Computer simulation of active sound intensity vector field in enclosure of irregular geometry

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Summary

The modal expansion method has been used to formulate expressions for real and imaginary parts of the complex sound intensity inside enclosures. Based on theoretical results, the computer program has been developed to simulate the active intensity vector field inside L-shaped enclosure. Calculation results have shown that a distribution of the active intensity is strongly influenced by the modal localization and the typical objects in the active intensity field are energy vortices and saddle points positioned irregularly inside the room. It was found that an increase in a sound attenuation results in the change of vortex positions and can cause the formation of new vortices. An influence of the wall impedance on the quantitative relation between the active and reactive intensities was also studied and it was concluded that for very small sound damping the behavior of the sound intensity is basically only oscillatory.

1. Sound intensity in enclosures

The sound intensity is very useful for studying the energetic properties of a sound field, therefore, it has received much attention in the past [1–4]. A concept of the sound intensity is based on a complex representation of the sound pressure for the monochromatic sound field. This pressure can be fully described by means of the spatially dependent real-valued amplitude P and phase function φ

$$p(\mathbf{r},t) = P(\mathbf{r})e^{\mathrm{j}[\varphi(\mathbf{r})+\omega t]}, \tag{1}$$

where $\mathbf{r} = (x,y,z)$ is the position vector and $\omega = 2\pi f$ is the angular frequency, with f being the frequency in Hz. The corresponding particle velocity $\mathbf{u}$ is expressed in terms of the gradient of the pressure according to the Euler's equation of motion, thus the formula for the complex particle velocity is given by

$$\mathbf{u}(\mathbf{r},t) = \frac{1}{\rho\omega}\left[-P(\mathbf{r})\nabla\varphi(\mathbf{r}) + \mathrm{j}\nabla P(\mathbf{r})\right]e^{\mathrm{j}[\varphi(\mathbf{r})+\omega t]}, \tag{2}$$

where ρ is the air density. The complex sound intensity vector $\mathbf{I}_c$ is expressed by the pressure p and the particle velocity $\mathbf{u}$ in the following form

$$\mathbf{I}_c(\mathbf{r}) = \tfrac{1}{2}p\mathbf{u}^* = \mathbf{I}(\mathbf{r}) + \mathrm{j}\mathbf{Q}(\mathbf{r}), \tag{3}$$

where the superscript * denotes the complex conjugate and $\mathbf{I}$, called the active intensity or acoustic intensity, and $\mathbf{Q}$, termed as the reactive intensity, are given by

$$\mathbf{I}(\mathbf{r}) = -\frac{P^2(\mathbf{r})\nabla\varphi(\mathbf{r})}{2\rho\omega},\quad \mathbf{Q}(\mathbf{r}) = -\frac{P(\mathbf{r})\nabla P(\mathbf{r})}{2\rho\omega}. \tag{4a,b}$$

An inspection of Eq. (4b) clearly shows that the reactive intensity is proportionally dependent on a gradient of the potential energy density w_p, thus the vector $\mathbf{Q}$ is always irrotational. On the contrary, the active intensity $\mathbf{I}$ is a rotational vector because the curl of the right-hand side of Eq. (4a) is nonzero and equals

$$\nabla\times\mathbf{I} = \frac{\omega}{c^2}\frac{\mathbf{I}\times\mathbf{Q}}{w_p}, \tag{5}$$

where c is the sound speed. When a weak sound damping is assumed, the steady-state pressure response of the room can be expanded in terms of rigid-walled modes, determined by specified sets of eigenfunctions Φ_m and the modal frequencies ω_m, and the associated damping coefficient r_m for each of these modes [5]

$$p(\mathbf{r},t) = \sqrt{V}\sum_{m=0}^{\infty}(\alpha_m + \mathrm{j}\beta_m)\Phi_m(\mathbf{r})\,e^{\mathrm{j}\omega t}, \tag{6}$$

where V is the room volume and

$$\alpha_m = \frac{(\omega_m^2-\omega^2)Q_m}{(\omega_m^2-\omega^2)^2+4r_m^2\omega^2},\qquad \beta_m = -\frac{r_m\omega Q_m}{(\omega_m^2-\omega^2)^2+4r_m^2\omega^2}, \tag{7}$$

where Q_m is the modal source strength. It is easy to see that Eq. (6) can be rewritten in the same form as Eq. (1), where P and φ are expressed by

