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STRUCTURAL HEALTH MONITORING – A REVIEW WITH THE EMPHASIS ON LOW-FREQUENCY METHODS

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Structural Health Monitoring (SHM) is a fast-developing, interdisciplinary field of research having its roots in vibroacoustics and non-destructive testing and evaluation. Fast development of the area is due to the fact that SHM is heavily stimulated by the engineering problems of maintenance and safe operation of technical infrastructure. The use of SHM is slowly becoming a standard in high-cost, modern infrastructure. Therefore, the possibility of application should always be on the horizon of any related research work. Thus far, the majority of SHM applications have been demonstrated in civil, aerospace and mechanical engineering. This paper reviews the main achievements and points out the current trends in this field with the emphasis on low-frequency methods.

1. Introduction

The point of this publication is to make the Reader acquainted with the historical milestones and up-to-date trends in the field of Structural Health Monitoring. A concise and informative definition of Structural Health Monitoring can be found in Sohn *et al.* [72]: "The process of implementing a damage detection strategy for aerospace, civil, and mechanical engineering infrastructure is referred to as *Structural Health Monitoring* (SHM). The SHM process involves the observation of a system over time, using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of the system's health." The damage state of a system can be considered as a five-step process, as discussed in Rytter [70]. It can be described by answering the following questions:

1. Is there any damage in the system (existence)?
2. Where is the damage in the system (location)?
3. What kind of damage has occurred (type)?
4. How severe is the damage (extent)?
5. How much useful life remains (prognosis)?

The importance of diagnostics was probably first appreciated by the community of mechanical engineers dealing with *rotating machinery*, where damage involves high risk to staff and high cost of repair e.g. in turbines. NATKE, CEMPEL [61] present *vibroacoustics* as a tool for machine diagnostics, which still remains a challenge in practice as shown in EISENMANN, EISENMANN [19]. The specific feature of rotating machinery is that it is self-exciting and relatively compact (not of huge dimensions), which is often not the case in SHM. The machine diagnostics is usually a low-frequency problem.

Many SHM methods originate from the *Non-Destructive Testing and Evaluation* (NDT/E) methods i.e. ultrasonic testing, radiographic testing, acoustic emission, penetrant testing, magnetic particle inspection, eddy currents, and optical holography, which are successfully applied in the industry for local detection of flaws in structural components. Some NDT/E methods require external excitation, for instance ultrasonic testing, others do not, for instance acoustic emission. The NDT/E methods usually operate in high frequencies.

The objective of SHM is to create a monitoring system (possibly for the whole structure, which is sometimes of complicated topology and considerable dimensions), able to track changes in structural condition continually and raise appropriate alerts if a defect is detected. As a consequence of the evolution from the two major streams, i.e. machine diagnostics and NDT/E, the SHM methods for identifying structural damage can be roughly split into low-frequency methods (non-ultrasonic) and high-frequency methods (ultrasonic), respectively. As examples, low-frequency methods are used in civil engineering for examining stiffness degradation of a bridge, whilst high-frequency methods are used in aerospace engineering for crack identification in a wing.

As the author deals with low-frequency methods in his research, the focus of the article is naturally moved towards these methods. Special attention is paid to *system identification* as the basis for most low-frequency SHM methods currently used in engineering practice. Subsequent *damage identification* widely relies on the system identification methods.

The organization of the article is the following: Sec. 2 describes the measuring devices and techniques commonly used in SHM. Section 3 takes up the problem of system identification, which is a crucial issue in low-frequency SHM, because a well-recognized reference structure is needed for tracking subsequent changes in behaviour due to damage. Section 4 briefly characterizes the low-frequency (vibration-based) methods. Section 5 describes the NDT/E methods, which gave an impetus for the development of high-frequency (ultrasonic) SHM methods, the quick review of which is presented in subsequent Sec. 6. Section 7 reports the applications of artificial intelligence to perform *signal processing* or damage identification. Section 8 mentions examples of testing the SHM methods on real structures. Finally, Sec. 9 highlights modern trends in the field. A selection

of the essential references, including the periodically-held, key events on SHM [51, 25, 21, 74], the related events [3, 29, 42] and the relevant journals in the field [7, 17, 67] is enclosed.

2. Measuring devices and techniques in SHM

Almost all the SHM methods analyze a structural response due to excitation with an actuator. In order to capture the response, various sensors are mounted on the structure. This chapter briefly describes the most frequently used devices and techniques for making accurate measurements. The quality of the measurements is absolutely essential in SHM.

2.1. Piezoelectric transducers

Probably the most common measuring devices used in SHM are piezo-electric transducers because of their outstanding electromechanical properties, relatively low price, and both actuating and sensing capabilities. They have been used for years in classical NDT/E methods like ultrasonic testing or acoustic emission. An in-depth presentation of piezo-electric sensors is given in Gautschi [27].

The direct piezoelectric effect, utilized in sensors, is present when a mechanical deformation of the piezoelectric material produces a proportional change in the electric polarization of that material (electric charge appears on opposite faces of the material). The converse piezoelectric effect, utilized in actuators, means that an acting external electric field induces proportional mechanical stress in the piezoelectric material (the material is deformed when an electric voltage is applied).

A rather restricted number of piezoelectric materials have been found suitable for transduction elements in piezoelectric sensors. Basically, natural (e.g. quartz, tourmaline) and synthetic single crystals (e.g. gallium orthophosphate, crystals of the CGG group), piezoelectric ceramics (e.g. lead-zirconate-titanate – denoted as PZT), and thin films (e.g. polyvinylidene fluoride – denoted as PVDF) can be used. Piezoelectric materials used in sensors combine excellent mechanical properties with a high piezoelectric sensitivity at a low production cost. They have a number of advantages, which makes them particularly suitable for dynamic measurements. Piezoelectric sensors have extremely high stiffness (their deflections are usually in the μm range), high natural frequency (hundreds of kHz), wide measuring range and wide operating temperature range. The crystal-based sensors have very high stability whereas the ceramic-based ones can be produced in commercial quantities. Quasi-static measurements are possible with sensors having single crystals as transduction elements.

Piezoelectric sensors can directly measure the force, strain, acceleration, and pressure.

2.2. Fibre optics

Another widespread sensor with a growing number of applications in SHM is an optical fibre. An extensive review of various types of optical fibre sensors is presented in Udd [82] and Briley [9].

The phenomenon of guiding light by a transparent cylinder has been known since the antique era (Egyptians). The early light-guiding materials (glass) had very high optical loss parameters i.e. hundreds or thousands decibel per kilometre. A real technology breakthrough occurred in 1966, when Kao and Hockham [44] envisaged the fabrication of a glass fibre with optical loss lower than 20 dB/km and its potential application in telecommunications.

Modern optical fibres are composed of high-silica glass doped with some oxides to achieve a required refraction index. An optical fibre consists of a core encapsulated in a cladding with a smaller refraction index. This enables total internal refraction at some incidence angle of entering light. Optical fibres can be generally divided into two types: multi-mode and single-mode. Of special importance in strain sensor technology are single-mode optical fibres. Single-mode fibre limits its guidance capability for a chosen wavelength to one mode thanks to small diameter of the core and small difference between the core and cladding refraction indices. In practice however, instead of just one mode, two orthogonally-polarized, strongly-coupled modes are carried by most commercially produced fibres. This effect, called *birefringence*, is often preferable since it helps the fibre to maintain the polarization of a guided wave and to transmit it long distance. Light launching into a single-mode fibre of small core is difficult and favours a light source with highly directional output, e.g. light emitting diode or injection laser diode.

The major division of the fibre optic sensors distinguishes the interferometric sensors for outside application and fibre Bragg grating sensors for inside application (embedded in the structure).

The most widespread interferometric sensors interrogate a measurand-induced change of phase in the light propagating along a single-mode optical fibre. Several different layouts of interferometric sensors may be used, depending on arrangement of optical paths. The most common configurations, based on phase change analysis, are the Michelson and Fabry-Perot interferometers. Interferometric optical fibre sensors provide high-sensitivity measurements. They require a special signal recovery technique (demodulation) to perform absolute measurements. Development of low-cost fabrication methods that do not compromise the strength and fatigue life of the optical fibres should facilitate the wide use of the sensors.

