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Time-domain identification of damage in skeletal structures using strain measurements and gradient-based optimization

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Abstract

This paper presents an improved numerical tool for identification of damage in skeletal structures. The problem of identification has been formulated in the time domain within the framework of the Virtual Distortion Method (VDM). VDM generally belongs to fast structural reanalysis methods and can be applied to Structural Health Monitoring problems, among others. The major computational asset of VDM is the influence matrix, containing all the local-global inter-relations for a structure due to given perturbations e.g. initial strain or external force. A non-linear least squares problem with strains, entering the objective function, is the subject of consideration. Strains are used in order to have relatively smooth variations (compared to accelerations) of the analyzed signal in time. The change of stiffness is the design variable. Analytical gradients are implemented in the optimization code based on the Levenberg-Marquardt algorithm with some penalty function terms. The efficiency of the software tool is demonstrated for a numerical example of a 2D truss structure. A breakthrough in terms of computational time reduction has been observed compared to the previously used steepest-descent optimization. The presented software assumes the feasibility of reliable measurements of strains in time for real skeletal structures (e.g. truss bridges). Future research will include experimental verification of the idea with piezoelectric sensors acting as tensometers.

1 Introduction

The Structural Health Monitoring (SHM) has evolved over the last decade into an industry-stimulated field of interdisciplinary research. This paper fits in the vibration-based SHM [1] providing a global view on the dynamic behaviour of a structure as a whole. A calibrated model of the structure is necessary at start. Then, the identification of damage is performed by subsequent updating of selected parameters of the reference model. The basis for analysis is the Virtual Distortion Method (VDM), which belongs to fast reanalysis methods of structural mechanics [2].

The first VDM-based approach to identification of damage was proposed in [3]. The feasibility of the idea was presented using some numerical examples and a simple experimental validation. A serious drawback of the approach was a considerable computational time due to a very simple optimization algorithm used. One way to overcome the problem was to develop an analogous VDM-based approach in the frequency domain [4]. An alternative proposition, described in the present paper, is to equip the time-domain identification algorithm with a more sophisticated optimization method i.e. the Levenberg-Marquardt method plus some penalty function terms [5].

This presentation of the developed VDM-based time-domain approach of damage identification in skeletal structures is limited to numerical tests only. However, the authors firmly believe the approach has a great potential to be applied to monitoring of real engineering structures, in particular truss bridges. The experimental verification of the successful numerical analysis will be the subject of future tests, both in laboratory as well as in field. A monitoring campaign started on a railway bridge in 2007 [6] will be continued to demonstrate the validity of the proposed approach. Piezoelectric strain sensors will be used for collecting the dynamic response of the bridge because of relatively smooth variation of strain in time, which should be of benefit for a time-domain analysis.

The organization of the paper is as follows. Section 2 makes the reader briefly acquainted with the VDM method. Section 3 presents the formulation of the damage identification problem and discusses the related optimization issues. Section 4 provides a detailed insight into numerical results of damage identification for a 2D truss structure. Section 5 conludes the paper.

2 Virtual Distortion Method in brief

2.1 General idea

The VDM belongs to fast reanalysis methods in structural mechanics. This implies an initial FEM response, which is further modified by introduction of proper fields of virtual distortions. A review of selected reanalysis methods, including VDM, is provided in [2].

The VDM has been used in various problems of structural design (e.g. prestress), optimization (e.g. topology remodelling) and control (e.g. damping of vibrations) thus far. It has also proved to work in SHM. The first time-domain formulation of the damage identification problem assumed only stiffness degradation as a damage modelling parameter. Low-frequency impulse excitation (e.g. windowed sine pulse) was applied and dynamic time responses of a structure were captured by piezoelectric transducers, whose deformations were proportional to mechanical strains. For the sake of comparison with the first formulation, the considerations in this paper are also limited to the stiffness changes for modelling damage, although mass changes can also be included in such analysis. An overview of various VDM applications can be found in [7].

The Virtual Distortion Method (VDM) is conceptually similar to the initial strains approach. Local imposition of an initial strain leads to violation of equilibrium conditions and the solution proceeds in iterations. The VDM approach is able to produce such a solution in one step thanks to defining all local-global interrelations in a structure at start of numerical simulation. The collection of all the local-global responses, including information about structural topology, materials and boundary conditions, is called the influence matrix within the framework of VDM. This matrix makes an essential distinction between the VDM and initial strains approach.

