-echoes for A/ cylindrical scatterers bone model, B/ model consists of two populations of cylindrical scatterers and C/ mixed model

6. CONCLUSIONS

The proposed mixed model of trabecular bone allows each of the parameters important in the diagnosis of osteoporosis, including the physical dimensions and shape of the scatterers to be examined separately. It was shown that the physical dimensions, such as size and shape of the individual scatterers, exerted influence on frequency dependence of scattered signals. The importance of thin interconnecting trabeculae in the bone structure model was demonstrated.

The backscattering coefficient determined from the backscatter envelope has changed when the population of thin cylinders was replaced with the population of spherical scatteres to create the *mixed model* of bone.

It was shown that introducing of spherical scatterers increased significantly the value of the exponent *n* of the power-law function approximating frequency dependence of backscattering coefficient. The mixed model approximates well the exponent values closely to those calculated from the experimentally measured bone backscattering. The results presented above indicate that the model proposed can potentially provide clinically useful information about the bone status and can be applied as a tool for investigating the ultrasound scattering in cancellous bone.

Acknowledgements

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