Fibre Bragg grating (FBG) sensors are highly sensitive devices as well. Their manufacturing process is automated and ensures no strength loss of the optical fibre. The intracore Bragg grating fibre optic sensor relies on the narrow-band

reflection from a fibre segment of periodic variations (gratings) in the core index of refraction of a single-mode fibre. If FBG sensors are used with the ratiometric demodulation system, they can build robust, absolute-measurement, low-cost sensing system, which can be integrated on an optoelectronic chip, easily interconnected with the structure.

Interferometric optical fibre sensors and intracore FBG sensors have a great potential to become widely used instruments for strain-like measurements.

2.3. Other sensors

Other sensors used in SHM are: electro-magnetic acoustic transducers (EMAT) widely used in ultrasonic testing (see Sec. 5), micro-electro mechanical systems (MEMS), and laser interferometers – used in optical holography (see Sec. 5).

3. System identification

System identification is the process of developing a faithful mathematical model of a physical system using experimental data. The point of vibration-based system identification is determination of modal characteristics of a structure using either known or unknown excitation. It is a *sine qua non* stage of the vast majority of low-frequency SHM analyses, which is always carried out before the commencement of the damage identification procedure. As a result of system identification a reference model for SHM is determined, thereby enabling the tracing of subsequent states of damage.

System identification theory stems basically from the control theory. In this work a brief summary of system identification is given, emphasizing the methods used in SHM. Simple peak picking method was the origin of modal testing in the frequency domain. However, some subsequent and more sophisticated methods turned into the time domain, using only the output measurements for extracting modal parameters. Thorough treatment of the subject can be found in Juang [41], Ljung [49] and an overview in Peeters [64].

3.1. Useful models

The equations of motion of a linear-dynamic mechanical system of n_2 independent variables are second-order differential equations of the form:

$$M\ddot{w} + \zeta\dot{w} + Kw = f(w,t). \tag{3.1}$$

For the sake of performing efficient computational analysis, it is convenient to reshape Eq. (3.1) into the following compact form:

$$\dot{x} = A_c x + B_c u, \tag{3.2}$$

where:

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}\zeta \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0 \\ M^{-1}B_2 \end{bmatrix},$$

$$x = \begin{bmatrix} w \\ \dot{w} \end{bmatrix}, \qquad f(w,t) = B_2 u\,(t),$$

A_c $(n \times n)$, $n = 2n_2$, is the state matrix and B_c $(n \times r)$ is the input matrix, with r being the number of inputs.

The response of the dynamic system can be measured by various sensors e.g. accelerometers, velocity sensors, strain gauges, and stored in the output vector $y(t)$ of q entries:

$$y = C_a\ddot{w} + C_v\dot{w} + C_d w, \tag{3.3}$$

where C_a, C_v, C_d are output matrices corresponding to acceleration, velocity and displacement, respectively. The matrices contain conversion factors between the measured units (e.g. meters) and electrical units (e.g. volts) indicated by the measuring equipment.

Solving (3.1) for $\ddot{w}$ and substituting the result into (3.3) yields:

$$y = Cx + Du, \tag{3.4}$$

where: $C = \left[C_d - C_a M^{-1}K \;\; C_v - C_a M^{-1}\zeta\right]$, $D = C_a M^{-1}B_2$, $C(q \times n)$ is the output matrix and $D(q \times r)$ is the direct transmission (throughput) matrix.

Equations (3.2) and (3.4) constitute a continuous-time, deterministic state-space model of a dynamical system. In practice however, the measurements are available at discrete time instants. Thus the discrete model, sampled in equally spaced instants 0, Δt, $(k+1)\Delta t, \ldots$, yields:

$$x_{k+1} = Ax_k + Bu_k, \tag{3.5}$$

$$y_k = Cx_k + Du_k, \tag{3.6}$$

where: $A = e^{A_c \Delta t}$, $B = B_c \int\limits_0^{\Delta t} e^{A_c \tau} d\tau$, $\tau = (k+1)\,\Delta t - \tau'$.

Assuming a system to be in a steady state and solving for the output y_k in terms of the previous inputs u_i $(i = 0, 1, \ldots, k)$ with zero initial condition, produces:

$$y_k = \sum_{i=1}^{k} Y_i u_{k-i} + Du_k, \tag{3.7}$$

where Y_k are the *Markov parameters*:

$$Y_k \equiv CA^{k-1}B, \quad Y_0 \equiv D, \tag{3.8}$$

Note that $Y_0 = D$. Equation (3.7) is called the *weighting sequence description* because the contribution to the output at time step k is made by the input at time step k and all previous steps $k-1$, $k-2$, $\ldots, 1, 0$, weighted by the *pulse response sequence* (Markov parameters).

Another useful model can be obtained by adding and subtracting the same term Gy_k in Eq. (3.5). Then, substituting y_k from (3.6), gives:

$$x_{k+1} = \bar{A}x_k + \bar{B}\bar{u}_k, \tag{3.9}$$

where: $\bar{A} = A + GC$, $\quad \bar{B} = \begin{bmatrix} B + GD & -G \end{bmatrix}$, $\quad \bar{u}_k = \begin{bmatrix} u_k \\ y_k \end{bmatrix}$.

Equations (3.9) and (3.6) constitute a discrete-time, deterministic *state-space observer* model with the matrix G being the observer gain matrix.

For a steady state and zero initial conditions, the state-space observer model can be expressed as:

$$y_k = \sum_{i=1}^{k} \bar{Y}_i \bar{u}_{k-i} + Du_k, \tag{3.10}$$

where:

$$\begin{aligned} \bar{Y}_k &\equiv C\bar{A}^{k-1}\bar{B} = \left[C\left(A+GC\right)^{k-1}\left(B+GD\right) \;\; -C\left(A+GC\right)^{k-1}G \right], \\ \bar{Y}_0 &\equiv D, \end{aligned} \tag{3.11}$$

$\bar{Y}_k$ are the *observer Markov parameters*. Note that $\bar{Y}_0 = D$. Equation (3.10) is commonly called the linear difference model or ARX *model* where AR refers to the AutoRegressive part (output data) and X refers to the eXogeneous part (input data). The ARX model corresponding to the state-space observer model is analogous to the weighting sequence description corresponding to the state-space model (cf. (3.7)).

If we decide to account for noise in modelling and measurements, the following discrete-time, deterministic-stochastic state-space model has to be considered:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k, \\ y_k &= Cx_k + Du_k + v_k, \end{aligned} \tag{3.12}$$

where w_k is the process noise due to modelling imperfections and v_k is the measurement noise due to equipment imperfections. If the input u_k is unknown

e.g. in the case of ambient vibration, the corresponding terms disappear from Eqs. (3.12), and the discrete-time, stochastic state-space model yields:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k, \\ y_k &= Cx_k + v_k. \end{aligned} \tag{3.13}$$

Both noise components are assumed to be zero mean $E[w_k] = E[v_k] = 0$, white, independent of the actual state $E[x_k\ w_k^T] = E[x_k\ v_k^T] = 0$, with the following covariance matrices:

$$E\left[\begin{pmatrix} w_m \\ v_m \end{pmatrix} \begin{pmatrix} w_n^T & v_n^T \end{pmatrix} \right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{mn}. \tag{3.14}$$

The stochastic process is also zero mean $E[x_k] = 0$ and stationary $E[x_k\ x_k^T] = \Sigma$, meaning that the state covariance matrix Σ is constant in time. The set of i output covariance matrices and the next state-output covariance matrix are defined as:

$$R_i = E\left[y_{k+i}\, y_k^T\right], \qquad \Gamma = E\left[x_{k+1}\, y_k^T\right]. \tag{3.15}$$

With the definitions (3.15), the following relations can be derived:

$$\Sigma = A\Sigma A^T + Q, \qquad \Gamma = A\Sigma C^T + S, \tag{3.16}$$

$$R_i = CA^{i-1}\Gamma, \qquad R_0 = C\Sigma C^T + R. \tag{3.17}$$

Equation (3.17) resembles the definition of Markov parameters in Eqs. (3.8) or (3.11). This formal similarity is crucial as it allows to use the system realization theory of deterministic time-invariant model in stochastic considerations. Therefore, it is the basis for developing stochastic, output-only system identification methods.