2.2 Influence matrix

The concept of the influence matrix for truss structures in static analysis will be presented first. According to the VDM formulation [8], each component of the influence matrix D_{ij} describes strains in a truss member i caused by the unit *virtual distortion* $\varepsilon_j^0 = 1$ (i.e. unit axial tensile strain for truss structures) applied to the member j. The unit virtual distortion is imposed in numerical calculations as a pair of *self-equilibrated compensative forces* of reverse signs applied to the nodes of the strained element. Thus to build the full influence matrix D_{ij}, m solutions (m – the number of truss members) of a linear elastic problem (with m different right-hand sides) have to be found:

$$\sum_N K_{MN}\, u_N = f_M \tag{1}$$

with K_{MN} being the stiffness matrix, u_N - displacement vector, f_M - force vector in global co-ordinates. The N global displacements are transformed to i local strains by means of the geometric matrix G_{iN}:

$$\varepsilon_i = \sum_N G_{iN}\, u_N \tag{2}$$

Most often the influence matrix collects the responses in strains. However, storage of any other required responses, i.e., displacements or forces, is also useful. Note that the static influence matrix for statically determinate structures becomes identity (zero redundancy means no interrelations between members) and the VDM loses its major tool (only in statics).

Analogously, the influence matrix can be built in dynamic analysis (e.g. handled by the Newmark integration scheme), where a pair of self-equilibrated forces equivalent to unit strain is applied to a truss member (like in statics) in the first time step only. Such a perturbation introduced to the structure is called an impulse virtual distortion (see Fig. 1). Structural response to such impulse is examined over a descretized period of time. Thus, compared to statics, the influence matrix is 3 dimensional in dynamics, with 2 spatial and 1 time dimension. Generally, the dynamic influence matrix is time-dependent, however for harmonic excitation, it becomes quasi-static, because only amplitudes of responses are stored.

One should mention that the influence matrix collecting strain responses is sufficient for modelling stifness modifications in a structure. For modelling inertia modifications, the displacement responses due to an out-of-balance force are required [1]. If the response in displacements is considered as output and harmonic excitation serves as input, the influence matrix is equivalent to the FRF matrix at fixed frequency.

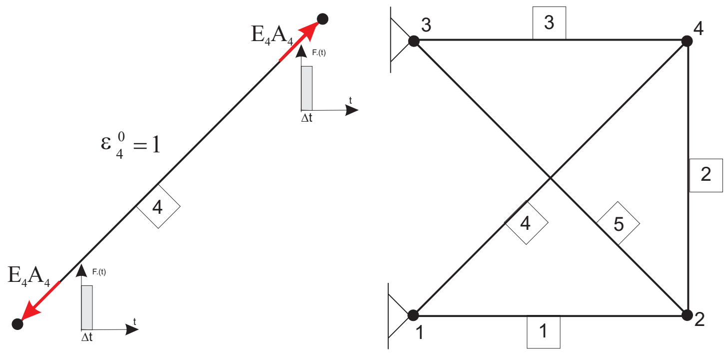

Figure 1: Impulse virtual distortion applied in an element.

2.3 VDM-based parameter modifications

Let us consider an elastic truss structure subjected to external load, provoking a *linear response* ε_i^L, p_i^L in each member i. For modelling parameter changes in the structure we introduce a field of virtual distortions ε_j^0, which provokes a *residual response* in terms of strains and forces:

$$\varepsilon_i^{\mathrm{R}} = \sum_j D_{ij}\, \varepsilon_j^0 \tag{3}$$

$$p_i^{\mathrm{R}} = E_i A_i \sum_j (D_{ij} - \delta_{ij})\, \varepsilon_j^0 \tag{4}$$

The overall response of the structure is a superposition of both the responses:

$$\varepsilon_i = \varepsilon_i^{\mathrm{L}} + \varepsilon_i^{\mathrm{R}} \tag{5}$$

$$p_i = p_i^{\mathrm{L}} + p_i^{\mathrm{R}} = E_i A_i\, (\varepsilon_i - \varepsilon_i^0) \tag{6}$$

