Scatter assessment of rotor-shaft vibration responses due to uncertain residual unbalances and bearing properties*

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Abstract

The main objective of the presented study is an evaluation of the effectiveness of various methods for estimating statistics of rotor-shaft vibration responses. The computational effectiveness as well as the accuracy of statistical moment estimation are essential for efficient robust design optimization of the rotor-shaft systems. The most important sources of the observed response scatter are inherently random rotor-shaft residual unbalances as well as stiffness and damping properties of the journal bearings. A relevant representation of these parameters leads to multidimensional stochastic models. The compared stochastic moment estimation methods include sampling techniques, the dimension reduction method and the polynomial chaos expansion method. Two problems of the rotor-shaft vibration analysis are considered: a typical single-span rotor-shaft of the 8-stage centrifugal compressor driven by the electric motor and a large multi-bearing rotor-shaft system of the steam turbogenerator. It is shown that methods that provide a satisfactory balance between the estimation accuracy and computational effectiveness are sampling techniques. Methods employing polynomial chaos expansion perform well in the case of reduced stochastic models. On the other hand, low accuracy of the methods based on Taylor series expansion very often renders these techniques unsuitable for the robust design optimization of vibrating rotor shafts.

Keywords: vibrations, robustness, numerical analysis, stochastic phenomena

1. Introduction

In exploitation of rotating machines some of the observed phenomena are considered to be particularly undesired from the viewpoint of effectiveness and safety. Excessive stress concentrations and rubbing effects occurring between stators and rotors attached to flexible shafts subjected to lateral vibrations can be given as examples of such a detrimental behavior. The modern, responsible and heavily affected rotating machines must assure possibly high level of reliability, durability and safety in operation. For these reasons their design process should be performed very thoroughly in order to obtain relatively small magnitude of unavoidable dynamic excitation, e.g. due to residual unbalance, gas-pressure forces or electromagnetic forces.

While aiming at realistic modeling of rotor-shaft systems the actual stochastic nature of important model parameters should be taken into account. The main objective of the presented study is to investigate methods that allow for efficient scatter estimation of the rotor-shaft vibration responses. The scatter is basically caused by inherently random rotor-shaft residual unbalances and by uncertain journal bearing parameters. By evaluating mean values as well as variances of the responses of interest one can not only assess a typical performance of the rotating machine, but also its sensitivity with respect to parameter imperfections. Efficient methods of stochastic moments estimation are a crucial component of robust design optimization (RDO) algorithms (a comprehensive survey of RDO formulations and solution techniques is given in e.g. [15, 6, 1]). The goal of the rotor-shaft robust design optimization is to find the optimal design that is not sensitive with respect to parameter imperfections even when the rotor-shaft is subjected to considerable bending or torsional resonant vibrations.

In the current paper there is examined feasibility of various methods to compute statistical moments of the rotor shaft vibration responses. The investigated methods include sampling techniques, i.e. the classical Monte Carlo as well as Latin hypercube sampling, the Taylor series expansion method (the perturbation approach), the so-called dimension reduction methods proposed by Xu and Rahman [17] and the polynomial chaos expansion method [3, 2]. It must be emphasized that problems concerning the propagation of uncertainty in analysis of complex systems have already been addressed by many authors in numerous papers, see e.g. [4, 5, 8, 11]. However, these issues do not seem to have been investigated for the rotor shaft systems where the stochastic model is typically given by a big number of random variables describing residual unbalances and bearing properties.

The paper consists of four main sections. In Sec. 2 each of the studied scatter analysis techniques is shortly described. Sec. 3 introduces the employed hybrid mechanical model of the rotor-shaft system, which thanks to its high computational efficiency is particularly convenient for stochastic analyses. Finally, in sections 4 and 5 the effectiveness of the selected methods for statistical moment estimation is compared using two problems of the rotor-shaft vibration analysis. The first example deals with a typical single-span rotor-shaft of the 8-stage centrifugal compressor driven by an electric motor. In the second example a model of a large multi-bearing rotor-shaft system of the steam turbogenerator is considered.

2. Statistical moment assessment

It is fairly typical in mechanical and civil engineering that some quantities which describe a structural system and applied loads should be modeled as random variables, $X_1, \ldots, X_n$. They are

*This research was financially supported by European Regional Development Fund within the framework of the Innovative Economy Programme, project no. POIG.01.03.01-14-209/09.

called the basic variables and constitute a random vector $\mathbf{X}$ whose samples $\mathbf{x}$ belong to the Euclidian space with the probability measure defined by the joint probability density function (PDF) $f_{\mathbf{X}}(\mathbf{x})$. Assuming a rotor-shaft vibration response Y is a function of the basic variables, $Y = h(\mathbf{X})$, in the current study we focus on estimating the mean value μ_Y and the variance $\mathbb{V}\mathrm{ar}(Y) = \sigma_Y^2$ of Y, which are given, respectively, by

$$\mu_Y = \mathbb{E}[Y] = \int_{-\infty}^{\infty} h(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x}, \tag{1}$$

$$\sigma_Y^2 = \mathbb{E}[(Y-\mu_Y)^2] = \int_{-\infty}^{\infty} [h(\mathbf{x})-\mu_Y]^2 f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x}. \tag{2}$$

The following methods for computing the above moments are investigated:

Simulation methods – They employ samples of basic random variables $\mathbf{X}$ to assess the values of μ_Y and σ_Y^2. The commonly used unbiased estimators are formulated as follows:

$$\mu_Y \approx \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y^{(i)} = \frac{1}{N}\sum_{i=1}^{N} h(\mathbf{X}^{(i)}), \tag{3}$$

$$\sigma_Y^2 \approx s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y^{(i)}-\bar{Y})^2 = \frac{1}{N}\sum_{i=1}^{N}[h(\mathbf{X}^{(i)})-\bar{Y}]^2. \tag{4}$$

Realizations $\mathbf{x}^{(i)}$, $i = 1,\ldots,N$, of the random vector $\mathbf{X}$ are drawn from the distribution of $\mathbf{X}$ and the moments are computed using the well known estimators. The simulation methods differ mainly by the way the samples are obtained. One may distinguish two major sampling techniques: random sampling (RS) and descriptive sampling [10]. Under some assumptions, the so-called Latin hypercube sampling (LHS) [9] can be classified as a descriptive sampling technique. In the performed study the efficiency of RS as well as LHS are examined.