$$P(\mathbf{r}) = \sqrt{V\left[\Sigma \alpha_m \Phi_m(\mathbf{r})\right]^2 + \left[\Sigma \beta_m \Phi_m(\mathbf{r})\right]^2},$$
$$\varphi(\mathbf{r}) = \arctan\left[\frac{\Sigma \beta_m \Phi_m(\mathbf{r})}{\Sigma \alpha_m \Phi_m(\mathbf{r})}\right] \tag{8}$$

and the symbol Σ denotes a sum over m from zero to infinity. Inserting of Eqs. (8) into Eqs. (4) yields the following equations

$$\mathbf{I} = \frac{V}{2\rho\omega}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\alpha_n\beta_m - \alpha_m\beta_n)\Phi_m(\mathbf{r})\nabla\Phi_n(\mathbf{r}),$$
$$\mathbf{Q} = -\frac{V}{2\rho\omega}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\alpha_m\alpha_n + \beta_m\beta_n)\Phi_m(\mathbf{r})\nabla\Phi_n(\mathbf{r}), \tag{9}$$

which confirm that in each enclosure both kinds of the sound intensity simultaneously exist. Taking the curl of the active intensity vector gives

$$\nabla\times\mathbf{I} = \frac{V}{2\rho\omega}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(\alpha_n\beta_m - \alpha_m\beta_n) \cdot\nabla\Phi_m(\mathbf{r})\times\nabla\Phi_n(\mathbf{r}). \tag{10}$$

This result is of particular importance since it indicates that when the room is excited by a monochromatic sound source there is a circulating energy flow in a steady-state sound field.

2. Numerical simulation and analysis

The simulation was carried out for L-shaped room. The plane view of this room is depicted in Fig. 1. The dimensions of the room are the following (in meters): $d_1 = d_2 = 3$, $l_1 = 4$, $l_2 = 6$, $h = 3$. A sound source driving the room had the power of 10^{-3} W and was modeled by a point source located at the position (in meters): $x_0 = 4$, $y_0 = 4$, $z_0 = 1$. It was assumed that the absorbing material is uniformly distributed on the room walls and its absorption properties describes the specific impedance $\xi = Z/\rho c$, where Z is the real-valued wall impedance. In a numerical study the first steps towards calculating the intensity $\mathbf{I}$ were a computation of the eigenfunctions Φ_m and a determination of the modal parameters: ω_m, r_m and Q_m. By means of the computer program a considerable amount of modes was found ($m_{\max} = 240$). Eigenfunctions Φ_m corresponding to these modes were computed by a numerical solution of two-dimensional wave equation [6], where the finite difference method (FDM) and the forced oscillator method (FOM) have been used. The simulation was performed for the source frequency of 70 Hz, which is close to the frequency of 15th eigenmode.

The purpose of a numerical study was to investigate how the sound damping in the room system affects the active intensity $\mathbf{I}$. In order to perform this task, a shape of vector field $\mathbf{I}$ was first studied in detail for the specific impedance ξ of 20 and 100, which are appropriate values of ξ for the moderate and small sound attenuation. It was found that the random-incident absorption coefficient corresponding to these values of ξ are respectively equal to 0.297 and 0.0734. Figure 3 illustrates vector fields $\mathbf{I}$ on the observation plane situated at a constant height from the floor ($z = 1.8$ m) calculated numerically for the assumed values of ξ. As seen, simulated distributions of active intensity streamlines are very complex and characteristic objects in the vector field $\mathbf{I}$ are energy vortices positioned irregularly inside the room. This type of vortices form such patterns of flow where acoustic energy flows continuously around closed paths. For the assumed source frequency vortices are present mainly in the bottom part of the room because at this frequency the sound field is dominated by the 15th mode which is localized in the bottom part of the room.

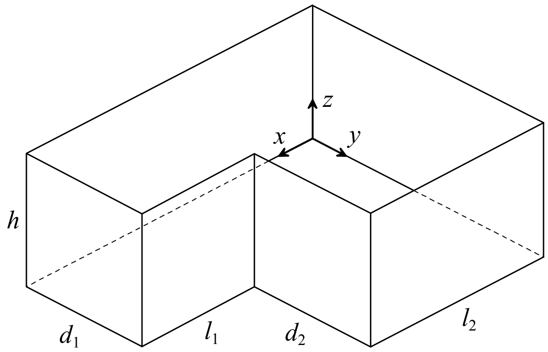

Figure 1. Room system under consideration having a shape resembling the capital letter L.