Let us now take into account structural stiffness modifications exemplified by changes of Young's modulus of a member with a modified value $\hat{E}_i$. On the one hand, Equation (6) provides us with element forces in the *distorted structure* i.e. the *original structure* influenced by virtual distortions supposed to model stiffness changes. On the other hand, the element forces in the actually *modified structure* can be expressed as follows:

$$\hat{p}_i = \hat{E}_i\, A_i\, \hat{\varepsilon}_i \tag{7}$$

The main postulate of the VDM in static remodelling requires that local strains (including plastic strains) and forces in the distorted and modified structures are equal:

$$\varepsilon_i = \hat{\varepsilon}_i \tag{8}$$

$$p_i = \hat{p}_i \tag{9}$$

which leads to the following relation:

$$E_i\, A_i\, (\varepsilon_i - \varepsilon_i^0) = \hat{E}_i\, A_i\, \varepsilon_i \tag{10}$$

Equation (10) provides the coefficient of the stiffness change μ_i for each truss element i as the ratio of the modified parameter $\hat{E}_i$ to the initial one E_i:

$$\mu_i = \frac{\hat{E}_i}{E_i} = \frac{\varepsilon_i - \varepsilon_i^0}{\varepsilon_i} \tag{11}$$

Note that the coefficient μ_i may be equivalently expressed as the ratio of the initial to modified cross-sectional area of a truss element (if the related mass modification is disregarded). If $\mu_i = 1$ we deal with an intact structure. Variation of the coefficient in the range $0 \leq \mu_i \leq 1$ means degradation of stiffness and in the range $\mu_i \geq 1$ increase of stiffness. Substituting (3), (5) into (11) we get a set of equations for ε_j^0, which must be solved for an arbitrary number of modified elements (usually small compared to all elements in the structure), described by a coefficient μ_i different than 1:

$$\sum_j \left(\delta_{ij} - (1-\mu_i)\, D_{ij}\right) \varepsilon_j^0 = (1-\mu_i)\, \varepsilon_i^{\mathrm{L}} \tag{12}$$

In dynamics, a residual response is a discrete convolution of the influence matrix and virtual distortions. The time-dependent residual strain and force vectors can be expressed as follows (cf. (3), (4)):

$$\varepsilon_i^{\mathrm{R}}(t) = \sum_{\tau=0}^{t} \sum_j D_{ij}(t-\tau)\, \varepsilon_j^0(\tau) \tag{13}$$

$$p_i^{\mathrm{R}}(t) = E_i A_i \sum_{\tau=0}^{t} \sum_j \left(D_{ij}(t-\tau) - \delta_{t\tau}\delta_{ij}\right) \varepsilon_j^0(\tau) \tag{14}$$

All the relations for statics are valid for dynamics as well. While most quantities vary in time, the stiffness change coefficient, defined analogously to (11), remains time independent:

$$\mu_i = \frac{\hat{E}_i}{E_i} = \frac{\varepsilon_i(t) - \varepsilon_i^0(t)}{\varepsilon_i(t)} \tag{15}$$

The set of equations to be solved for distortions in dynamics looks analogously to (12):

$$\sum_j \left(\delta_{ij} - (1-\mu_i)\, D_{ij}(0)\right) \varepsilon_j^0(t) = (1-\mu_i)\, \varepsilon_i^{\neq t}(t) \tag{16}$$

where $\varepsilon_i^{\neq t}(t)$ denotes the strains cumulated without the effect of the current-step virtual distortion:

$$\varepsilon_i^{\neq t}(t) = \varepsilon_i^{\mathrm{L}}(t) + \sum_{\tau=0}^{t-1} \sum_j D_{ij}(t-\tau)\, \varepsilon_j^0(\tau) \tag{17}$$

Note that the matrix on the left-hand side of (16) is constant in time. Only the right-hand side vector of (16) has to be modified in every time step, according to (17). Similarly to (12), the set (16) may be local if structural remodelling is performed. In identification problems however, in which the location of a damaged/modified member is sought, the set (16) concerns all elements potentially changed (usually the whole structure).

3 VDM in SHM of skeletal structures

3.1 Problem formulation

The proposed approach assumes that strain measurements are collected in time by piezo-transducers. The fluctuations of strains in time are much smoother compared to accelerations, therefore the time-domain analysis based on strains can be much easier handled by the VDM.