By applying a steady-state *Kalman filter* to the stochastic state-space model, another representation called the *forward innovation* model can be obtained:

$$\begin{aligned} z_{k+1} &= Az_k + Ke_k, \\ y_k &= Cx_k + e_k, \end{aligned} \tag{3.18}$$

where the white noise sequence e_k are called innovations and K is the so-called Kalman filter gain matrix. Note that the state vector z_k is in a different basis than x_k (cf. (3.13)). The corresponding general model (non-state-space) is the *Auto Regressive Moving Average* (ARMA) model with the AR term related to outputs and the MA term related to white noise input:

$$y_k + \alpha_1 y_{k-1} + ... + \alpha_{n_\alpha} y_{k-n_\alpha} = e_k + \gamma_1 e_{k-1} + ... + \gamma_{n_\gamma} e_{k-n_\gamma}. \tag{3.19}$$

For an ARMA model deduced from a state-space model, the AR order n_α is equal to the MA order n_γ.

3.2. Fundamentals of system identification

In order to transform time-dependent linear differential equations (the equations of motion) into the algebraic domain (frequency domain), we need the *Laplace transform* defined as:

$$L\left[y\left(t\right)\right] = \int_0^\infty y\left(t\right) e^{-st} dt, \tag{3.20}$$

where the scalar $s = \sigma + j\omega$ is a complex variable. The Laplace transform applies to the continuous time analysis and is customarily called the *s-transform*. Two-sided Laplace transform covering the whole range $(-\infty, \infty)$ is called *continuous Fourier transform*.

In practice, the data for system identification are sampled at discrete time instants $k\Delta t$ $(k = 0, 1, 2, \ldots)$, and an analogous *z-transform* in the discrete time is defined as:

$$Z\left[y\left(k\Delta t\right)\right] = \sum_0^\infty y\left(k\Delta t\right) z^{-k}. \tag{3.21}$$

Note that the s-transform of the sampled time signals can be obtained from its z-transform by substituting $z = e^{s\Delta t}$.

It is known that any periodic signal can be expressed in terms of a *Fourier series*, being a linear combination of cosine and sine terms. An arbitrary discrete-time input signal of the frequency ω, captured for m samples at Δt-spaced instants $\tau = 0, 1, \ldots, m-1$, can be expressed in terms of the following Fourier series:

$$u\left(\tau\right) = \sum_{i=-\infty}^{\infty} \hat{U}\left(i\right) e^{j(\omega\tau\Delta t)i} = \sum_{i=-\infty}^{\infty} \hat{U}\left(i\right) e^{j\left(\frac{2\pi\tau}{m}\right)i}, \tag{3.22}$$

$$i = -\infty, \ldots, 0, 1, \ldots, \infty,$$

where $\hat{U}\left(i\right)$ are the scaling coefficients (weighting amplitudes) for the frequencies in the decomposition.

For performing numerical analysis, the infinite series must be truncated, which is equivalent in practice to consideration of a limited frequency range. Thus the input signal becomes:

$$u\left(\tau\right) = \sum_{k=0}^{m-1} U\left(k\right) e^{j\left(\frac{2\pi\tau}{m}\right)k}, \qquad k = 0, 1, \ldots, m-1. \tag{3.23}$$

In signal processing, the weighting amplitudes $U(k)$ need to be determined, which can be obtained through the inverse transform of (3.23):

$$U(k) = \frac{1}{m} \sum_{n=0}^{m-1} u\,(n)\, e^{-j\left(\frac{2\pi k}{m}\right)n}, \qquad n = 0,\, 2,\, \ldots,\, m-1. \tag{3.24}$$

The formula (3.24) is called the *Discrete Fourier Transform* (DFT) of the sampled input sequence $u(n)$. The Inverse Discrete Fourier Transform (IDFT) is naturally the formula (3.23).

Note that $U(k)$ is periodic, such that $U(k) = U(k+m)$. As the sequence $u(n)$ is real, the equality $U(-k) = U^*(k)$ holds, where * means a complex conjugate. One can show that the values of $U(k)$ $(k = m/2,\, m/2+1, \ldots, m)$ are the complex conjugates of the values $U(k)$ $(k = m/2,\, m/2-1, \ldots, 0)$. Only three values $U(0)$, $U(m/2)$ and $U(m)$ are always real, the other ones are complex. This implies that $U(k)$ should be uniquely determined over the interval $[0, m/2]$, corresponding to $[0,\, \pi]$. Consequently, the maximum frequency captured by the DFT (due to truncation of the infinite series) depends upon the time interval Δt as follows:

$$f_{nyq} = \frac{m}{2} \frac{1}{m\Delta t} = \frac{1}{2\Delta t}. \tag{3.25}$$

This is the *Nyquist frequency*. Hence, the highest frequency that can be estimated by the sampling rate $1/\Delta t$ Hz is only half that rate.

Applying the DFT directly in numerical analysis would require $\mathrm{O(m^2)}$ operations in contrast to only O(mlog(m)) ones performed by the efficient *Fast Fourier Transform* (FFT) algorithm. There exists a number of the FFT implementations, the most popular of which is the Cooley–Tukey [14] version. Performing the z-transform (3.21) of the weighting sequence description (3.7), one transfers the input/output relation to the frequency domain:

$$y(z) = \sum_{\tau=0}^{\infty} Y_\tau z^{-\tau} u\,(z), \qquad z = e^{j\omega\Delta t}, \tag{3.26}$$

where

$$G(z_k) = \sum_{\tau=0}^{\infty} Y_\tau z_k^{-\tau}, \qquad z_k = e^{j\frac{2\pi k}{m}} \tag{3.27}$$

is the *Frequency Response Function* (FRF). The infinite series in (3.27) can be truncated if the system is asymptotically stable.

A periodic input $u(n)$ will produce a similar periodic output $y(n)$ scaled in amplitude and shifted in phase. Thus having DFTs (cf. (3.24)) $U(k)$, $Y(k)$ of

the input $u(n)$ and output $y(n)$ signal for a single-input single-output (SISO) system, the Frequency Response Function can be simply expressed as:

$$G(z_k) = \frac{Y(k)}{U(k)}. \tag{3.28}$$

The relation between the state-space model and the FRF is given by:

$$G(z_k) = C(z_k I_n - A)^{-1} B + D. \tag{3.29}$$

For a multi-input multi-output (MIMO) system with N data records available, the FRF becomes:

$$G(z) = \frac{\sum_{i=1}^{N} Y_i(k) U_i^*(k)}{\sum_{i=1}^{N} U_i(k) U_i^*(k)}. \tag{3.30}$$

An equivalent of FRF for the Laplace transform (continuous time) is the *transfer function.*

In system identification one uses the correlation functions for the input and output signals. The *cross-correlation* function R_{yu} between the output and input (delayed by the time $i\,\Delta t$), and the *auto-correlation* functions of the input R_{uu} and output R_{yy} signals are defined as (cf. (3.15)):

$$\begin{aligned} R_{yu}(i) &= \frac{1}{m}\sum_{k=0}^{m-1} y(k)\, u^T(k-i) = E\left[y_k\, u_{k-i}^T\right], \\ R_{uu}(i) &= \frac{1}{m}\sum_{k=0}^{m-1} u(k)\, u^T(k-i) = E\left[u_k\, u_{k-i}^T\right], \\ R_{yy}(i) &= \frac{1}{m}\sum_{k=0}^{m-1} y(k)\, y^T(k-i) = E\left[y_k\, y_{k-i}^T\right]. \end{aligned} \tag{3.31}$$

One can show the validity of the following relation:

$$R_{yu}(i) = \sum_{k=0}^{\infty} Y_k R_{uu}(i-k) \tag{3.32}$$

which is similar in form to (3.7) and is therefore called the *correlation weighting sequence description* for linear systems.

The DFTs S_{yu}, S_{uu}, S_{yy} of the correlation functions (3.31) are called *cross-spectral* and *auto-spectral densities*, respectively. For SISO systems, the FRF can be expressed in terms of the densities as (cf. (3.28)):

$$G(z_k) = \frac{S_{yu}(k)}{S_{uu}(k)}, \tag{3.33}$$

Spectral methods frequently use the formula, which expresses the output spectrum in terms of the input one and the related FRFs, as:

$$S_{yy} = G(z_k) S_{uu} G^*(z_k). \tag{3.34}$$

The spectral density of the signal multiplied by an appropriate factor will produce the power carried by the signal and can be plotted in the *power spectral density* (spectral power distribution). Due to the finite representation of the DFT, a phenomenon called *spectral leakage* occurs, resulting in blurring the input frequencies in the power spectral density. To minimize the leakage, various *windowed signals* are used (e.g. Hamming, Bartlett, etc.), which means introducing some weight in Eq. (3.24).