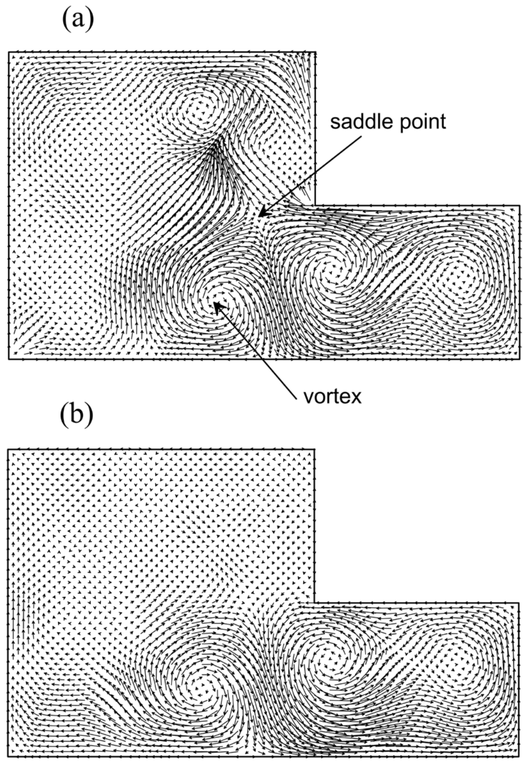

Figure 2. Active intensity $\mathbf{I}$ on the observation plane $z = 1.8$ m for the source frequency of 70 Hz and the specific impedance ξ equal to: (a) 20, (b) 100.

A comparison between vector fields in Figs. 2a,b evidently shows that the sound attenuation inside the room affects considerably the intensity **I** distribution when a transition from moderate to small sound damping occurs. The modification of the vector field primarily consists of changing the number and position of vortices. It is well illustrated by graphs in Fig. 3a,b where mapped distributions of the magnitude |**I**| of active intensity are depicted. As is evidenced from these figures, centers of vortices correspond exactly to local minima of |**I**| and this property results from the fact that in the vortex point the sound pressure should be zero theoretically [2]. Equations (4a,b) show that in this case the active and reactive intensities **I** and **Q** are zero at the vortex center. Another singular point of the active intensity field is the saddle point which can be thought of as a stagnation point because it represents the point where there is separation of the vortex region and the region where the intensity vectors form open lines. In Figs. 2a and 3a the exemplary vortex and saddle point are indicated by arrows.

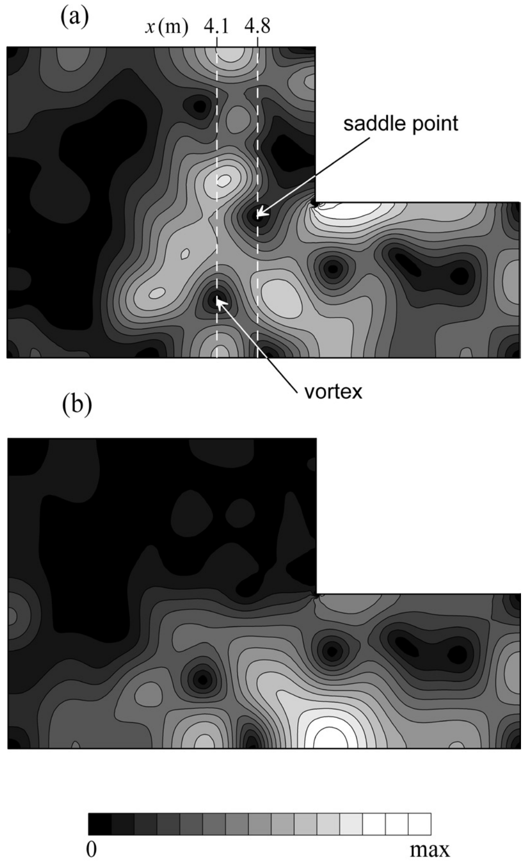

Figure 3. Magnitude |**I**| of active intensity for the source frequency of 70 Hz and the specific impedance ξ equal to: (a) 20, (b) 100.