Taylor series expansion method – An alternative method of estimating stochastic moments of random functions is based on expanding these functions into Taylor series around the mean values of random variables. In the expansion the terms of order higher than two are usually neglected and the stochastic description of variables is given only by the vector of mean values and the covariance matrix. Such approach is called the stochastic perturbation method [7]. Contrary to sampling techniques that reduce to computing the random function values for many realizations of random variables the major component of perturbation methods is sensitivity analysis, i.e. computing gradients and higher order derivatives of the functions of interest. In our study only the first and second order perturbation methods are considered.

Dimension reduction method (DRM) – Also this method is based on expanding the random function into Taylor series around mean values of random variables, see [17]. However, contrary to the perturbation approach, DRM does not require computing values of partial derivatives. The method allows for significant reduction of computational cost with respect to numerical integration of equations (1) and (2). By DRM the multivariate function $h(\mathbf{X})$ is approximated by a sum of less dimensional functions depending only on $s < n$ variables with the other variables fixed to their mean values. >From the point of view of computational efficiency the two versions of the method are particularly attractive. These are the univariate dimension reduction (UDR) and the bivariate dimension reduction (BDR) methods .

Polynomial chaos expansion method (PCE) – Provided the variable $Y = h(\mathbf{X})$ has a finite variance, it can be expanded onto the so-called "polynomial chaos" basis as follows [2]:

$$Y = h(\mathbf{X}) = \sum_{\boldsymbol{\alpha}\in\mathbb{N}^n} a_{\boldsymbol{\alpha}}\psi_{\boldsymbol{\alpha}}(\mathbf{X}), \tag{5}$$

where $a_{\boldsymbol{\alpha}}$ are unknown deterministic coefficients and $\psi_{\boldsymbol{\alpha}}$ are multivariate polynomials, orthogonal with respect to the joint PDF $f_{\mathbf{X}}(\mathbf{x})$. In practice, for computational efficiency the series in Eq. (5) is truncated after a finite number of terms. Most often, the polynomials, which degree $|\boldsymbol{\alpha}| = \sum_{i=1}^{n}\alpha_i$ is higher than a given degree p, are eliminated from the series. The number of $a_{\boldsymbol{\alpha}}$ coefficients that have to be computed is equal to

$$M = \binom{n+p}{p}. \tag{6}$$

The unknown coefficients are usually computed by the regression approach. Since the number of coefficients in truncated expansion grows rapidly with the number of variables n and the polynomial degree p, therefore, in order to reduce the computational burden and to improve the approximation quality of the method Blatman and Sudret proposed in [2] an adaptive sparse PCE. The iterative algorithm allows to eliminate these of expansion coefficients which are not significant in approximation of function $h(\mathbf{X})$. A version of the sparse PCE algorithm implemented by the authors of the current paper was used for assessing statistical moments of the rotor-shaft vibration response.

3. Description of the hybrid mechanical model of the rotor-shaft system

In order to obtain sufficiently reliable results of numerical simulations together with a reasonable computational efficiency, the vibrating rotor-shaft system of a rotor machine is usually modeled by means of one-dimensional finite elements of the beam-type. Nevertheless, such models can still be characterized by relatively high number of degrees of freedom in the range between hundreds and even thousands, which may substantially increase the computational cost of sampling methods. Thus, in case of large finite-element models proper algorithms reducing number of degrees of freedom have to be employed in order to shorten computer simulation times. It is to remember that such reductions of degrees of freedom are troublesome and can lead to computational inaccuracies. In order to avoid the abovementioned drawbacks of the finite element approach and to maintain the obvious advantages of this method, in this paper, similarly as in [12, 13, 14], the dynamic analysis of the entire rotating system is performed by means of the one-dimensional hybrid structural model consisting of continuous visco-elastic macro-elements and discrete oscillators. This model is employed here for eigenvalue analyses as well as for numerical simulation of lateral vibrations of the rotor-shaft. In the model successive cylindrical segments of the stepped rotor-shaft are substituted by flexurally and torsionally deformable cylindrical macro-elements of continuously distributed inertial-visco-elastic properties. A typical i-th continuous visco-elastic macro-element is presented in Fig. 1.

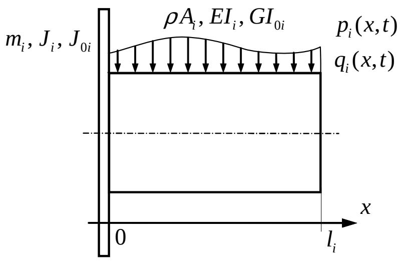

Figure 1: Flexurally and torsionally deformable continuous visco-elastic macro-element.