Historically, the system identification methods started from the input-output deterministic approach to turn towards the output-only stochastic approach nowadays. The first step of classical input-output system identification is determination of the Markov parameters. Depending upon the data available, the methods of determining Markov parameters are divided into time domain or frequency domain methods. The data for extracting the Markov parameters may be either input and output signals in the time domain, measured with relatively simple equipment or FRFs in the frequency domain, measured directly with sophisticated spectrum analyzers. The initial approach of determining the Markov parameters is the time domain, off-line approach consisting of three steps:

1) performance of FFT (cf. (3.24)) for input and output signals measured, for some period of time (transformation of the problem into the frequency domain),
2) calculation of FRF (cf. (3.28)) on the basis of the FFTs from step 1),
3) performance of inverse FFT on the FRF (cf. (3.27)) to get the Markov parameters (back transformation of the problem into the time domain).

An alternative for the off-line approach is an on-line one for determining observed Markov parameters in the time domain. It is called recursive least squares method, which can be arranged in a special modular structure called Least Squares Lattice Filter (see Juang [41]), especially suitable for on-line application. The method is quite sophisticated, however the possibility of on-line use is its greatest asset. A review of up-to-date recursive identification methods is given in Ljung, Soderstrom [50].

If the FRF data is available directly, the frequency-domain state-space identification is possible thanks to expressing the FRF in the form of the *matrix fraction description*:

$$G(z_k) = \bar{Q}^{-1}(z_k)\,\bar{R}(z_k), \tag{3.35}$$

where

$$\begin{aligned} \bar{Q}(z_k) &= I_q + \bar{Q}_1 z_k^{-1} + ... + \bar{Q}_p z_k^{-p}, \\ \bar{R}(z_k) &= \bar{R}_0 + \bar{R}_1 z_k^{-1} + ... + \bar{R}_p z_k^{-p}. \end{aligned} \tag{3.36}$$

The multipliers $\bar{Q}_p,\ \bar{R}_p$ are nothing else but the observer Markov parameters (cf. (3.11)). Premultiplying Eq. (3.35) by $\bar{Q}(z_k)$ and rearranging, the following set of equations is obtained:

$$G(z_k) = -\bar{Q}_1 G(z_k)\, z_k^{-1} - ... - \bar{Q}_p G(z_k)\, z_k^{-p} + \bar{R}_0 + \bar{R}_1 z_k^{-1} + ... + \bar{R}_p z_k^{-p} \tag{3.37}$$

The linear algebraic set of Eqs. (3.37) can be solved for the observer Markov parameters $\left[-\bar{Q}_1\ ...\ -\bar{Q}_p\ \bar{R}_0\ ...\ \bar{R}_p\right]$ (the unknown vector) in the sense of the least squares norm.

For output-only stochastic system identification, the input is not included in the model description (cf. (13)), therefore the Markov parameters relating output with input cannot be determined. Only output is measured in this approach and it entirely replaces Markov parameters in mathematical calculations. However, thanks to the formal similarity of state-output covariances (3.17) to Markov parameters (3.8), the classical theory of system realization for time-invariant deterministic models can be applied.

A *realization* of the system is the computation of the state, input and output matrices A, B, C (sometimes the direct transmission matrix D, too) from the Markov parameters for the deterministic model and from the outputs for the stochastic model. Any system has an infinite number of realizations since there exist an infinite number of state-space representations that describe the same input-output relationship. A model with the smallest state-space dimensions is called *minimum realization*.

System realization starts with formation of the generalized αq x βr Hankel matrix (a matrix in which every antidiagonal stores the same element), composed of Markov parameters in the case of deterministic models:

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & ... & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & ... & Y_{k+\beta} \\ ... & ... & ... & ... \\ Y_{k+\alpha-1} & Y_{k+\alpha} & ... & Y_{k+\alpha+\beta-1} \end{bmatrix}. \tag{3.38}$$

Choosing $\alpha \geq n$ and $\beta \geq n$ (the order of the system), the Hankel matrix is of rank n. Using the definition (3.8) of Markov parameters, the Hankel matrix can be expressed as:

$$H(k-1) = O_\alpha A^{k-1} Q_\beta, \tag{3.39}$$

where:

$$O_\alpha = \begin{bmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{\alpha-1} \end{bmatrix}, \qquad Q_\beta = \begin{bmatrix} B & AB & A^2B & \dots & A^{\beta-1}B \end{bmatrix}. \tag{3.40}$$

The block matrix O_α is the observability matrix and the block matrix Q_β is the controllability matrix. The *Eigensystem Realization Algorithm* (ERA) starts with SVD factorization of the Hankel matrix for $k = 1$, yielding:

$$H(0) = UTV^T, \tag{3.41}$$

where U and V are orthonormal matrices and T is a diagonal matrix with n non-zero components arranged in decreasing order.

Considering only the parts of matrices U, T, V corresponding to the n non-zero components of T and taking into account that $H(0) = O_\alpha Q_\beta$ (cf. (3.39)), the observability and controllability matrices can be expressed in terms of the SVD matrices (cf. (3.41)) as:

$$O_\alpha = U_n T_n^{1/2}, \qquad Q_\beta = T_n^{1/2} V_n^T \tag{3.42}$$

Thus from the shifted Hankel matrix formula $H(1) = O_\alpha A Q_\beta$ (cf. (3.39)), the state matrix A can be determined as:

$$A = T_n^{-1/2} U_n^T H(1) V_n T_n^{-1/2}, \tag{3.43}$$

Note that the input matrix B is the first r columns of the controllability matrix Q_β and the output matrix C is the first q rows of the observability matrix O_α (cf. (3.40)). The triplet A, B, C determined in this way is a minimum realization of the system.

Let us decide to use only some data stored in the observability matrix O_α. If we delete the first p rows or the last p rows from matrix O_α, we get the following reduced observability matrices (cf. (3.40), (3.42)):

$$O_\alpha^{\downarrow p} = \begin{bmatrix} C \\ CA \\ \dots \\ CA^{\alpha-2} \end{bmatrix} = U_n^{\downarrow p} T_n^{1/2}, \qquad O_\alpha^{\uparrow p} = \begin{bmatrix} CA \\ \dots \\ CA^{\alpha-2} \\ CA^{\alpha-1} \end{bmatrix} = U_n^{\uparrow p} T_n^{1/2}. \tag{3.44}$$

In order to speed up the computations of A, instead of using Eq. (3.43), another formula based on definitions (3.44) can be used:

$$A = T_n^{-1/2} \left[U_n^{\downarrow p}\right]^{+} U_n^{\uparrow p} T_n^{-1/2}, \tag{3.45}$$

where $^{+}$ means the pseudo-inverse[1)]. This procedure of computing A is called the *principal component algorithm.*

Considering only the parts of matrices U, T, V corresponding to the remaining zero components of T and using Eq. (3.41), two *canonical-form realizations* can be obtained:

$$\sum_{i=1}^{\alpha} U_{0i}^T C A^{i-1} = 0, \tag{3.46}$$

$$\sum_{i=1}^{\beta} A^{i-1} B V_{0i} = 0. \tag{3.47}$$

Equation (3.46) is the basis for the $q(\alpha-1)$-dimensional *observable canonical-form realization.* In general, this realization is not controllable and therefore it is not a minimum realization. In the observable realization, the matrix A is a function of U_0, B is a function of Markov parameters $Y_{\alpha-1}$, and C is identity. On the other hand, Eq. (3.47) is the basis for the $r(\beta-1)$-dimensional *controllable canonical-form realization.* In general, this realization is not observable and therefore it is not a minimum realization either. In the controllable realization, the matrix A is a function of V_0, B is identity, and C is a function of Markov parameters $Y_{\beta-1}$. From the computational point of view one should choose the canonical-form realization of smaller dimension.

The above considerations are basically the core of the system realization theory for deterministic, time-invariant systems (idealistic case) developed by Kalman [43], Ho, Kalman [31]. With the advent of practical engineering applications of the system identification, the Kalman theory was extended by Akaike [2] and Aoki [1] for stochastic systems taking into account the state and measurement noise components (cf. (3.12)). Another practical requirement i.e. the use of ambient excitation (e.g. wind, traffic), pushed the system identification towards the output-only analysis. For observer identification, which is the case in practical situations, the related ARX model (cf. (3.10)) for deterministic input-output analysis is replaced by the ARMA model (cf. (3.19)) for stochastic output-only analysis.