To illustrate properties of acoustic field at the vortex and the saddle point, in Fig. 4 changes in the normalized pressure amplitude P/P_{max} and normalized magnitudes $|\mathbf{I}|/|\mathbf{I}|_{max}$, $|\nabla\times\mathbf{I}|/|\nabla\times\mathbf{I}|_{max}$, $|\nabla\varphi|/|\nabla\varphi|_{max}$ with the y-coordinate along a line segments covering the vortex center ($x = 4.1$ m) and the saddle point ($x = 4.8$ m) are depicted. In Fig. 3a these line segments are indicated by white dashed lines. Calculation results in Figs. 4a,b confirm that the pressure amplitude P and the magnitude of active intensity vector **I** reach the minimum values at the vortex center. In contrast to this property, the magnitude $|\nabla\varphi|$ of the gradient of the pressure phase peaks at this point (Fig. 4d). Theoretically, the value of $|\nabla\varphi|$ is going to infinity at the vortex center [2] which means that in the vortex region there is a jump of the pressure phase. At the saddle point, magnitudes of three acoustic variables **I**, $\nabla\times\mathbf{I}$, $\nabla\varphi$ reach their minimum values (Figs. 4f,g,h) because these quantities are zero at the saddle point theoretically [2]. The behavior of acoustic field at the saddle point was studied and physically interpreted by Chien and Waterhouse [3]. They found that at the saddle point there is a zero of the particle velocity or the phases of velocity and pressure differ by odd multiples of $\pi/2$ or a combination of the previous two situations occurs. From Eqs. (2), (4a) and (5) it follows that under the condition $\nabla\varphi = 0$, the active intensity **I** and $\nabla\times\mathbf{I}$ are zero but the complex velocity amplitude is non-zero because a real part of this amplitude vanishes only. However, Eqs. (1) and (2) imply that in this case the phases of velocity and pressure differ by $\pi/2$ which is the criterion for the saddle point.

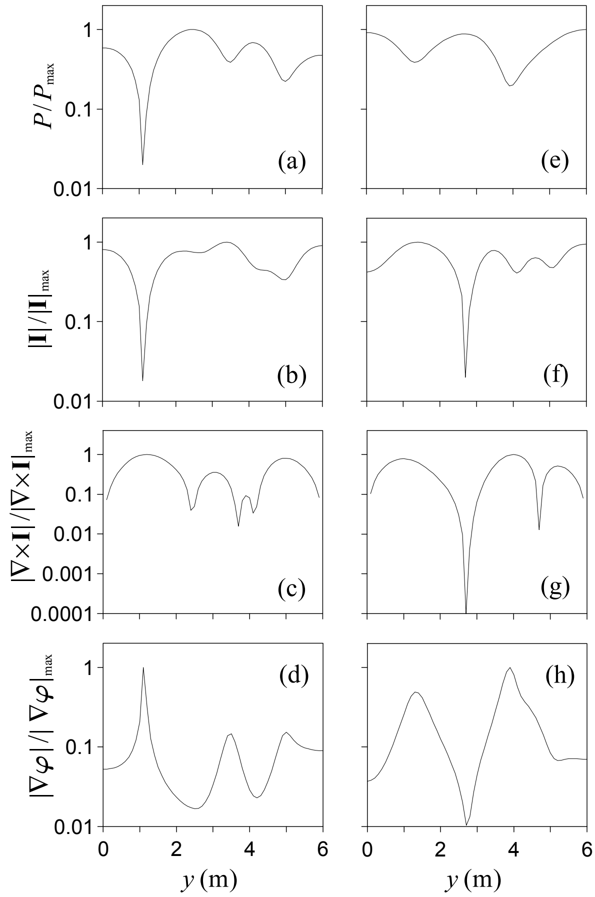

Figure 4. Changes in P/P_{max}, $|\mathbf{I}|/|\mathbf{I}|_{max}$, $|\nabla\times\mathbf{I}|/|\nabla\times\mathbf{I}|_{max}$ and $|\nabla\varphi|/|\nabla\varphi|_{max}$ with y-coordinate for $x = 4.1$ m (a)-(d) and $x = 4.8$ m (e)-(h) for the frequency of 70 Hz.