Thus we pose the damage identification task as a nonlinear least squares minimization problem with the objective function expressed in strains:

$$F(\boldsymbol{\mu}) = \sum_k \frac{\sum_t \left(\varepsilon_k(t) - \varepsilon_k^{\mathrm{M}}(t)\right)^2}{\sum_t \left(\varepsilon_k^{\mathrm{M}}(t)\right)^2} \tag{18}$$

The function (18) collects responses from k sensors, placed in those elements where non-zero strains $\varepsilon_k^{\mathrm{M}}(t)$ of high signal-to-noise ratio are measured. The strain $\varepsilon_k(t)$ in an arbitrarily selected location k is influenced by virtual distortions ε_j^0, which may be generated in any element j of the structure (cf. (13)). One should also note that the modification coefficient μ_i, quantifying potential damage and used as a variable in optimization, depends upon the virtual distortions ε_j^0 nonlinearly (cf. (15)).

Natural constraints are imposed on the modification coefficient μ_i, which is non-negative by definition (cf. (15)):

$$\mu_i \geq 0 \tag{19}$$

If degradation of a member is considered, another constraint has to be imposed on μ_i:

$$\mu_i \leq 1 \tag{20}$$

Using (5), (13), the gradient of the objective function (18) with respect to the optimization variable μ_i is expressed as:

$$\nabla_i F = \frac{\partial F}{\partial \mu_i} = \frac{\partial F}{\partial \varepsilon_k}\frac{\partial \varepsilon_k}{\partial \varepsilon_j^0}\frac{\partial \varepsilon_j^0}{\partial \mu_i} = \sum_k \frac{\sum_t 2\left(\varepsilon_k(t) - \varepsilon_k^{\mathrm{M}}(t)\right)}{\sum_t (\varepsilon_k^{\mathrm{M}}(t))^2} \sum_{\tau=0}^{t} D_{kj}(t-\tau)\frac{\partial \varepsilon_j^0(\tau)}{\partial \mu_i} \tag{21}$$

The partial derivatives $\frac{\partial \varepsilon_j^0(t)}{\partial \mu_i}$ can be easily calculated by differentiating relation (16) with respect to μ_i:

$$\sum_j \left(\delta_{kj} - (1-\mu_k)D_{kj}(0)\right)\frac{\partial \varepsilon_j^0(t)}{\partial \mu_i} = -\delta_{ik}\varepsilon_k(t) + (1-\mu_k)\frac{\partial \varepsilon_k^{\neq t}}{\partial \mu_i} \tag{22}$$

Note that the left-hand side matrices in (16) and (22) are alike, which simplifies computations. Only the right-hand sides vary.

3.2 Optimization issues

In the previously investigated time-domain approach [3], the vector $\boldsymbol{\mu}$ of the optimization variables μ_i was updated iteratively according to the steepest descent formula:

$$\boldsymbol{\mu}_{n+1} = \boldsymbol{\mu}_n - \alpha F(\boldsymbol{\mu}_n)\frac{\nabla F(\boldsymbol{\mu}_n)}{\|\nabla F(\boldsymbol{\mu}_n)\|^2} \tag{23}$$

Subscript n denotes here the values in the current iteration and $n+1$ in the subsequent iteration. The constant α was varied in the range $0.1 \div 0.3$; the constraints were disregarded, which could lead to meaningless results in the presence of significant measurement noise. Since the steepest descent method converges linearly at best, the performance was very poor and several hundreds of iterations were necessary to accomplish the identification.

The considered identification problem is essentially a least squares parameter-fitting problem. Therefore, it is perfectly suited for the Levenberg-Marquardt method, which exhibits near the minimum a much quicker quadratic convergence rate [5]. The method is used for unconstrained optimization, hence the inequality constraints (19) and (20) have to be imposed in the form of penalty function terms. This approach seems to be readily applicable to many SHM-related optimization problems with a moderate number of variables (e.g. up to a thousand) and relatively weak constraints, see e.g. an application to load identification in [9]. If more optimization variables are necessary, handling of the approximated Hessian can become numerically too costly. In this case, other quasi-Newton methods with simpler and less accurate Hessian approximations (like BFGS) may be preferable.

