VIBRATIONS
IN PHYSICAL SYSTEMS

Volume XXV

Editors

Czesław CEMPEL, Marian W. DOBRY

Poznan 2012

VIBRATIONS IN PHYSICAL SYSTEMS
Vol. 25, 2012

ISBN 978-83-89333-43-8

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PUBLISHED AND PRINTED
Agencja Reklamowa COMPRINT
ul. H. Rzepeckiej 26 A, 60-465 Poznan, Poland
Tel. +48 602 26 64 26, e-mail: comprint@data.pl, www.comprint.com.pl

The volume was published in the cooperation of the Institute of Applied Mechanics at Poznan University of Technology and Polish Society of Theoretical and Applied Mechanics.

This journal has been indexed in the list of Ministry of Science and Higher Education (6 points).

POZNAN UNIVERSITY OF TECHNOLOGY
MECHANICAL ENGINEERING AND MANAGEMENT FACULTY
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XXV Jubilee Symposium

VIBRATIONS
IN PHYSICAL SYSTEMS

2012

XXV JUBILEE SYMPOSIUM VIBRATIONS IN PHYSICAL SYSTEMS

Będlewo (near Poznań), May 15-19, 2012

Present edition of Symposium Vibrations in Physical Systems is under auspices of

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FINANCIAL SUPPORT

Ministry of Science and Higher Education
Mechanical Engineering and Management Faculty

XXV Symposium *Vibrations in Physical Systems,* Poznan – Bedlewo, May 15-19, 2012

Numerical Analysis of Vibration in a Brake System for High Speed Train

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Abstract

The paper is devoted to a computer simulation of brake pad/brake disc dynamic interaction. The main purpose of the studies is a numerical analysis of friction pair dynamics, aiming at describe generation of vibration related to the process of transition phenomena associated with braking. To investigation will be used environment of Automatic Dynamic Analysis of Mechanical Systems. In the analysis is used two-dimensional friction model. The results show the slip-stick and creep phenomenon.

Keywords: vibrations excited by friction, self-excited of vibrations, slip-stick, creepage.

1. Introduction

Self-excitation of vibrations due to dry friction commonly found in brake systems. Many research connected with determination of causes of vibrations have been undertaken. In questions of that nature it is necessary to determine criteria for modelling such phenomena, considering the principal factor such as dynamic non-linear friction. Developing models of interact friction pair of brake systems in an era of increasing vehicle speed and increase their weight is very important. In the case of braking systems for high speed train introduced sintered pads (Fig. 1).

Figure. 1. Brake system for high speed train and sintered brake pads (*source: www.gobizkorea.com*).

Such a segmented structure can faster conduct and radiate the heat from the brake pad. The dynamic interaction between elements of the brake pad may increase or decrease the vibration depending on configuration. It seems to be dependent on mode of pad elements vibration (in-phase or out of phase).

Elaboration of a method of decrease and damping of unfavourable vibrations requires, among others, building of correct numerical model simulating, as far as possible, accurately the system under investigation. To identify the interactions of friction pairs requires understanding the dynamics of the friction transient process [1, 2]. Numerous investigation in research units all over the world are engaged in analysis of the

self-excited vibration in brake systems. Problem of “squeaking” brakes has been investigated to considerable extent in automotive industry. Author in paper [3] gave comprehensive preview on that phenomenon with regard to vibrations and contact forces. He presented both experimental and numerical study. Majority of projects connected with modelling of brake systems was based on finite elements method. The authors of paper [4] adopted the model with two degrees of freedom for disc brakes where the disc and pads are modelled as particular kinds of connections through the interface of friction and stability. In their approach also analysis of limit cycles was executed. Some methods of brake discs vibration analysis are presented in the studies [5] and [6]. Brake disc components were modelled with application of the plate finite elements. The influence of non-linearity of contact forces on generation of low and high frequency noise in brake discs was subjected to experimental and analytical investigation by authors [7]. The researchers observed that time-frequency analysis is very useful in identification of character of generated noise.

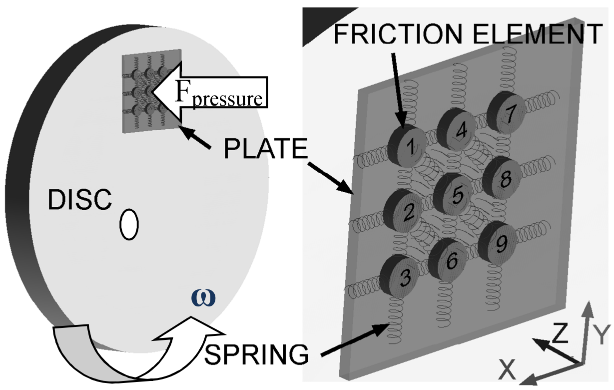

Figure. 2. Model of brake system.