In this figure symbols A_i, I_i and I_{0i}, $i = 1, 2, ..., n_e$, denote respectively the cross-sectional area, the diametral and polar geometric moment of inertia and n_e is the total number of macro-elements in the considered hybrid model. The transverse and torsional external loads continuously distributed along the macro-element length l_i are respectively described by the two-argument functions $p_i(x, t)$ and $q_i(x, t)$, where x is the spatial coordinate and t denotes time. With an accuracy that is sufficient for practical purposes, in the proposed hybrid model of the rotor-shaft system, some heavy rotors or coupling disks can be represented by rigid bodies attached to the macro-element extreme cross-sections, as shown in Fig. 1. Here, symbols m_i, J_i and J_{0i} denote respectively the mass and the diametral and polar mass-moments of inertia of this rigid body. Each journal bearing is represented by a dynamic oscillator of two degrees of freedom, where apart from the oil-film interaction also the visco-elastic properties of the bearing housing and foundation are taken into consideration. This bearing model makes possible to represent with a relatively high accuracy kinetostatic and dynamic anisotropic and anti-symmetric properties of the oil-film in the form of constant or variable stiffness and damping coefficients. An example of such a hybrid model of the centrifugal compressor rotor-shaft is presented in Fig. 2. The rotor-shaft is supported on two journal bearings, where the additional support in its mid-span caused by the aero-dynamic cross-coupling effect is also taken into consideration. The complete mathematical formulation and solutions for such hybrid models of the rotor-shaft systems can be found in [12, 13, 14].

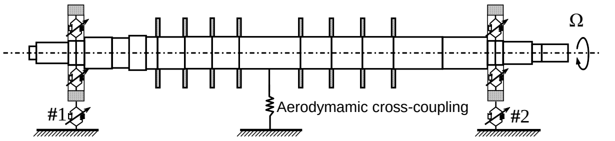

Figure 2: Hybrid mechanical model of the compressor rotor-shaft.

4. Numerical example: the centrifugal compressor rotor-shaft

4.1. Model description

In order to create an adequate geometrical and mechanical representation, the stepped-rotor shaft of this compressor of the total length 2.8 m and total weight 485 kg has been modeled by means of $n_e = 27$ continuous macro-elements. All geometrical parameters of the successive real rotor-shaft segments together with their material constants as well as the average stiffness and damping coefficients of the oil-film in the bearings of this compressor have been taken from [16].

In the first step of dynamic analysis the eigenvalue problem must be solved in order to obtain fundamental natural frequencies and the corresponding eigenfunctions of bending and torsional vibrations. As it follows from the comparison performed for the constant nominal rotational speed 5626 rpm, the shear effect taken into consideration in the case of Timoshenko's beam theory results in a little bit smaller natural frequency values than these determined by means of Rayleigh's beam model. Here, in the frequency range $0 \div 400$ Hz containing the first 10 bending eigenforms, which is the most important from the engineering viewpoint, the respective differences do not exceed 2%. The eigenfunctions corresponding to these natural frequencies and determined using both beam theories respectively overlay each other.

According to the above, one can conclude that in this frequency range an application of Rayleigh's rotating beam theory seems to be sufficiently accurate for further simulations of forced vibrations. For the considered compressor rotor-shaft regarded here as dynamically isolated from the driving motor by means of the low-stiffness elastic coupling the torsional eigenvalue problem has been solved using the analogous hybrid (discrete-continuous) model described e.g. in [12]. The obtained in this way the lowest torsional natural frequency values 597.8 and 1212.4 Hz are far away above the fundamental first 10 bending natural frequencies, therefore assumed that during the investigated entire dynamic process flexural deformations of the shaft are predominant and the shaft torsional dynamic deformations seem to be negligible. According to the above, the rotor-shaft can be regarded as a torsionally rigid body rotating with a rotational speed gradually varying in time during start-ups and run-downs. However, the shaft bending vibrations induced by the system residual unbalance are taken into consideration. For the assumed residual static unbalances uniformly distributed along each cylindrical rotor-shaft segment and for the concentrated static unbalances of each rigid body representing rotor-disks the external excitation is expressed by means of the following forcing terms:

$$\begin{aligned} p_i(x,t) &= \varepsilon_i \rho A_i \Omega^2(t) \sin(\Theta(t) + \psi_i), \\ &\qquad \text{for } \ 0 < x < l_i, \ i = 1, 2, \ldots, n_e, \\ P_k(t) &= \varepsilon_k m_k \Omega^2(t) \sin(\Theta(t) + \psi_k), \\ &\qquad \text{for } \ x = 0, \ \ k = 1, 2, \ldots, K. \end{aligned} \tag{7}$$

where ε_i, ε_k denote the proper eccentricities caused by admissible manufacturing errors, ψ_i, ψ_k are the respective phase shift angles of the unbalance circumferential location with respect to the shaft rotation axis, K denotes the total number of rigid disks in the model. For the assumed hybrid model of the investigated compressor rotor-shaft in the frequency range $0 \div 1000$ Hz 14 bending eigenmodes have been considered to calculate forced vibration amplitudes with sufficiently high computational accuracy.

4.2. Assessment of statistical moments

The stochastic model of the compressor rotor-shaft contains 64 random variables $\mathbf{X} = \{X_1, \ldots, X_{64}\}$. The stiffness as well as damping coefficients of the two journal bearings are represented by 16 normally distributed variables with the coefficients of variation equal to 10% for the stiffness and 15% for the damping coefficients. It was assumed that the distribution of residual unbalances of the rotor-shaft segments can be represented by a weighted sum of 4 principal eigenmodes with the most probable contribution from the first eigenmode, with the value of the corresponding weight coefficient up to 0.8, and the contributions from subsequent modes controlled by the maximal values of their weight coefficients equal to 0.1, 0.08 and 0.02, respectively. Therefore, the weight coefficients are modeled by uniformly distributed random variables in the ranges $(0, 0.8]$, $(0, 0.1]$, $(0, 0.08]$ and $(0, 0.02]$. The random magnitude of the unbalances is obtained by setting the maximal value of such constructed distribution function to be equal to a realization of log-normal random variable with the expectation 0.15 mm and the standard deviation 0.02 mm. The remaining random parameters are: 27 uniformly distributed phase shift angles in the range $0 \div 2\pi$, 8 uniformly distributed rotor-disk unbalances in the range $0 \div 1$ mm and finally 8 uniformly distributed rotor disk phase shift angles in the range $0 \div 2\pi$.