Note that the uniqueness of the solution (unique global minimum of the objective function), provided the structure is statically indeterminate, can be in practice ensured by considering sufficiently large number of time samples. In the opposite case of statically determinate structures, the dynamic effects of the test load have to be large enough to provoke sufficiently different responses of the intact and damaged structures.

3.2.1 Augmented objective function and approximated Hessian

The augmented objective function F_c includes quadratic penalty terms and takes the following form

$$F_c(\boldsymbol{\mu}) = F(\boldsymbol{\mu}) + c\sum_i \left[\mu_i^2 \mathbf{1}_{\mu_i<0} + (1-\mu_i)^2 \mathbf{1}_{\mu_i>1}\right] \tag{24}$$

where $F(\boldsymbol{\mu})$ is the original objective function, see (18)). The gradient of the augmented function can be expressed as

$$\nabla_i F_c(\boldsymbol{\mu}) = \nabla_i F(\boldsymbol{\mu}) + 2c\left[\mu_i \mathbf{1}_{\mu_i<0} + (1-\mu_i)\mathbf{1}_{\mu_i>1}\right] \tag{25}$$

where the derivative $\nabla_i F(\boldsymbol{\mu})$ of the original function is defined in (21).

The Levenberg-Marquardt method approximates the exact Hessian $\mathbf{H}(\boldsymbol{\mu})$ by assuming small residuals and/or local linearity of the response:

$$\left[\varepsilon_k(t) - \varepsilon_k^{\mathrm{M}}(t)\right] \frac{\partial^2 \varepsilon_k(t)}{\partial\mu_i \partial\mu_j} \approx 0 \tag{26}$$

which yields the approximate Hessian $\tilde{\mathbf{H}}(\boldsymbol{\mu}) = \left[\tilde{H}_{ij}(\boldsymbol{\mu})\right]$ in the form

$$\tilde{H}_{ij}(\boldsymbol{\mu}) = \frac{\partial^2}{\partial\mu_i \partial\mu_j} F_c(\boldsymbol{\mu}) \approx 2\sum_k \frac{\sum_t \frac{\partial\varepsilon_k(t)}{\partial\mu_i}\frac{\partial\varepsilon_k(t)}{\partial\mu_j}}{\sum_t \left[\varepsilon_k^{\mathrm{M}}(t)\right]^2} + 2c\delta_{ij}\left(1 - \mathbf{1}_{\mu_i\in[0,1]}\right) \tag{27}$$

Note that the approximate Hessian obtained by (27) is always positive-semidefinite.

3.2.2 Iteration step

The optimization variables are updated iteratively according to

$$\boldsymbol{\mu}_{n+1} = \boldsymbol{\mu}_n + \Delta\boldsymbol{\mu}_n \tag{28}$$

where the vector of updates $\Delta\boldsymbol{\mu}_n$ is the solution of the following set of linear equations

$$\left[\tilde{\mathbf{H}}(\boldsymbol{\mu}_n) + \lambda\mathbf{I}\right]\Delta\boldsymbol{\mu}_n = -\nabla F_c(\boldsymbol{\mu}_n) \tag{29}$$

Table 1: Levenberg-Marquardt method: computations in a single optimization step

1. Derivatives of the response $\partial\varepsilon_k(t)/\partial\mu_i$
2. Gradient of the augmented objective function $\nabla F_c(\boldsymbol{\mu}_n)$
3. Approximate Hessian of the augmented objective function $\tilde{\mathbf{H}}(\boldsymbol{\mu}_n)$
4. Repeat
   (a) Optimization step $\Delta\boldsymbol{\mu}_n$ by solving $\left[\tilde{\mathbf{H}}(\boldsymbol{\mu}_n) + \lambda\mathbf{I}\right]\Delta\boldsymbol{\mu}_n = -\nabla F_c(\boldsymbol{\mu}_n)$
   (b) Actual improvement in the augmented objective function $F_c(\boldsymbol{\mu}_n) - F_c(\boldsymbol{\mu}_n + \Delta\boldsymbol{\mu}_n)$
   (c) Expected improvement of the modified objective function $0.5\Delta\boldsymbol{\mu}_{\mathbf{n}}\left[\lambda\mathbf{I}\Delta\boldsymbol{\mu}_n - \nabla F_c(\boldsymbol{\mu}_n)\right]$
   (d) Actual-to-expected improvement ratio κ

$$\kappa = \frac{F_c(\boldsymbol{\mu}_n) - F_c(\boldsymbol{\mu}_n + \Delta\boldsymbol{\mu}_n)}{0.5\Delta\boldsymbol{\mu}_{\mathbf{n}}\left[\lambda\mathbf{I}\Delta\boldsymbol{\mu}_n - \nabla F_c(\boldsymbol{\mu}_n)\right]}$$

