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ANALYSIS OF NON-EXPONENTIAL SOUND DECAY IN AN ENCLOSURE COMPOSED OF TWO CONNECTED RECTANGULAR SUBROOMS

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Acoustically coupled rooms have recently been drawing more and more attention in the architectural acoustics community. Determination of decay times in these coupled rooms from computer-based models has become increasingly significant. In computational modelling of coupled enclosures the geometrical acoustics methods do not apply at lower frequencies, and models must account for the wave-nature of sound fields, especially modal effects. In this investigation a combination of modal analysis with finite-difference algorithm was used to predict a decay in the sound field energy in an enclosure composed of two connected rectangular subrooms. Random incidence absorption coefficients inside subrooms have been modified in this study to create different configurations of absorbing materials on subrooms walls. When one of these subrooms was much more absorbent than the other one, calculation results have shown that, at any frequencies, there existed both the initial sudden and the late slow decrease in the sound field energy. This effect was characterized by two metrics: early decay time (EDT) and late decay time (LDT) determined from a sound level decay from −50 to −60 dB. The analysis of numerical data confirmed that the most influential factor in realizing the non-exponential sound decay is a phenomenon of eigenmodes localization.

Keywords: coupled rooms, reverberation, double-sloped sound decay, localization of modes.

1. Introduction

A coupled-room systems typically are made up of two or more volumes that are connected through an acoustically transparent opening known as a coupling aperture. Examples of such systems are theatres with boxes which communicate with a main room through small apertures only, or churches with several naves and chapels. In an enclosure composed of two connected rooms the most dramatic evidence of coupling is noted if one of the rooms has a high sound absorption and the second one is more reverberant. In this case it is possible for the interesting acoustical phenomenon to occur, namely the sound pressure has a non-exponential decay and the curve describing a drop of pressure level consists of two distinctive parts which refer to the early and late stages of the decay.

The acoustics of coupled-room systems and the coupling phenomenon have been investigated intensively in the past. HARRIS and FESHBACH [1] have applied wave theory to the problem of two connected rooms and found explanations of some discrepancies noted by earlier researchers between measurements and predictions from geometrical acoustics. The energy flux between rooms has been investigated in detail by CREMER and MÜLLER [2] for cases in which the rooms are coupled to an open area and when they are coupled through a door or window. The acoustics of large buildings with coupled spaces has been examined by ANDERSON and BRATOS–ANDERSON [3] to predict the decay of the sound energy density and the reverberation time. They reported a non-exponential sound decay in different locations of St Paul's Cathedral in London and investigated in particular the early decay of a sound that contributes mostly to its subjective perception. The ability of Bayesian probability theory to evaluate decay times from the non-exponential sound decay was demonstrated by XIANG and GOGGANS [4]. The effect of three architectural parameters: the ratio of the volume between the main and secondary rooms, the ratio between the rooms' absorption, and the size of a coupling aperture on the non-exponential decay of sound in a system of two connected rooms, has been studied by BRADLEY and WANG [5]. They proposed a new objective method of quantifying the double-sloped effect (DSE) and carried out tests to determine the subjective response to the DSE.

The paper was dedicated to the analysis of a sound decay in two connected rectangular subrooms having a horizontal section presented in Fig. 1. A theoretical model of a reverberant sound field is based on the modal expansion of a sound pressure for a weakly damped room system [6] and the numerical implementation is an extension of computational methods presented by the author in papers [7–9]. The effect of non-exponential sound decay, which occurs when one of the subrooms is more absorbent than the second one, was characterized by early reverberation time (EDT) and late re-

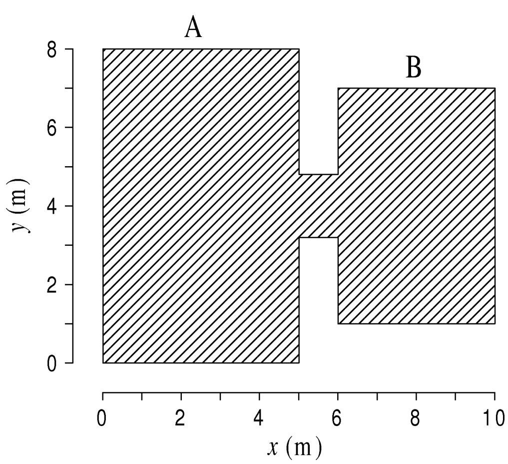

Fig. 1. Horizontal section of room composed of two connected subrooms denoted by A and B.

verberation time (LDT) predicted from a sound level decay from -50 to -60 dB. Calculated frequency dependences of average values of EDT and LDT in both subrooms have shown that the most influential factor in producing the non-exponential sound decay is the effect of modal localization.

2. Analysis of computational results

A numerical simulation of reverberant sound field was performed for the room geometry often occurring in practice when two rectangular subrooms with the same heights are joined together, thus they form a simple coupled-room system. Horizontal dimensions of room can be easily deduced from Fig. 1 and the room height was assumed to be 3 m. Shapes of modes and their frequencies were computed by use of a forced oscillator method with a finite difference algorithm [8]. Two positions of a sound source were assumed (in meters): $x = 2$, $y = 5$, $z = 1$ (subroom A) and $x = 8$, $y = 5$, $z = 1$ (subroom B). A numerical procedure [9] made possible to predict a decay of sound pressure level at any location in the room interior and to calculate fit curve describing time-average decay of pressure level by use of regression method.