The selected method of modeling the unbalances can be justified by a technological process of the rotor-shaft manufacturing.

The predominant unbalance amplitude distribution of the successive stepped shaft segments according to the first lateral eigenvibration mode is substantiated by the machining processes typical for the considered rotor-shaft type. Namely, the rotor-shaft usually clamped at both ends can be forced to bending vibrations by the cutting tool, when an excitation of the first eigenmode is the most probable and an excitation of the next eigenmodes seems to be of a secondary importance. The assumed uniform distribution of phase shift angles of these unbalances follows from the shaft segment-to-segment machining steps usually set as mutually independent during the entire cutting process. However, the uniform distribution of the gravity center eccentricities together with their phase shift angles of the rotor-disks can be substantiated by their commonly applied shrink-fit connections with the shaft, which usually requires final balancing of the entire rotor-shaft system upon its 'on-site' assembly.

The rotor-shaft vibrational response $Y(\mathbf{X})$, which mean value and standard deviation are to be estimated, is the maximal vibration amplitude. For most of the realizations of the vector $\mathbf{X}$ this maximal lateral rotor-shaft displacement occurs in the mid-span of the rotor-shaft.

In order to establish reference values of the estimated statistics a random sampling with $N = 100\,000$ sample points was performed. The obtained values are $\bar{Y} = 0.4808$ mm and $s = 0.2616$ mm for the mean and standard deviation, respectively, see Eqs. (3) and (4).

In Fig. 3 there is shown a scatter plot prepared for a 5000 point random subset of the 100 000 point sample. As can be observed, unfavorable realizations of the random variables describing uncertain parameters of the rotor-shaft system may lead to lateral displacements of the rotor shaft axis up to 1.7 mm. The corresponding histogram, presented in Fig. 3, gives some indication of the kind of probability distribution, the maximal vibration amplitude obeys. The positively skewed histogram can be well approximated by the Weibull probability density function.

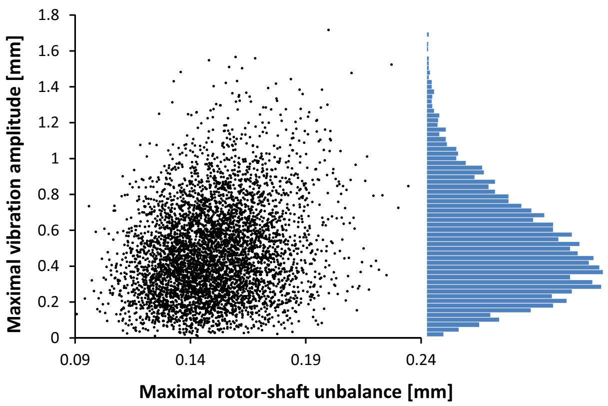

Figure 3: The compressor rotor shaft example. Scatter and corresponding histogram of maximal rotor-shaft lateral vibration amplitude.

The correlation coefficients between the maximal vibration amplitude and the random variables are illustrated in the form of a bar chart in Fig. 4. By examining their values it could be concluded that there are no random variables significantly correlated with the rotor-shaft response. Lacking statistically dominating relations it is not a straightforward task to eliminate some of the variables from the stochastic model. On the other hand, several random variables seem to influence the output more than the other. These are the maximal rotor-shaft residual unbalance and the direct vertical stiffness and damping coefficients of the journal bearings. Nevertheless, in the performed comparative study the complete set of 64 random variables is taken into account.

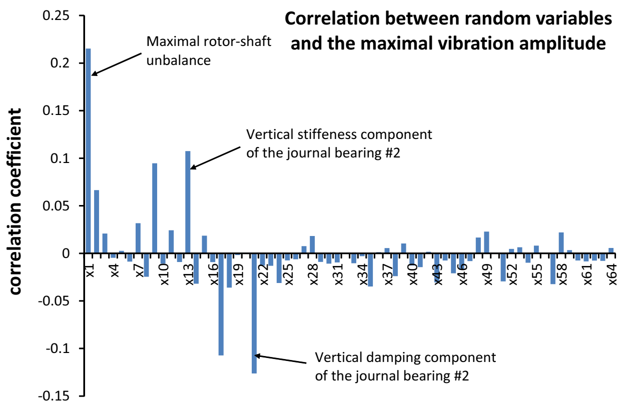

Figure 4: The compressor rotor shaft example. Correlation coefficients between random variables and the rotor-shaft response.

The methods based on the Taylor series expansion, described in [8, 17], entirely failed to estimate the mean and standard deviation of the vibration amplitude. For example, the first order estimation of the mean leads to $\bar{Y} = 2.78$ mm, i.e. to the value many times larger than the reference one. Neither the second order expansion nor the univariate or bivariate reduction approaches provide a substantial improvement of the first order results.