So far numerical results were used to reveal details of the active intensity vector field but did not provide information about the intensity **I** magnitude but more importantly, about a quantitative relation between magnitudes of the active and reactive intensities **I** and **Q**. The problem will be analyzed by use of the parameter $|\mathbf{I}|_{max}/|\mathbf{Q}|_{max}$ denoting the ratio of maximal magnitudes of **I** and **Q** on the observation plane for a selected value of the specific impedance ξ. Dependencies of $|\mathbf{I}|_{max}/|\mathbf{Q}|_{max}$ on ξ at the assumed source frequencies for moderate and large values of ξ are depicted in Fig. 5, where points denote simulation data. In the range of moderate values of the specific impedance ($\xi < 100$), the ratio $|\mathbf{I}|_{max}/|\mathbf{Q}|_{max}$ reaches a maximum value, which is slightly less than unity, and then continuously decreases with increasing ξ (Fig. 5a). A graphic reconstruction of the sound intensity field has shown that when ξ is moderate, changes in a distribution of the active intensity vector field are observed. For large values of ξ the ratio $|\mathbf{I}|_{max}/|\mathbf{Q}|_{max}$ fast decreases and its dependence on ξ is well approximated by the formula

$$|\mathbf{I}|_{max} / |\mathbf{Q}|_{max} = a/\xi, \tag{11}$$

where a is the constant (Fig. 5b). In this case a reconstruction of the sound intensity field has revealed that patterns of the active and reactive intensities do not change with the specific impedance. On the other hand, a great diminution of the parameter $|\mathbf{I}|_{max}/|\mathbf{Q}|_{max}$ for large values of ξ indicates also that for the rooms with nearly rigid walls there is, in fact, no flow of acoustic energy because the behavior of the sound intensity inside the room space is essentially only oscillatory.

3. Summary and conclusions

In this study the modal expansion method was applied to simulate a low-frequency distribution of the active intensity vector field inside L-shaped room. In the theoretical section, a concept of the complex sound intensity was briefly presented and modal representations of the steady-state sound intensity were given. Calculated distributions of the active intensity vector have shown that the characteristic objects in the active intensity field are energy vortices and saddle points located in an irregular manner inside the room. Simulations revealed that, according to the theory, the pressure amplitude and the magnitude of active intensity have minima at the vortex center, where there is a peak of magnitude of the pressure phase gradient. On the other hand, at the saddle point there are minima of magnitudes of the active intensity, the curl of the active intensity and the gradient of the pressure phase. An impact of the specific impedance on vortex positions was also investigated and it was found that an increase in a sound damping results in the change of vortex positions and can cause the formation of new vortices when the sound damping is moderate. Finally, an influence of the specific impedance on a quantitative relation between the active and reactive intensities was analyzed. It was concluded that the ratio between maximal magnitudes of active and reactive intensity varies inversely with the specific impedance when there is no change in patterns of active and reactive intensities. It was found also that when the sound damping is negligible, there is essentially only oscillatory sound energy flux inside a room space.

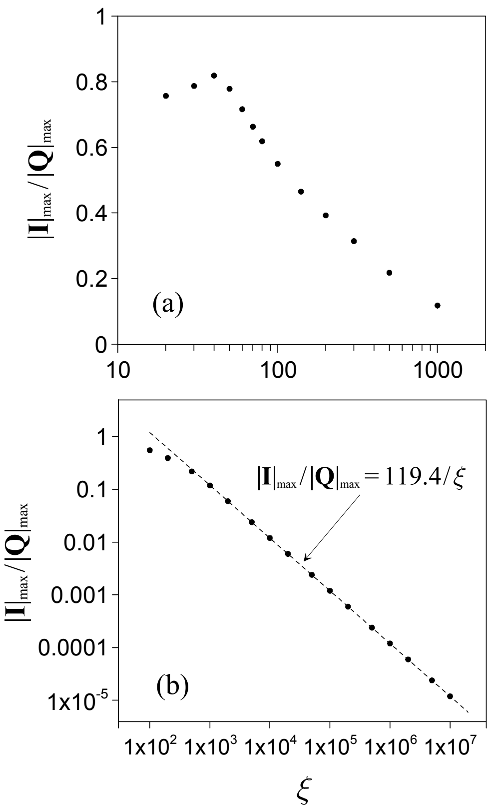

Figure 5. Dependence of $|\mathbf{I}|_{max}/|\mathbf{Q}|_{max}$ on the specific impedance ξ for moderate (a) and large (b) values of ξ. Source frequency of 70 Hz.

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