   (e) If $\kappa < \kappa_{\min}$, then $\lambda := k\lambda$, else if $\rho > \rho_{\max}$, then $\lambda := \lambda/k$

   until $\kappa \geq 0$

The coefficient λ changes during the optimization in order to tune the step length to the actual quality of the approximation $\mathbf{H} \approx \tilde{\mathbf{H}}$. For a large λ, the system (29) becomes effectively the steepest descent formula; small λ makes it close to the second order Gauss-Newton formula, which finds the minimum of the $\tilde{\mathbf{H}}$-based quadratic approximation to the augmented objective function

$$\tilde{F}_c(\boldsymbol{\mu}_n + \Delta\boldsymbol{\mu}_n) \approx \tilde{F}_c(\boldsymbol{\mu}_n) + \Delta\boldsymbol{\mu}_n^{\mathrm{T}}\nabla\tilde{F}_c(\boldsymbol{\mu}_n) + \frac{1}{2}\Delta\boldsymbol{\mu}_n^{\mathrm{T}}\tilde{\mathbf{H}}(\boldsymbol{\mu}_n)\Delta\boldsymbol{\mu}_n \tag{30}$$

Computations necessary in each optimization step are listed in Table 1.

4 Numerical example

A simple 2D truss structure has been chosen for demonstration of the performance of the improved VDM-based numerical tool. The truss, depicted in Fig. 2, consists of 20 steel elements of $E = 210\,\mathrm{GPa}$, $\rho = 7800\,\mathrm{kg/m^3}$ and uniform cross-sectional area $A = 1\,\mathrm{cm}^2$, grouped in 4 sections of 1 m by 1 m.

The considered damage scenario assumes three elements with reduced stiffness, characterized by the following modification coefficients $\mu_1 = 0.8$, $\mu_{14} = 0.4$ and $\mu_{18} = 0.6$. Only one strain sensor has been placed in element No. 6. The structure is excited at node No. 9 by the impulse force of magnitude $P = 1\,\mathrm{kN}$ applied in the first step of Newmark analysis. The corresponding responses of the intact and damaged structures are shown in Fig. 3. No noise has been accounted for.

The results of identification for the assumed damage scenario are presented in Fig. 4 using both the steepest descent (SD) and Levenberg-Marquardt (LM) approaches. The quality of identification results is very good, however the accuracy of the LM method is higher, which can be analyzed in detail in Fig. 5. Anyway, the most apparent difference between the two optimization approaches is manifested in Fig. 6. At the assumed drop of the objective function by 5 orders of magnitude to terminate the analysis, the LM method converges

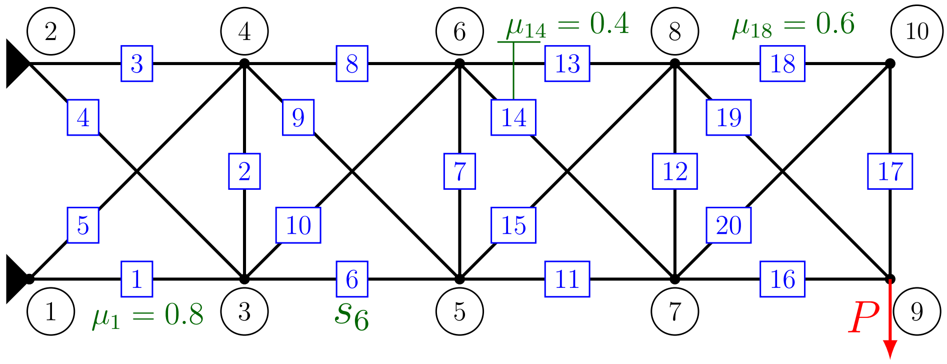

Figure 2: 2D truss structure with the applied damage scenario and sensor s_6.