A reason for this unsatisfactory performance may be the way the residual unbalances of the rotor-shaft are modeled. It was assumed that the phase shift angles of the unbalance circumferential location with respect to the shaft rotation axis are modeled by independent uniformly distributed random variables. Therefore, accounting for the number of rotor-shaft segments, it is extremely unlikely that all the phase shift angles of the rotor-shaft unbalances as well as rotor-disk unbalances take the same values. This, however, is the case for the first order perturbation approach. All the unbalances are in phase with the phase shift angles equal to the same expected value π. Since the first symmetrical eigenmode determines distribution of residual unbalances, harmonized phase shift angles are the source of a significant excitation leading to excessive lateral vibrations. Below, only the results of sampling methods and sparse polynomial chaos expansion technique are presented.

In order to assess the estimation error of the methods employing random sampling and Latin hypercube sampling, the mean value and standard deviation of the rotor-shaft response were computed by RS and LHS for different sample sizes ranging from $N = 120$ to $N = 2400$. For both sampling method and for each N the estimation was performed 300 times using independently generated samples. This allowed to obtain the estimation error statistics, i.e. the mean and standard deviation. The results are shown in Figs. 5 and 6. As it can be seen, in the considered example there is no qualitative difference between RS and LHS and neither of them is visibly superior with respect to the other. It is interesting to observe that even for relatively small samples, i.e. $N = 340 < 6n$, the mean percentage estimation error is less than 3.5%, which seems to be acceptable for the purpose of robust design optimization. According to Eq. (6) assuming the degree of polynomials $p = 2$ and for $n = 64$ random variables, there are 2145 unknown coefficients in the truncated polynomial chaos equation (5). The coefficients have to be determined by the linear regression approach described in [2]. Obviously, the solution depends on the design of experiments used to fit the PC response surface. In the performed study LH based designs of experiments were used for this purpose. In consequence, due to the random nature of such a design, by repeating the analysis for various Latin hypercubes it was possible to obtain estimation error statistics. Two cases were considered: The first one adopting the LH design with $N = 2150$ experimental points, which is slightly more than the number given by Eq. (6), and the second case with $N = 2400$. The employed sparse PC algorithm allowed to reduce the number of PC expansion terms to about 690. The reduction leads to an improvement of the linear regression results since a smaller number of coefficients is determined using the same number of experiments. The mean estimation error of the sparse PC method is compared in Figs. 5 and 6 with the sampling techniques.

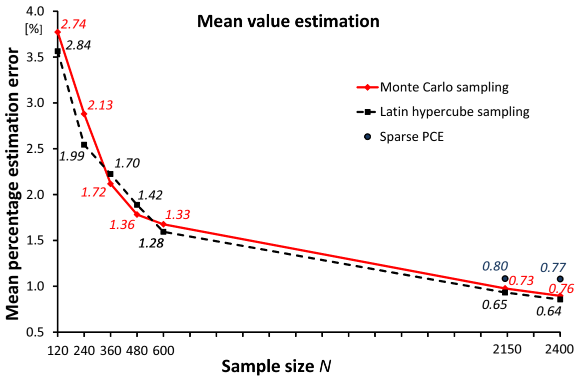

Figure 5: The compressor rotor shaft example. Mean relative percentage error of the mean value estimation of the maximal rotor-shaft vibration amplitude obtained using RS, LHS and sparse PCE. The graph point labels stand for standard deviations of the errors.

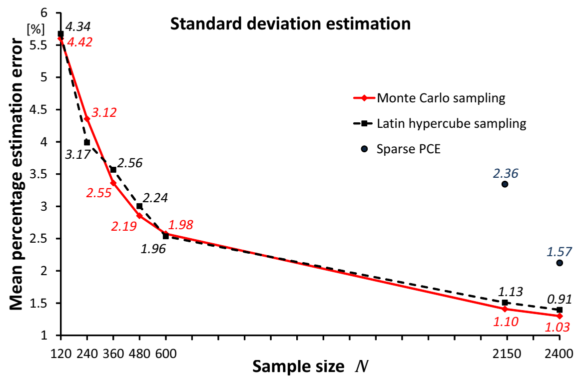

Figure 6: The compressor rotor shaft example. Mean relative percentage error of the standard deviation estimation of the maximal rotor-shaft vibration amplitude obtained using RS, LHS and sparse PCE. The graph point labels stand for standard deviations of the errors.

In the case of the mean value estimation the error of the PC method is comparable with RS and LHS results. On the other hand, the corresponding error of the standard deviation estimation is significantly higher than the one for sampling techniques. Taking into account the reduced number of terms in the PC expansion, the similar results can be obtained for smaller LH design, say for $N \approx 700$. This, however, requires an a priori knowledge of the functional relationship between the random variables and the rotor-shaft response, which is usually not available.

5. Numerical example: the steam turbo-generator rotor-shaft

5.1. Model description

The presented methodology of vibration analysis is applied here in the second numerical example of a rotor-shaft system of the typical 200 MW steam turbo-generator consisting of the single high- (HP), intermediate- (IP) and low-pressure (LP) turbines as well as of the generator-rotor (GEN). The rotor-shaft system is supported by seven journal bearings, as shown in Fig. 7. For the purpose of this study it seems to be sufficient to model the considered stepped-rotor shaft of the total length 25.9 m by means of $n_e = 49$ continuous macro-elements, as an initial approximation of its geometry. All geometrical parameters of the successive real rotor-shaft segments as well as their material constants have been determined using the detailed technical documentation of this turbo-generator. The average stiffness and damping coefficients of the oil film in the bearings as well as the equivalent masses and stiffness and damping coefficients of the bearing housings are obtained by means of measurements and identification performed on the real object.

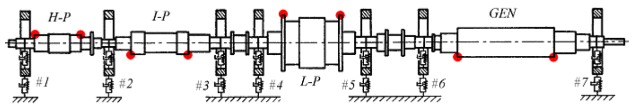

Figure 7: Hybrid mechanical model of the steam turbo-generator rotor-shaft system.