The on-line, stochastic system identification can be best realized through the use of the *Kalman filter.* The role of the Kalman filter is to give an optimal

[1)] The Moore–Penrose pseudo-inverse of a non-square matrix A is defined as $A^{+} = A^T A^{-1} A^T$.

prediction of the next state vector x_{k+1} by using observations of the outputs up to time k and the related matrices. The predictions are denoted by $\hat{x}_{k+1}$. Given the known dynamics and output measurements, and assuming the initial values $\hat{x}_0 = 0$, $P_0 \equiv E\left(\hat{x}_0 \hat{x}_0^T\right) = 0$, the Kalman filter estimates $\hat{x}_{k+1}$ for a general, non-steady state are expressed by recursive formulas, describing the system, *Kalman filter gain matrix* and the state covariance matrix, respectively:

$$\begin{aligned} \hat{x}_{k+1} &= (A - KC)\,\hat{x}_k + (B - KD)\,u_k + Ky_k, \\ K_k &= \left(\Gamma - AP_kC^T\right)\left(\Lambda_0 - CP_kC^T\right)^{-1}, \\ P_{k+1} &= AP_kA^T + \left(\Gamma - AP_kC^T\right)\left(\Lambda_0 - CP_kC^T\right)^{-1}\left(\Gamma - AP_kC^T\right)^T. \end{aligned} \tag{3.48}$$

For output-only analysis, the term related to the input u_k disappears. There is a close relation between the state-space observer model (cf. (3.9), (3.6)) and the Kalman filter. The arbitrary matrix G for the state-space model can be chosen in such a way that the estimated state (the observer) approaches the theoretical one in the quickest possible way (producing minimum error). It turns out that the Kalman filter gain matrix $K = -G$ is the quickest observer. To be found in observer identification is the Kalman filter state sequence:

$$\hat{X} \equiv (\hat{x}_i\ \hat{x}_{i+1}\ ...\ \hat{x}_{i+j-1})\,. \tag{3.49}$$

In stochastic identification, the Hankel matrix is composed of output measurements only:

$$H \equiv \frac{1}{\sqrt{j}} \left[\begin{array}{cccc} y_0^{\mathrm{ref}} & y_1^{\mathrm{ref}} & ... & y_{j-1}^{\mathrm{ref}} \\ y_1^{\mathrm{ref}} & y_2^{\mathrm{ref}} & ... & y_j^{\mathrm{ref}} \\ ... & ... & ... & ... \\ y_{i-1}^{\mathrm{ref}} & y_i^{\mathrm{ref}} & ... & y_{i+j-2}^{ref} \\ \hline y_i & y_{i+1} & ... & y_{i+j-1} \\ y_{i+1} & y_{i+2} & ... & y_{i+j} \\ ... & ... & ... & ... \\ y_{2i-1} & y_{2i} & ... & y_{2i+j-2} \end{array}\right] \equiv \left(\frac{Y_{0|i-1}^{\mathrm{ref}}}{Y_{i|2i-1}}\right), \tag{3.50}$$

where the *reference sensor* block is allocated first. The reference sensors are crucial for system identification as they are placed in optimal locations able to represent all modes of vibration well. In practice, not all outputs are measured at the same time. For large structures all sensors are grouped into a number of

set-ups, arranged in such a way that the common part of neighbouring set-ups are just the reference sensors.

For the sake of computational efficiency for large systems, only a subspace of all outputs can be considered in stochastic system identification. A useful projection of the row space of non-reference outputs onto the row space of the reference outputs can be defined as:

$$\wp_i^{\mathrm{ref}} \equiv \frac{Y^{\mathrm{ref}}}{Y} = YY^{\mathrm{ref^T}} \left(Y^{\mathrm{ref}} Y^{\mathrm{ref}^T}\right)^+ Y^{\mathrm{ref}}. \tag{3.51}$$

The $H(0)$ matrix in deterministic approach is a product of the observability and controllability matrices (cf. (3.39)). The main theorem of the stochastic subspace identification formulated in Van Overschee, De Moor [84] states that similarly, the projection matrix $\wp^{\mathrm{ref}}$ can be expressed as the product of the observability matrix and the Kalman filter state sequence (cf. (3.40), (3.49)):

$$\wp_i^{\mathrm{ref}} = \begin{pmatrix} C \\ CA \\ CA^2 \\ ... \\ CA^{i-1} \end{pmatrix} (\hat{x}_i \; \hat{x}_{i+1} \; ... \; \hat{x}_{i+j-1}) = O_i \hat{X}_i \,. \tag{3.52}$$

For proving this, it is assumed that the number of measurement sessions approaches infinity ($j \to \infty$). The QR factorisation of the Hankel matrix (3.50) enables determination of the projection matrix $\wp^{\mathrm{ref}}$. Thus the Kalman filter state sequence can be calculated as:

$$\hat{X}_i = O_i^+ \wp_i^{\mathrm{ref}}. \tag{3.53}$$

Using the Kalman filter sequence, the stochastic output-only model (3.13) can be expressed as follows:

$$\begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} \hat{X}_i + \begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix} . \tag{3.54}$$

The noise residuals ρ_w, ρ_v are uncorrelated with $\hat{X}$, thus Eq. (3.54) can be solved in the least squares sense to give the state and output matrices:

$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} \hat{X}_i^+ \,. \tag{3.55}$$

Alternatively, the stochastic system identification may be performed quite analogously to the deterministic approach. The deterministic Hankel matrix of outputs is replaced by a stochastic Toeplitz matrix (a matrix in which every diagonal is composed of the same element) of output covariances and the formula to determine the state matrix A looks formally the same as the deterministic solution (3.43).

3.3. Modal analysis for an identified system

The modal decomposition of the state matrix A yields:

$$A = \Psi \Lambda_d \Psi^{-1} \,. \tag{3.56}$$

For general viscous damping, Λ_d is the diagonal matrix containing complex conjugate eigenvalues:

$$\Lambda_d = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda^* \end{bmatrix} \quad \lambda_i,\ \lambda_i^* = -\xi_i \Omega_i \pm j\sqrt{1-\xi_i^2}\,\Omega_i \,, \tag{3.57}$$

where Ω_i is the i-th undamped natural frequency (rad/s) and ξ_i is the i-th damping ratio. Ψ is the matrix of eigenmodes:

$$\Psi = \begin{bmatrix} \Theta & \Theta^* \\ \Lambda\Theta & \Lambda^*\Theta^* \end{bmatrix} . \tag{3.58}$$

where Θ contain eigenvectors of the system for the case of non-proportional damping.

It can be shown that the (A, B, C) realization of the system can be transformed to the modal realization $(\Lambda_d,\, \Psi^{-1}B,\, C\Psi)$, identifying all modal parameters. The diagonal matrix Λ_d contains information about the eigenvalues. The matrix $\Psi^{-1}B$ defines the initial modal amplitudes (the so-called modal participation matrix) and the matrix $V = C\Psi$ – the mode shapes at sensor points. Note that the output-only methods will never provide information about modal amplitudes as the input matrix B is not identified in these methods.

By transformation of eigenvalues λ_i from the discrete to the continuous time, the system poles μ_i informing about the modal damping factors σ_i and the damped natural frequencies ω_i can be obtained using the following relation (cf. (3.5), (3.6)):

$$\mu_i = \sigma_i + j\omega_i = \frac{\ln \lambda_i}{\Delta t} \,. \tag{3.59}$$

The damping ratios ξ_i are then defined as:

$$\xi_i = \frac{-\sigma_i}{\sqrt{\omega_i^2 + \sigma_i^2}}\,. \tag{3.60}$$

The *Modal Assurance Criterion* (MAC) is frequently used to evaluate the correctness of the performed modal analysis:

$$\mathrm{MAC}_{p,\,p+1} = \frac{\left|v_p^H v_{p+1}\right|^2}{\left(v_p^H v_p\right)\left(v_{p+1}^H v_{p+1}\right)}\,, \tag{3.61}$$

where the superscript ^{H} means the complex conjugate transpose. The MAC coefficient varies between 0 and 1 and indicates whether there is a good correlation between the mode shapes for the identified model order p and the mode shapes for the one-order-higher model.

3.4. Methods of system identification

3.4.1. Frequency domain. The frequency domain methods use formula (3.34), relating the output and input spectra. The methods are often called *non-parametric* system identification methods since no physical parameters are directly identified through them.