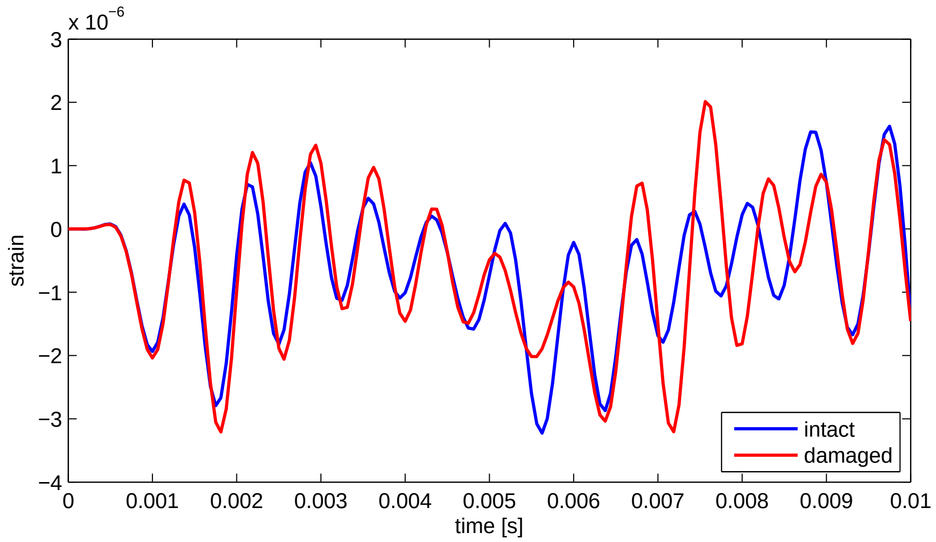

Figure 3: Strain responses of the intact and damaged structures collected by sensor s_6.

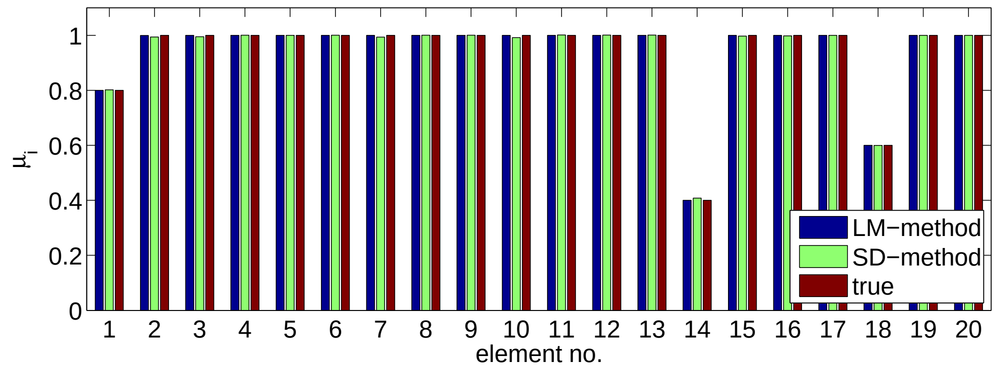

Figure 4: Results of damage identification using the LM and SD methods.

to the optimum in 5 s while the SD method in 1540 s. So the LM outperforms SD by more than 2 orders of magnitude, considering the computational time criterion, which is the expected breakthrough.

The VDM-based damage identification algorithm (in the previous and proposed versions) has been implemented using three compilers: Java, C++ and Fortran. The results presented in Figures 3–6 have been obtained with C++. Comparisons of performance of all the compilers in terms of computational effort and identification accuracy (worst value of all structural elements selected) for both the LM and SD methods