As in the previous numerical example, first the eigenvibration analyzes have been performed for the nominal rotational speed 3000 rpm, using the two abovementioned rotating beam theories. The shear effect taken into consideration in the case of Timoshenko's beam also results in a little bit smaller natural frequency values than these determined by means of Rayleigh's beam model. Here, in the frequency range $0 \div 150$ Hz, which is the most important from the engineering viewpoint, the respective differences slightly exceed 3%. The eigenfunctions corresponding to these natural frequencies and determined using both beam theories also respectively overlay each other. Therefore, similarly as in the first example, one can conclude that in this frequency range an application of Rayleigh's rotating beam theory for simulations of forced vibrations seems to be sufficiently accurate, too.

Since typical steam turbo-generators are the devices operating almost permanently in steady-state, out-of-resonance working conditions during a majority of their life, their start-ups and run-downs are rather rare exploitation phases. Thus, in the considered case simulations of passages through lateral vibration resonance zones are not necessary. Therefore, the dynamic and stochastic analyses of the steam turbo-generator rotor-shaft system are going to be carried out only for the steady-state, out-of-resonance operation with the constant nominal rotational speed 3000 rpm corresponding to the excitation of bending vibrations by means of residual unbalances with the synchronous frequency equal to 50 Hz. Here, for the assumed hybrid model of this object in the frequency range of a practical interest $0 \div 500$ Hz, 22 bending eigenmodes have been considered in computing forced vibration amplitudes.

5.2. Assessment of statistical moments

The uncertain parameters of the rotor-shaft system are represented by 59 random variables. As in the first numerical example, the stiffness and damping coefficients of 7 journal bearings are modeled by normal random variables. However, here the common coefficient of variation is taken equal to 5%. Thus, there are 56 random variables that correspond to the journal bearings, i.e. 7×(4 stiffness coefficients + 4 damping coefficients). The remaining 3 variables account for random values of the residual unbalances.

The rotor-shaft system of the considered turbo-generator consists of the 3 units described in 5.1, which are independently manufactured and then mutually connected during on-site assembly process of the entire device. Each of them is characterized by a combined cross-sectional structure consisting of the load carrying shaft core and of the strip created by the turbine blade rims or generator windings, respectively, attached along this core by means of a shrink-fit connection. Thus, the residual unbalance distributions of the HP-IP and LP turbines as well as of the generator rotor are in principle not related to the machining process applied as in the case of the centrifugal compressor rotor-shaft, but it is more complicated in character. Taking this into account, it seems to be reasonable to assume that for each given rotor-shaft unit its unbalance is proportional to the successive shaft segment diameters with the common proportionality factor for all segments in the entire unit. For the 3 rotor-shaft units, this assumption results in 3 variables that model the uncertainty of residual unbalances. The 3 proportionality factors are given by realizations of log-normally distributed random variables. Based on the technical data for the considered turbo-generator rotor-shaft system, the mean values of the 3 uncertain factors were estimated as: $5.6 \cdot 10^{-5}$ for the HP and IP turbines, $2.0 \cdot 10^{-5}$ for the LP turbine and $3.2 \cdot 10^{-6}$ for the generator rotor. The coefficient of variation of these variables was assumed equal to 10%. According to this assumption, each rotor-shaft unit is characterized by the common phase shift angle for all unbalance amplitudes corresponding to successive shaft cylindrical segments. The obtained in this way 3 phase shift angles for each abovementioned rotor-shaft units are not random, but they are determined from respective identification measurements performed for the real object and assumed equal to zero for the HP-IP turbine, 2.79 rad for the LP turbine and zero for the generator rotor unit.

To get reference values for the mean and standard deviation of maximal rotor-shaft lateral displacement a thorough random sampling with the sample size $N = 100\,000$ was performed. The obtained estimations are $\bar{Y} = 0.0972\,\mathrm{mm}$ and $s = 0.0091\,\mathrm{mm}$. These values are many times smaller than the corresponding ones from the compressor rotor-shaft example.

When analyzing the simulation results it is interesting to check correlations between the random variables and the considered vibrational response. The values of correlation coefficients are graphically presented in Fig. 8. Contrary to the previous numerical example, here one can easily select variables that strongly influence the value of the vibration amplitude. A random scatter of the values of these variables directly translates into the scatter of the rotor-shaft response. In particular, variable X_{57}, which represents the random factor of residual unbalance magnitude for the HP and IP turbines, is strongly positively correlated with the response. The corresponding correlation coefficient $\rho = 0.84$ indicates that this variable is the major source of the response variance. Thus, a natural choice is to reduce the stochastic description of the rotor shaft by keeping only the variables significantly correlated with the vibrational response. If considering only the three variables marked in Fig. 8, the estimated moments are equal to $0.0975\,\mathrm{mm}$ and $0.00838\,\mathrm{mm}$ for the mean and standard deviation, respectively. The obtained values provide a very accurate assessment of the moments computed for the full model consisting of 59 variables. If adding two more variables into the reduced model, i.e. variables X_9 and X_{19} corresponding to stiffness coefficients of journal bearings #3 and #5, the estimations of the mean value and standard deviation change to $0.0974\,\mathrm{mm}$ and $0.00870\,\mathrm{mm}$, respectively. Since the variance of the reduced model is more than 90% of the variance of the full model, the performance of stochastic moment estimation methods was examined for two cases: the full model characterized by 59 variables and the reduced one characterized by 5 variables.