3.4.1.1. Peak Picking. The earliest engineering application utilizing correlation and spectral analysis for determination of modal parameters is the *Peak Picking* (PP) method. As the name suggests it identifies eigenfrequencies of a system, occurring as peaks in a spectrum plot. With the strongly limiting assumption of low damping and well-separated eigenmodes (no close eigenfrequencies), the spectrum matrix in the PP method is an approximation of the observed mode shape, scaled by a constant factor. However, if the column corresponds to a degree of freedom (dof), which is not excited by a certain mode (the dof is in the nodal point of the mode), such mode cannot be identified. The PP method also estimates the damping ratios, but it is unfortunately not accurate. The principal assumptions of the PP method are often not fulfilled, which leads to misinterpretations. Namely, the PP method tends to identify operational deflection shapes instead of eigenmodes. This is incorrect because for closely-spaced modes, the deflection shape is a superposition of many modes.

3.4.1.2. Complex Mode Indication Function. The *Complex Mode Indication Function* (CMIF) method consists in performing the SVD of the spectrum matrix. A complete review of the CMIF with applications can be found in Allemang and Brown [3]. The observation behind the method is the fact that the

spectrum matrix (or equivalently the FRF) at a certain frequency is practically influenced by only a few eigenmodes. The number of the influencing eigenmodes determines the rank of the spectrum matrix. At resonance, for well-separated modes, only one mode has influence on the spectrum matrix, so its rank in SVD is one. If there exist several closely-spaced modes at the same frequency, this fact is reflected by the increased rank of the spectrum matrix. Thus the CMIF method can be considered as an SVD extension of the PP method, able to identify closely-spaced modes. By performing the SVD, the CMIF method decomposes the initial system into single-degree-of-freedom systems. Ewins [20] describes the relevant parameter estimation methods for such systems.

3.4.1.3. Maximum Likelihood. The most advanced spectrum-driven method is the *Maximum Likelihood* (ML) method, described in Pintelon, Schoukens [68]. Unlike PP or CMIF, the ML estimates the parameters of a model by minimizing an error norm. It is thus a parametric method, solving a non-linear optimization problem in an iterative way. Despite the complexity of the approach due to the nonlinear formulation and the related high computational burden, the ML methods proved to be a robust tool, identifying modal parameters from large and noisy data. Guillaume *et al.* [29] shows that the ML can operate both with FRFs (input/output) or spectra (output-only), which makes the method versatile.

3.4.2. Time domain. The time domain methods follow strictly the system realization theory, i.e. identify the system by determining the triplet (A, B, C). Thanks to the modal analysis (see Sec. 3.3), all modal parameters can be identified by the time-domain methods. Therefore these methods are often referred to as *parametric* identification methods.

3.4.2.1. Determnistic approaches. Two most popular traditional (input-output) approaches for system identification in the time domain were developed simultaneously in the early 80's. One of them called the Eigensystem Realization Algorithm (ERA) was elaborated in the NASA Langley Research Center by Juang [41] in the field of aerospace engineering. It uses the SVD factorization of the Markov parameter Hankel matrix (cf. (3.43), (3.45)) for finding the triplet (A, B, C) to identify the system. The deterministic ERA algorithm is based on both the input and output data. The canonical form of ERA (cf. (3.46), (3.47)) is very similar to another approach developed in the field of structural engineering called the *Polyreference Time Domain* (PTD), proposed by Vold, Russell [85]. The PTD was probably the most widespread time-domain input-output method of system identification applied in engineering before the advent of output-only methods. It encompasses two particular earlier versions called

Ibrahim Time Domain (see IBRAHIM, MIKULCIK [40]) and *Least Squares Complex Exponential* methods. The relations between the methods can be studied in ALLEMANG [4].

3.4.2.2. Stochastic approaches. Stochastic methods in the time domain began their fast development with proving the validity of formula (3.17), which enabled the use of classical realization theory in output-only considerations. Many stochastic methods can be derived from deterministic ones by substituting the impulse responses with output covariances. For instance, the *Instrumental Variable* (IV) method formally corresponds to the PTD method after performing such substitution. It identifies only the AR parameters, which results in a linear problem solved in the least squares sense, however the ARMA model structure is preserved. Advanced covariance-driven methods use only a subset of sensors (reference sensors) for system identification and are therefore called stochastic subspace methods. *Stochastic Subspace Identification-cov* (SSI-cov) uses the SVD factorization and corresponds to the deterministic ERA after substituting the impulse responses with output covariances. Depending upon the selection of weighting matrices for the Hankel matrix, the SSI-cov has two versions, namely *Balanced Realization* (BR – no weighting) and *Canonical Variate Analysis* (CVA – the weighting suited for the less excited modes to be better identified). Another subspace method, the *Stochastic Subspace Identification-dat* (SSI-dat) uses direct outputs (i.e the Hankel matrix (3.50)) instead of their covariances. This is an advantage over SSI-cov, because the data is not squared up (cf. (3.31)), which leads to higher numerical accuracy. SSI-dat determines the projection matrix (3.51), (3.52) and then employs the Kalman filter state sequence (3.49), (3.53) in a robust and stable least squares algorithm, identifying the system matrices (cf. (3.55)).

4. Low-frequency SHM methods

The low-frequency SHM methods operate in the non-ultrasonic range of frequencies i.e. below 20 kHz. They strongly rely on modal analysis. Therefore, they are often called vibration-based SHM. Most of the low-frequency SHM methods require a finite element model and many of them use the state-space form (cf. (12)) of equations of motion. Collecting experimental or in-situ measurements is necessary for obtaining the real response of a structure. Then, the FE model is fit to the measured data by means of the system identification procedures (see Sec. 3) in order to provide a reliable reference model. Damage identification is usually performed by subsequent updating of the model. The low-frequency methods often use quite sophisticated mathematical tools, but the effort of implementing them in numerical algorithms pays off when it comes

to results interpretation, which is rather straightforward. Since the methods look at a structure globally and examine a relatively broad inspection zone (possibly the whole structure, e.g. a bridge), they tend to identify damage of considerable extension, e.g. stiffness degradation or corrosion. Presently, low-frequency SHM methods are first of all applied in civil and mechanical engineering.

Several damage-sensitive parameters of the model have been tried to diagnose structural health. The damage identification procedure consists of updating the model parameters to best match the experimental response of a damaged structure. FRISWELL, MOTTERSHEAD [23] describe difficulties in solving the resulting inverse problem inherent to model updating. Natural frequencies were used first to assess damage, but they are sensitive to structural defects of rather significant intensity. WILLIAMS, MESSINA [87] demonstrate the accuracy improvement of damage predictions through adding anti-resonance frequencies. Besides natural frequencies, FRFs are also used to identify damage. YANG, LEE [90] incorporate mode shapes to damage assessment algorithms as being more informative than frequencies. Multi-component objective functions, combining both frequencies and modal shapes, are constructed to improve the effectiveness of damage identification. As an alternative to mode shapes, ZIMMERMANN [93] proposes to analyse the so-called *Ritz vectors*. HO, EWINS [32] and MAECK, DE ROECK [52] identified modal curvature as a parameter very sensitive to damage. CERAVOLO *et al.* [11] and MODENA *et al.* [57] use damping-related coefficients as parameters in damage detection. ZHANG *et al.* [92] and WORDEN *et al.* [88] formulate SHM algorithms on the basis of modal strain energy, while FRITZEN, BOHLE [25] use modal kinetic energy. FRISWELL, MOTTERSHEAD [24] and FRITZEN, BOHLE [26] consider model reduction as an important issue in model updating, which may make the analysis feasible or not for large engineering structures. TEUGHELS [79] performed successful model reduction by the application of linear and parabolic damage functions. BASSEVILLE [5] deals with the problem of optimal placement of sensors, which plays an important role in effective identification.

SPINA [75] uses standard deterministic realization theory to perform damage identification via the ARX family models. By discovering the formal similarity between the Markov parameters and output covariances (cf. (3.17)), a link was found between the stochastic methods and the classical realization theory. Since then, the influence of noise could be effectively analysed in SHM methods and moved them out of laboratories to in-field applications. DE STEFANO *et al.* [18] apply an ARMA model to perform modal analysis of a bridge girder with unknown random excitation. BODEUX, GOLINVAL [8] use an ARMA model to test the performance of buildings during earthquakes. Two stochastic system identification methods (SSI-cov and SSI-dat) with an application to a wind-excited steel mast are described in PEETERS, DE ROECK [65]. A comparison of IV,

BR and CVA approaches (see 3.4.2.2) applied to three different industrial case studies (car – acoustics, aircraft, bridge) can be found in Hermans, Van Der Auweraer [30].