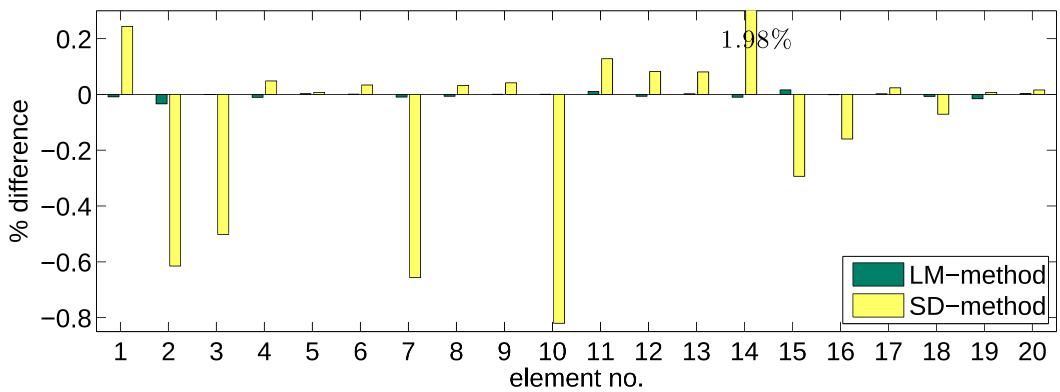

Figure 5: Accuracy of results by both methods with respect to the assumed (ideal) damage scenario.

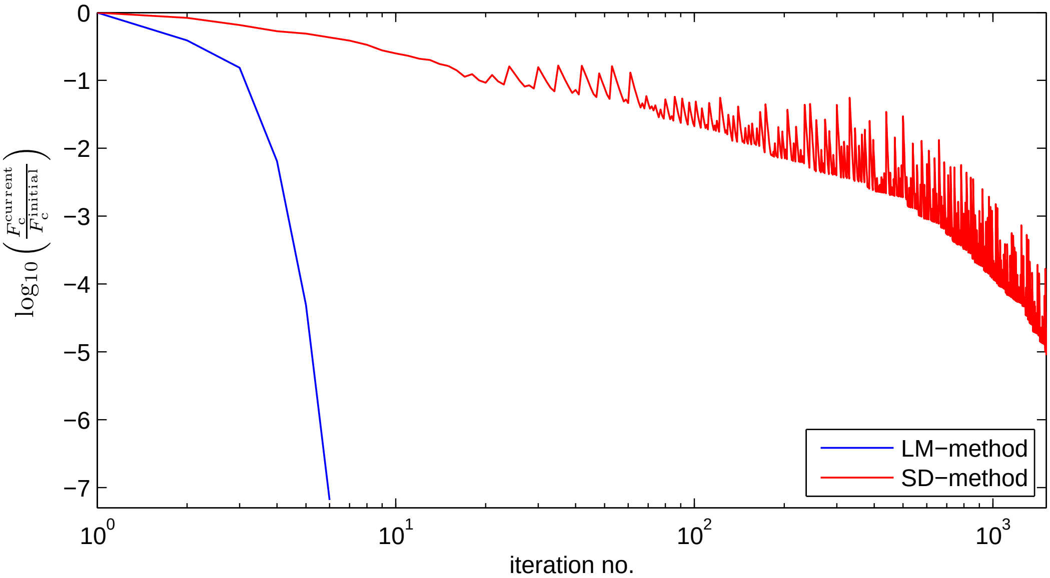

Figure 6: Convergence of the LM and SD methods at the assumed termination criterion.

have been gathered in Table 2. All computations were carried out on a 1.66 GHz Intel Core Duo processor.

5 Conclusions

The Virtual Distortion Method, which belongs to vibration-based methods of SHM by model updating, has been used to formulate the problem of damage identification in skeletal structures. Purely numerical results are presented. The inverse analysis is carried out in the time domain making use of "measured" strains, whose variations in time are relatively smooth (compared to accelerations), which is a great advantage from the viewpoint of signal processing. Damage is understood here as a degradation of structural stiffness. A loss of mass was not accounted for in order to be able to make comparisons with the previously proposed approach [3]. An essential modification has been done in the optimization part of the software i.e. the steepest-descent formula has been replaced with the Levenberg-Marquardt method with some penalty function terms. The major achievement of the presented numerical tool versus the previous one is a breakthrough in terms computational time reduction by roughly 2 orders of magnitude.

Further work will be focused on incorporation of mass modifications to the algorithm and verification of its capabilities by analyzing various damage scenarios applied to real truss structures both in lab and field tests.

Table 2: Results by various compilers

	Java	C++	Fortran
time [s] (LM)	57	5	10
time [s] (SD)	8182	1540	518
iterations (LM)	6	5	9
iterations (SD)	1711	1510	504
error [%] (LM)	~ 0.00	0.03	0.02
error [%] (SD)	2.14	1.98	~ 0.00

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