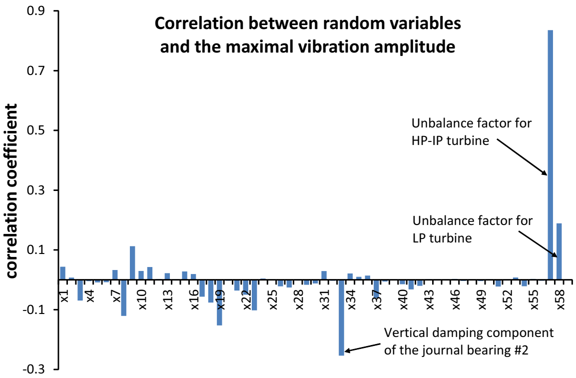

Figure 8: The turbo-generator rotor shaft example. Correlation coefficients between random variables and the rotor-shaft response.

Below, for the complete stochastic model, in Figs. 9 and 10 there are shown the mean percentage estimation errors for moments computed by RS, LHS and PC expansion methods. The error statistics are based on 300 repetitions of a given method for each value of N. As it can be seen, contrary to the previous numerical example, the Latin hypercube sampling provides the best estimation quality for each sample size.

Even though a very precise estimation of the mean value of the maximal rotor-shaft vibration amplitude can be obtained for relatively small samples, i.e. $N = 120 \approx 2n$, a proper estimation of the standard deviation of this rotor-shaft response requires an application of more sample points, where $N = 360 \div 600$ yields the error of approximately 2%.

The results of the PC expansion method are very accurate, especially in the case of mean value estimation. Unfortunately for a big number of random variables this approach is rather inefficient due to a size of the necessary design of experiments. When post-processing the sparse PC expansion results, it turned out that the reduction algorithm allowed to eliminate more than 1000 out of 1830 coefficients from the expansion (5). However, there is still more experimental points than required for sampling methods in order to guarantee an estimation that is accurate enough for the purpose of robust design optimization.

Again, as in the case of the compressor rotor-shaft, the methods based on the Taylor series expansion performed poorly when compared to other techniques. The dimension reduction methods, described in [17], completely failed to produce results of any relevance to the values of actual statistics. However, contrary to the previous numerical example, here the major reason for the observed discrepancies is not connected with the artificial excitation of the rotor-shaft vibrations due to the alignment of unbalance phase shift angles, which has been described in Sec. 4.2. It seems that for problems with a big number of random variables the values of stochastic moments obtained by DRM may be strongly affected by numerical integration errors, unavoidable when integrating non-polynomial functions using quadrature formulas. The “standard” perturbation methods yield much better estimations but still they are inferior when compared to simulation methods. The respective results are given in captions of Figs. 9 and 10. In fact, only the first order mean value estimation can be considered as a satisfactory compromise between the estimation accuracy and the computational cost.

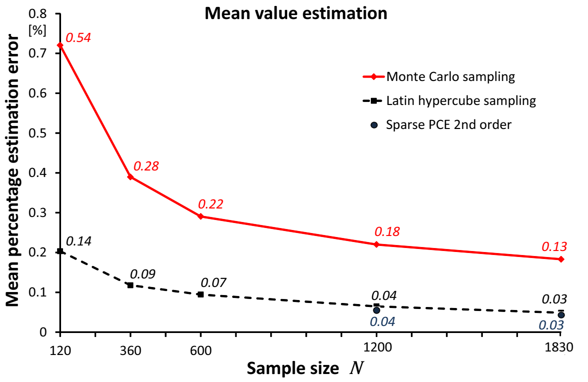

Figure 9: The turbo-generator rotor shaft example - full model. Mean relative percentage error of the mean value estimation of the maximal rotor-shaft vibration amplitude obtained using RS, LHS and sparse PCE. The graph point labels stand for standard deviations of the errors. The corresponding results for Taylor expansion methods [8]: first order – for $N = 61$ the error is 2.4%, second order – for $N = 1830$ the error is 2.3%.

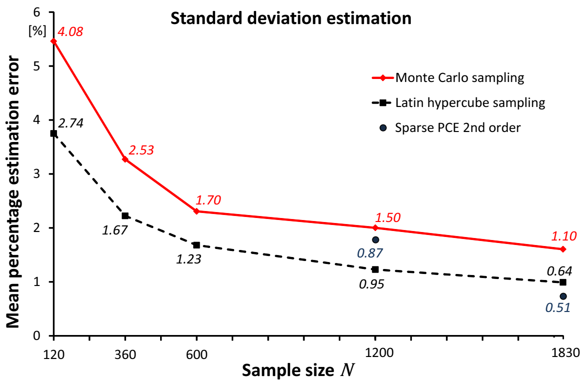

Figure 10: The turbo-generator rotor shaft example - full model. Mean relative percentage error of the standard deviation estimation of the maximal rotor-shaft vibration amplitude obtained using RS, LHS and sparse PCE. The graph point labels stand for standard deviations of the errors. The corresponding result for Taylor expansion methods [8]: first order – for $N = 61$ the error is 12.16%.

The same tests were performed for the reduced stochastic model consisting of 5 random variables. The results are shown in Figs. 11 and 12. They can be examined from two different perspectives. The first one is a comparison of estimation accuracy of the investigated methods for a given sample size. The other perspective is a selection of the sample size that provides sufficient estimation quality in order not to introduce an extensive numerical noise into the objective and constraint functions of the robust design optimization problem.