The Virtual Distortion Method (VDM), belonging to fast reanalysis methods (see Holnicki–Szulc, Gierlinski [33]), was applied to detecting damage via the solution of an inverse problem. Kołakowski *et al.* [47] demonstrate the effectiveness of VDM in the time domain whereas Świercz *et al.* [78] present its advantages in the frequency domain.

An interesting alternative to inverse methods in damage assessment, which are the vast majority of all approaches, is the Direct Stiffness Calculation proposed by Maeck [53]. The approach identifies structural stiffness directly as the quotient of the bending moment to beam curvature. With numerous sensors (accelerometers, optical fibres), both components of the quotient can be measured and theoretically no structural model is required.

5. NDT/E methods

Theoretical and practical aspects of the NDT/E methods can be these days quickly studied on the Internet, where the discussion panels addressing practical problems are of additional value (see e.g. http://www.ndt.net/ [38]). A selection of contemporary NDT/E methods is presented in Lewinska–Romicka [48].

5.1. Ultrasonic testing

Ultrasonic testing (UT) is the interrogation of materials using stress waves of the frequency higher than 20 kHz. This NDT/E method is capable of detecting both volume as well as planar (surface) defects. The essence of UT is to inject stress waves into the material to be examined and then monitor the transmitted or reflected signal. UT consists in scanning the surface of material with a probe. Piezoelectric probes are widely used to both induce and receive the stress waves. It is crucial that the stress waves propagate efficiently between the probe and the material, which requires a good acoustic coupling between them. This can be ensured by a coupling medium such as water, gels or greases. Electromagnetic acoustic transducers (EMAT) are also used to induce ultrasonic waves. No coupling medium is required for EMATs.

The type of defect detected by ultrasonic scanning depends upon the type of induced waves and their frequency. The theoretical detectable defect size is of the order of the wavelength. Hence, the high-frequency waves are more sensitive to defects. However, low frequency waves can penetrate the material to greater depths. So the choice of frequency is a compromise between sensitivity and penetration.

There are two major scanning techniques used in UT:

- Pulse echo technique – utilizes the phenomenon of reflection of ultrasonic waves. At a boundary of two different materials, a portion of the waves is reflected and the rest is transmitted. The portion reflected depends on the angle of incidence and the acoustic impedance (product of material density and wave velocity) of the two materials. The undeniable advantage of the pulse echo technique is the required access to only one surface of the tested object.
- Pitch and catch technique – requires access to two surfaces of the examined object. It is used for materials with high damping (pulse echo method not efficient) or for detecting near-surface defects. Two separate probes are placed on two opposite surfaces of the object. One of the probe is transmitting the signal whereas the other is receiving it. A material fault manifests itself in attenuation of energy of the transmitted wave. Amplitude analysis of the transmitted signal is the basis for quantification of defects.

The best detectability is observed for material discontinuities perpendicular to the injected ultrasonic beam.

The advantages of UT are:

- possibility of detecting both volume and planar defects,
- safe operation conditions for the staff,
- relatively low cost of testing.

The drawbacks of UT are:

- no detection of discontinuities positioned along the ultrasonic beam,
- difficulty in examining rough surfaces or irregular shapes,
- necessity of using the coupling medium for piezoelectric probes.

5.2. Radiographic testing

Radiographic testing (RT) is an NDT/E method detecting volume and planar defects. It is based on the measurement of the attenuation of electromagnetic radiation after having passed through a tested sample. The intensity of the incident radiation is reduced when passing through material, depending on its thickness, density, and the wavelength of radiation used. The penetrating power of the radiation increases with the growth of its energy (reduction of wavelength).

RT is performed using:

- the X radiation produced by Roentgen lamps (wavelength range $10^{-13} \div 10^{-9}$),
- the γ radiation produced by radioactive isotopes e.g. Se, Yb, Co (wavelength range $3 \times 10^{-14} \div 10^{-10}$).

The X or γ radiation triggers the photochemical phenomenon i.e. the decay of photo-emulsion. RT takes advantage of the fact that the difference in attenuation as the radiation passes through a defect, is sufficient to reveal the defect by exposition of a photographic film.

The advantages of RT are:

- possibility of testing objects of various, complicated shapes,
- good detectability of volume defects and planar defects along the radiation beam,
- no necessity of contact between the object and equipment.

The drawbacks of RT are:

- harmful operation conditions for the staff,
- poor detection of discontinuities positioned perpendicular to the radiation beam,
- relatively high cost of testing and bulky equipment.

5.3. Acoustic emission

Acoustic emission (AE) is the phenomenon of generating an elastic wave, in the range of ultrasound usually between 20 kHz and 1 MHz, by the rapid release of energy from the source within a material, which is deforming or fracturing under external loading. The energy comes from crack growth, dislocations or phase transformation. The elastic wave propagates through the solid to the surface, where it can be recorded by one or more piezoelectric sensors. In this way, information about the existence and location of possible sources is obtained.

AE differs from other NDT/E methods, which actively probe the structure. AE is a passive method, which listens to emissions from active defects and is very sensitive to defect activity when a structure is loaded beyond its service load in a test. AE analysis is a very useful method for the investigation of local damages in materials. One of the advantages compared to other NDT/E methods is the possibility to observe the damage the processes during the entire load history.

Operational environments are generally very noisy, and the AE signals are usually very weak. Hence, signal discrimination and noise reduction are very difficult, but extremely important for successful AE applications.

The advantages of AE are:

- no necessity of exciting the object or structure (service load is sufficient),
- ability of differentiating between stable and growing defects.

The drawbacks of AE are:

- difficulties in quantifying defects (commercial AE systems can only estimate qualitatively the extension of damage in the material and tell approximately how long the components will last),
- weak signals, therefore high sensitivity to environmental noise.

5.4. Magnetic particle inspection

Magnetic particle inspection (MT) consists in magnetizing an object made of a ferromagnetic material and examining the attenuation of magnetic field (flux), which reveals the places of defects. MT is suitable for planar defect detection only. By definition, MT is limited to ferromagnetic materials e.g. iron, ferromagnetic steel, cast iron, cast steel, iron-based alloys (Fe-Co, Fe-Ni).

The first stage of the MT method is magnetizing the tested material with a magnetic defectoscope, which is a device for inducing magnetic field in the material directly (flux defectoscopes) or indirectly by using electric current (current defectoscopes). Then, one of the following MT techniques may be used:

- magnetic powder inspection – determining the attenuation of magnetic flux using magnetic powders. As a result, a defectogram is obtained, in which the magnetic powder reflects the existing defects,
- magnetic transducer examination – measuring the attenuation of magnetic flux with magnetic transducers (induction transducers are the most widely used).

After the testing has been completed, the tested object has to be demagnetized.

The advantages of MT are:

- possibility of detecting planar (even narrow or shallow) discontinuities on/underneath the surface,
- low cost of equipment.

The drawbacks of MT are:

- harmful operation conditions for the staff (toxic powder components),
- limited applicability to ferromagnetic materials.

5.5. Eddy currents

Eddy current testing (ET) is a method for revealing surface defects in electrical conductors. The method is based on application of the Faraday's law of electromagnetic induction. The material is placed in varying magnetic field (produced by induction transducers), which induces varying electrical field in conductive materials. The amplitude and phase of the signal from the transducers

contains information about the geometry of material defects. The basic equipment is an ET defectoscope, providing power supply for induction transducers and analyzing their output signals.

The advantages of ET are:

- possibility of detecting planar discontinuities on/underneath the surface,
- applicability to coated (e.g. painted) surfaces.

The drawbacks of ET are:

- necessity of having extensive knowledge and experience by the operator,
- difficulties in interpretation of the signal affected by electric conductivity, magnetic penetrability and operating frequency.

5.6. Penetrant testing

Penetrant testing (PT) is a method of similar applicability to MT. It is used in materials in which MT or ET cannot be used. The essence of PT is application of a penetrant to a tested surface and subsequent application of a developer. The penetrant is usually an oil-base fluid and the developer is a dry powder or a powder suspended in water. The penetrant usually includes toxic additives e.g. fluorescent substance, therefore it is necessary to wash the examined object after the test.

The advantages of PT are:

- possibility of detecting planar discontinuities on/underneath the surface,
- possibility of detecting discontinuities in locations of rapid changes of cross-section (hardly possible by any other method),
- low cost of equipment.

The drawbacks of PT are:

- harmful operation conditions for the staff (toxic components),
- no applicability to coated (e.g. painted) surfaces,
- tedious (many stages) process of testing.