2. Model of brake system

The model (Fig. 2) consider consist of 11 moving parts (friction elements, disc and plate); therefore the number of parameters describing configuration is 11 x 3 = 33 used by the pre-processor to build the set of equations of motion. The model of brake system contains: one revolution joints, one translation joint, nine planar joint and one rotation joint motion. The total number is 11, what make together 20 degrees of freedom. The friction elements are flexibly connected between each other. Outer elements are flexibly connected with a plate. A force F directed along the Z axis operates by the plate on all friction pairs.

2.1. Equations of motion

The brake model is based on a Cartesian coordinates approach for the assembly of the equations of motion. The Euler parameters are used to represent the rotational degrees of freedom and Lagrangian formulation for the assembly and generation of equations of motion. The joints between bodies are expressed in a set of algebraic equations, subsequently assembled in a second derivative structure, obtaining finally a set of Differential Algebraic Equations in the following packable form:

$$\begin{bmatrix} \mathbf{M}(\mathbf{q}) & \mathbf{\Phi}(\mathbf{q})^T \\ \mathbf{\Phi}(\mathbf{q}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}, t) \\ \chi \end{bmatrix} \tag{1}$$

The symbols $\mathbf{M}$ is the mass matrix, $\mathbf{q}$, $\boldsymbol{\lambda}$ and $\mathbf{F}$ denote, respectively, the generalized coordinates, the Lagrange multipliers and the generalized forces applied to the rigid bodies vectors. Symbol χ the right-hand-side of the second derivative the constraint equations.

2.2. Description of contact

Model of contact used in our analysis lets you define a two-dimensional contact between a pair of geometric objects. Using the contact as a unilateral constraint, as a force that has zero value when no penetration between the specified elements exists, and a force that has a positive value when exists penetration between elements friction and disc. The model of contact describes the following formula (2).

Both the static and quasi-static equilibrium analysis modes use Newton-Raphson (NR) iterations to solve the nonlinear algebraic equations of force balance. The NR algorithm ensures that the system solution moves in the direction of most compliance (least stiffness). When a contact is active, the stiffness in the direction of the normal force is high, so the NR algorithm modifies the system states to decrease this force. If a contact is inactive, there is no stiffness in the direction of increasing contact.

$$F = \begin{cases} \max(k(x_p - x_k)^e - c\dot{x}, 0) & \text{for} \quad x_p \le x_k \\ 0 & \text{for} \quad x_p > x_k \end{cases}, \tag{2}$$

$$c(p) = \begin{cases} 0 & \text{for} \quad b \le 0 \\ c_{max}\left(\frac{3}{l^2}b^2 - \frac{2}{l^3}b^3\right) & \text{for} \quad 0 < b \le l \; . \\ c_{max} & \text{for} \quad b > l \end{cases} \tag{3}$$

To describe the contact model following parameters are used: c is damping parameters of contact, k is contact stiffness, x_p initial displacement of contact, x_k is displacement of the penetration (b=x_p- x_k), p is the maximal value of penetration. Parameter of damping coefficient c corresponds to energy dissipation during the contact. From the above mentioned formula (3) it follows that the value of coefficient c depends on the penetration b the friction element in the disc. Damping coefficient c can reach a maximum value after reaching required penetration l. During the further penetration the damping is constant ($c = c_{\text{max}}$).

2.3. Friction model

In order to determine the contact friction force in investigations used a velocity-based friction model of contact. The figure below shows how the coefficient of friction varies with slip velocity.

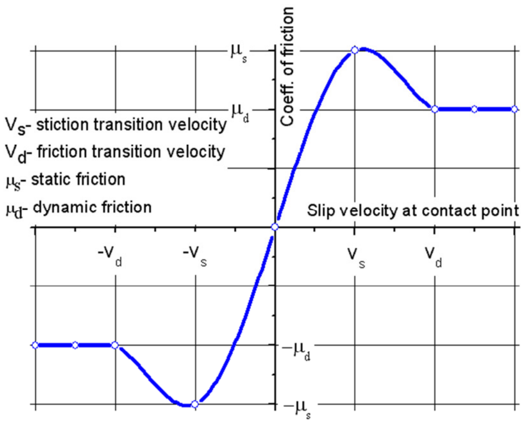

Figure. 3. Model of Friction.