If we compare the error values computed by RS and LHS techniques obtained for $N = 120$ using the complete and the reduced model, we may notice that the respective values match quite closely. This can be considered as a proof that the adopted reduced model is representative for the complete model and the eliminated variables do not bring much to the response scatter. Therefore, if such a reduction is possible, the non-sampling techniques, such as PC expansion, which are inefficient for multidimensional problems, may turn out to be competitive with respect to LHS. Especially in estimating the standard deviation, independently of the model used, in order to get a precise estimation, e.g. with the error of ca. 2%, samples of more than 300 point are necessary for Latin hypercube sampling. On the other hand, if the second order or the third order PC expansion method is employed, such an estimation accuracy is possible in the case of the reduced model even for $N = 100$.

The results of perturbation approach as well as univariate dimension reduction method are given in captions of Figs. 11 and 12. Also in this case they are visibly worse than the estimations provided by other methods.

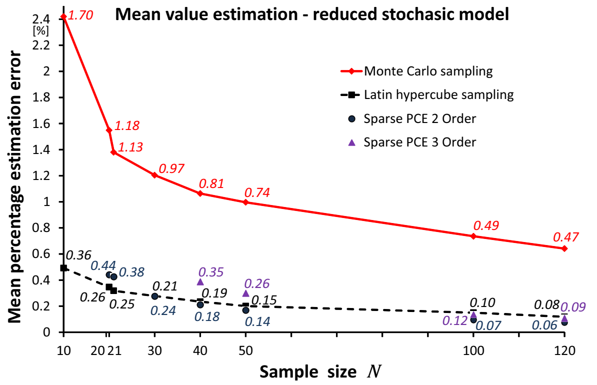

Figure 11: The turbo-generator rotor shaft example - reduced model. Mean relative percentage error of the mean value estimation of the maximal rotor-shaft vibration amplitude obtained using RS, LHS and sparse PCE. The graph point labels stand for standard deviations of the errors. The corresponding results for Taylor expansion methods [8]: first order – for $N = 6$ the error is 2.71%, second order – for $N = 15$ the error is 2.69%. Estimation by UDR [17]: for $N = 25$ the error is 1.59%.

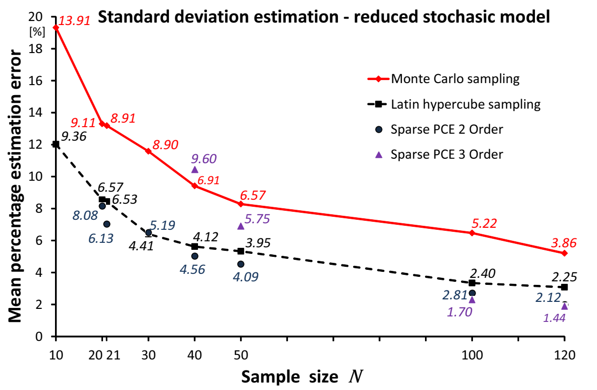

Figure 12: The turbo-generator rotor shaft example - reduced model. Mean relative percentage error of the standard deviation estimation of the maximal rotor-shaft vibration amplitude obtained using RS, LHS and sparse PCE. The graph point labels stand for standard deviations of the errors.The corresponding result for Taylor expansion methods [8]: first order – for $N = 6$ the error is 12.98%. Estimation by UDR [17]: for $N = 25$ the error is 10.45%.

6. Conclusions

The objective of this study was to examine feasibility of various stochastic moment estimation methods for their use in robust design optimization (RDO) of vibrating rotor-shaft systems. The observed scatter of the rotor-shaft vibrational responses is mainly due to the uncertainty of residual unbalances as well as random characteristics of stiffness and damping coefficients of the journal bearings. Since in popular RDO formulations the objective function and design constrains are defined in terms of mean values and variances of selected structural performance functions, the efficiency of stochastic moment estimation is crucial for numerical complexity and convergence of the RDO process.

The following methods were compared: the sampling methods (classical Monte Carlo and Latin hypercube sampling), the perturbation approach, the dimension reduction method and the polynomial chaos expansion method. To evaluate usefulness of a particular method, the mean and standard deviation of the maximal lateral vibration amplitude were estimated for two rotor shafts: the centrifugal compressor rotor-shaft and the turbo-generator rotor-shaft. The vibration analysis was carried out by means of the hybrid structural model consisting of one-dimensional beam-like continuous visco-elastic macro-elements and discrete oscillators. Such a hybrid model proved to be very computationally efficient and reliable, which is of a major importance in the context of stochastic analysis.

A proper representation of uncertain parameters of the rotor-shaft systems may lead to large stochastic models. In the analyzed examples they consisted of 64 and 59 random variables for the compressor rotor-shaft and the turbo-generator rotor-shaft, respectively. In the second case it was shown that the original model could be reduced to 5 variables, which are the main sources of the response scatter observed using the full model.

The methods based on the Taylor series expansion performed poorly in both considered cases. For multidimensional problems the dimension reduction technique seems to suffer from using inaccurate numerical integration scheme. On the other hand, the classical perturbation approach produced acceptable results only for the mean value estimation of the turbo-generator vibrational response. Better estimations were obtained when the reduced stochastic model was employed. Still, they were inferior with respect to the other investigated techniques.

The polynomial chaos expansion method provided stochastic moment assessment of comparable accuracy with the simulation techniques. However, even using the algorithm of eliminating insignificant expansion terms, i.e. sparse PC expansion, it may be of little practical use for problems involving many (tens, hundreds) random variables. On the other hand, if a reduced stochastic model is available, these methods may turn out to be more efficient than methods based on Latin hypercube sampling.

However, for RDO of the rotor-shaft systems, when the design changes during the optimization process and it is not possible to determine in advance the random variables influencing the response scatter, a "safe" solution seems to be the Latin hypercube sampling. Even for relatively small samples it gives the stochastic moment estimation that is sufficient for the purpose of the response surface based RDO.

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