5.7. Optical holography

Optical holography (OH) is an imaging method, which records the amplitude and phase of light reflected from an object as an interferometric pattern on film. It thus allows reconstruction of the full 3D image of the object. In OH, the test sample is compared by interferometric measurement in two different stress states. Loading can be mechanical (e.g. vibration), thermal, etc. The resulting interference pattern contours the deformation undergone by the specimen between the two recordings. Surface as well as sub-surface defects reveal distortions in the otherwise uniform pattern. In addition, the characteristics of the compo-

nent, such as vibration modes, mechanical properties, residual stress, etc., can be identified.

The light used to illuminate the surface of the specimen must be coherent, which means that it must also be monochromatic, and the only practical source is a laser. With fast development of charge coupled devices (CCD) and digital image processing, OH offers tremendous flexibility and real-time visualization. Automated defect detection and analysis is possible thanks to computerized analysis of patterns.

OH is a dynamically developing method of many advantages, however one serious drawback for the moment – its huge cost.

6. High-frequency SHM methods

In contrast to the low-frequency SHM methods, the high-frequency ones operate in the ultrasonic range of frequencies i.e. above 20 kHz. The physical background for the methods is the phenomenon of elastic wave propagation in solid media. The high-frequency SHM methods heavily utilize experimental instrumentation for all stages of analysis i.e. generation and detection of elastic waves as well as results interpretation. Especially in the last aspect, they differ from the low-frequency methods, because the interpretation of results (often performed on-line) may not be easy and in principle requires much experience. Most of the high-frequency methods can be placed in the category of pattern recognition at the stage of damage identification. The methods look at a local part of a structure and therefore are focused on precise identification of a relatively small defect (e.g. a crack) in a narrow inspection zone (e.g. the connection of a wing to the fuselage in an aircraft). The most numerous applications of the high-frequency SHM methods can be observed in the aerospace and also mechanical engineering.

A comprehensive review of the high-frequency SHM methods is given in Staszewski [76]. Detailed mathematical description of the phenomenon of elastic wave propagation is provided in Rose [69]. Muravin [60] and Holroyd [35] characterize acoustic emission (AE), which utilizes structure-borne stress waves generated by internal material defects under external load applied. It is a passive method – no excitation, except for the service load, is required. All other methods require high-frequency excitation. Ultrasonic testing (UT) relies on the transmission and reflection of bulk waves and utilizes various phenomena (e.g., wave attenuation, scattering, reflections, mode conversions, energy partitioning) for damage detection. Presently, the most frequently used damage detection method based on guided (between two boundaries of a plate) ultrasonic waves is the Lamb wave inspection, in which the symmetric and antisymmetric modes of 0-th order (below 0.5 MHz) are most frequently analyzed. Koehler

et al. [46] report that the shear horizontal waves with non-dispersive 0-th order mode are also effective in damage detection. MALLET *et al.* [56] identify structural defects by examining Lamb wave attenuation and mode conversions. BIEMANS *et al.* [7] and BETZ *et al.* [6] propose *acousto-ultrasonics* as another high-frequency SHM method, combining elements of AE, UT and Lamb wave inspection. Acousto-ultrasonics uses an impulse excitation, resulting in propagation of a number of mixed wave modes, difficult to analyse.

Piezotransducers are the most versatile devices used in SHM. The use of piezoelectric transducers for generation and detection of elastic waves in plates was first studied in TRACY, CHANG [81] and continued by many researchers. GIURGIUTIU *et al.* [28] use piezoelectric-wafer active sensors (PWAS) for Lamb wave inspection. PARK and INMAN [63] propose an interesting SHM method utilizing the direct and converse piezoelectric effect simultaneously, to obtain an impedance signature and identify damage by monitoring its changes.

Finite Element Method has been extensively used for modelling wave propagation. However, there are problems of accuracy and efficiency in the high-frequency regime. To overcome those, OSTACHOWICZ and KRAWCZUK [62] developed the Spectral Element Method providing high accuracy for a relatively coarse mesh thanks to defining the element stiffness matrix in the frequency domain. Except for the Finite Element Method, STAVROULAKIS [77] successfully uses the Boundary Element Method for high-frequency crack identification as well. LOPEZ–DIEZ *et al.* [51] employ the Statistical Energy Analysis as a tool for incipient damage detection in a spacecraft panel.

7. Artificial intelligence in SHM

In damage identification algorithms, two major approaches can be met – either the formulation is analytical and relevant formula-based methods are used e.g. sensitivity-based optimization, or the formulation avoids analytical formulas and then artificial intelligence (also named soft-computing) methods are employed.

In fact, most structural health monitoring methods include Artificial Intelligence (AI). The SHM methods make use of wavelet transformations (see MALLAT [55], HOU *et al.* [36], HOU, HERA [37]) at the stage of signal processing. Artificial neural networks (see CHANG *et al.* [12], YANA *et al.* [89]), genetic algorithms (see CHOU, GHABOUSSI [13]), and statistical analysis (see FARRAR *et al.* [21], MONACO *et al.* [58], SOHN *et al.* [73]) are employed at the stage of damage identification. The AI tools are efficient, although they disregard the physical interpretation of the analysed phenomena. Case-based reasoning (CBR) proposed by MUJICA et al. [59] is a worth-noting method, combining wavelet transformation with Kohonen-like neural networks (self-organizing maps). However, the use

of knowledge-based approaches (such as CBR) has not been extensively exploited for damage detection yet.

8. SHM applications

One of the earliest applications of an SHM method was proposed by Cawley for a pipeline [10]. Presently, SHM methods have found applications mainly in civil, aerospace and mechanical engineering.

The Civil Engineering applications are first of all bridges (see Fritzen, Bohle [25], [26], Wang *et al.* [86], Peeters *et al.* [66], Maeck, De Roeck [54], Teughels, De Roeck [80]) masts (see Peeters, De Roeck [65]), buildings (see Skjaerbaek *et al.* [71], Spina [75]) and historical buildings (see Del Grosso *et al.* [15]).

The aerospace applications encompass aircrafts (see Hunt, Hebden [39]), spacecrafts (see Lopez–Diez [51]), satellites (see Kabashima *et al.* [42]) and composites (see Zak *et al.* [91], Yana *et al.* [89]).

The mechanical applications concentrate on rotating machinery (see Uhl *et al.* [83], Kasarda *et al.* [45]).

Other applications include e.g. water networks (see Holnicki–Szulc *et al.* [34]).

9. Final remarks

The Structural Health Monitoring has started as a research field applied exclusively in the branches of engineering requiring high-precision standards, e.g. in spacecraft or rotating machinery design. However the problem of aging infrastructure has accelerated the development of the Structural Health Monitoring in civil engineering as well, especially for bridges. An important step forward for the low-frequency methods was the formulation of the stochastic identification approach, enabling the consideration of measurement noise. Another essential achievement was the use of output-only data, which enabled to perform the *Operational Modal Analysis* (OMA), basing solely on ambient excitation. The newest trend seems to combine the advantages of the OMA and methods dealing with the defined excitation, resulting in the *Operational Modal Analysis with eXogenous inputs* (OMAX).

Piezoelectric transducers are commonly used in present-day SHM as they have the rare feature of being the actuator and sensor in one device. Optical fibres are gaining more and more attention, especially in bridge applications. Wireless sensor communication is important in permanent monitoring. Once the wireless sensors are mounted, the collected data can be transmitted long distance and analysed in a central unit far away from the monitored structure. In view of

the abundance of recorded data on large structures, data condensation techniques have become a critical issue in present-day SHM (see SPENCER *et al.* [74]).

Thus far, the majority of research done in SHM is limited to linear problems. Nonlinear analysis has been recently proposed by FARRAR *et al.* [22], DEMARIE *et al.* [16], UHL *et al.* [83]. Another important and relatively unexplored problem is the distinction of damage identification from the influence of environment on the structure (see PEETERS *et al.* [67], SOHN *et al.* [73]). DERAEMAEKER and PREUMONT [17] propose to use spatial filters to handle the problem.

It seems that the SHM is becoming more and more important as a research area of practical value to the engineering community, whose development is in fact driven by maintenance requirements of industrial structures. The SHM community is getting better organized. Many European research centres, dealing with the area, were co-operating in the Structural Assessment Monitoring and Control (SAMCO – (http://www.samco.org/) network, EU-financed in the years 2002–2006, and still continued as a voluntary initiative. Another SHM-related organization is the European Association for the Control of Structures (http://dipmec.unipv.it/eacs/).

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