In this model:

$$\begin{aligned} &\mu(-v_s) = \mu_s \\ &\mu(v_s) = -\mu_s \\ &\mu(0) = 0 \\ &\mu(-v_d) = \mu_s \\ &\mu(v_d) = -\mu_d \\ &\mu(v) = -\text{sign}(v)\cdot\mu_d && \text{for } |v| > v_d \\ &\mu(v) = -\text{step}(|v|, v_d, \mu_d, v_s, \mu_s)\cdot\text{sign}(v) && \text{for } v_s < |v| < v_d \end{aligned} \tag{4}$$

$$\mu(v) = \text{step}(v, -v_s, \mu_s, v_s, -\mu_s) \qquad \text{for } -v_s < v < v_s\ .$$

Parameter v_s is the velocity at which full value of the static friction coefficient is applied. v_d is the velocity at which the value of the dynamic friction coefficient has fully transitioned from the static friction coefficient.

3. Computational example

The study on the numerical model provided us with some interesting observations shown in following graphs. For all results presented below assumed constant values of parameters: coefficients friction, stiffness of spring, force of pressure on the friction elements (normal force). They are μ_s=0.6, μ_d = 0.3, k_{spring} = 400 N/mm, F_{press}= 300 N. During the tests changed angular velocity of the disc ω, stiction friction transition velocity v_s and friction transition velocity v_d. Because the interaction of neighboring friction elements the amplitude of vibration of some the elements under consideration decreases with time (Fig. 3). Such behavior is confirmed by studies [1]. The authors introducing an external excitation, to reduce the vibration amplitude of the masses moving on the conveyor.

It was also found that reducing the angular velocity of the disc causes the phenomenon of stick-slip. Stick-slip phenomenon is also dependent on the value of the parameter v_d included in the model of friction. In case of increase in v_d above the 33 mm/s, this phenomenon disappears passing to periodic oscillations (Fig. 4).

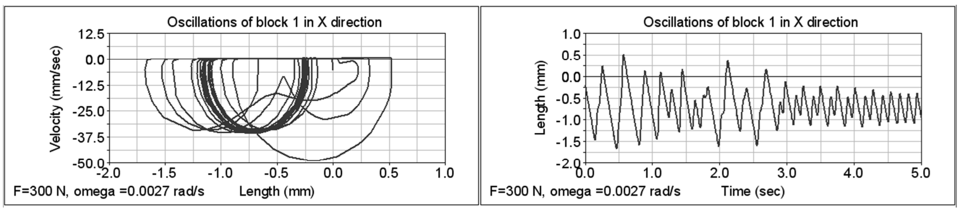

Figure. 3. Phase trajectory and time-histories of oscillation friction element no 1.

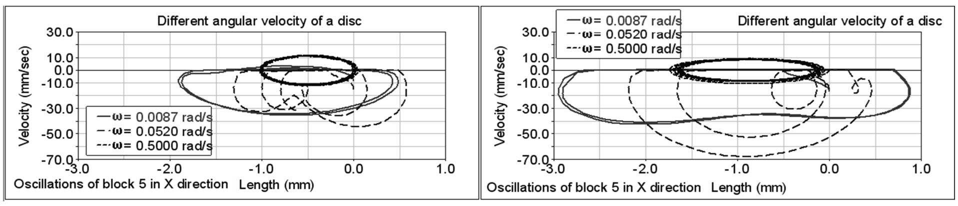

Figure. 4. Phase trajectory by different angular velocity of disc.

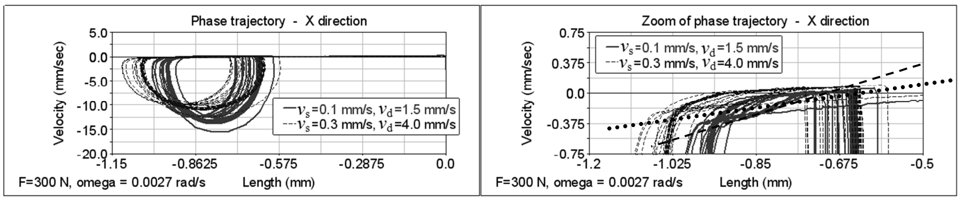

Figure. 5. Phase trajectory with creep effect at different velocity (v_s, v_d) describing curve of friction model– differences of creepage marked by dot and dashed lines.

In between the range of stick-slip and periodic vibration, creep phenomena can be observed (Fig. 5). This creepages is marked by dot and dashed lines. Differences in the creep process marked by straight lines on graphs phase trajectories (Fig. 5) are caused by different value of the velocity v_s and v_d of friction model. The values of these velocity affect on different slope of the curve of friction model (Fig. 3).

4. Conclusions

Stick-slip and creepage phenomena obtained in the numerical model of brake system has been presented. The studies also confirmed that introduction of an additional excitation to the system can reduce the amplitude of vibration of the system. In many papers the authors introduced external harmonic excitation into the system to reduce vibrations. In our case excitation generated by vibration of neighbouring friction elements of the model may reduce amplitude vibration depending on configuration of elements (Fig. 4).

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