Figure 2 shows examples of pressure level decays (black lines) together with fit curves (white lines) obtained in the observation point: $x = 8$ m, $y = 5$ m, $z = 1.8$ m, for the sound frequency of 52 Hz equal to an eigenfrequency of mode localized in subroom B (Table 1) and different random incidence absorption coefficients α_a and

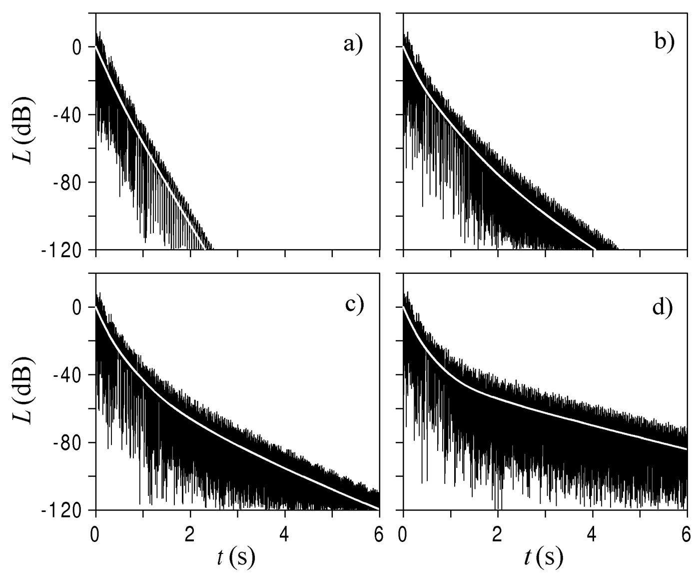

Fig. 2. Sound pressure level (black lines) and decay curves obtained via regression method (white lines) in observation point: $x = 8$ m, $y = 5$ m, $z = 1.8$ m, for absorption coefficients: a) $\alpha_a = \alpha_b = 0.15$, b) $\alpha_a = 0.22$, $\alpha_b = 0.046$, c) $\alpha_a = 0.23$, $\alpha_b = 0.031$, d) $\alpha_a = 0.24$, $\alpha_b = 0.016$. Sound source located in subroom A, sound frequency of 52 Hz.

α_b in subrooms A and B. In the first case, corresponding to equal values of α_a and α_b, a time-averaged decay of pressure level is linearly dependent on a time (Fig. 2a). It means that a sound pressure has an exponential decay and this situation occurs when a pressure time history is dominated by a decay of one mode or by decays of modes having the same or very similar damping coefficients. For growing difference between α_a and α_b the fitting curves become more and more nonlinear and the non-exponential sound decay is most evident for the largest difference between absorption coefficients (Fig. 2d). In this case the fit curve consists of two parts which refer to the rapid early decay and the slow late decay. In such a situation the time history of a pressure level characterizes the double-sloped effect (DSE) and it happens when a dominant mode is much more damped than neighbouring modes.

A steep initial sound decay may result in higher sound clarity, whereas a slow late decay leads to an increase in perceived reverberance [5], thus from a subjective viewpoint the standard reverberation time (the time for a sound to die away to a level 60 dB below its original level) appears to misleading measure of DSE. To quantify double-sloped profiles accurately the early decay time (EDT) and the late decay time (LDT) were computed, where LDT is the decay time from -50 to -60 dB in decay curve, multiplied by a factor of 6. A calculated distribution of LDT for a sound frequency of 52 Hz and a configuration of absorbing material that produces a high double-sloped effect is presented in Fig. 3. The data are plotted in a form of filled contour maps which are a two-dimensional representation of three-dimensional data. Since an eigenmode localized in an enclosure that has a small sound absorption is only weakly damped, the late decay time in subroom B is dramatically large because it corresponds to the delay time of localized eigenmode. In order to investigate this effect in the whole frequency range, from distributions of EDT and LDT in an observation plane the average values <EDT> and <LDT> of these delay times in subrooms A and B were computed.

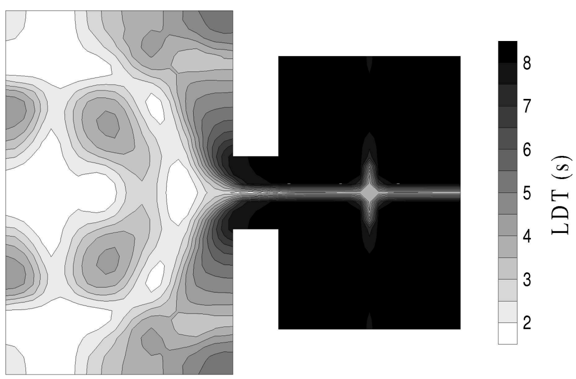

Fig. 3. Distribution of late decay time in observation plane $z = 1.8$ m for absorption coefficients: $\alpha_a = 0.24$, $\alpha_b = 0.016$. Sound source located in subroom A, sound frequency of 52 Hz.

Frequency dependencies of <EDT> and <LDT> for absorption coefficients α_a and α_b previously assumed and different locations of sound source are shown in Fig. 4.

The presented graphs are of great importance from a practical viewpoint because they show how the effect of modal localization can strongly influence the early and late sound decays if a difference between absorption coefficients in coupled subrooms is sufficiently great. Evident results of this phenomenon are local maxima of <EDT> and <LDT> because, as was proved by a detailed analysis of calculation data, they occur for sound frequencies which correspond to eigenfrequencies of modes localized in subroom B (Table 1). It is important to note that the effect of modal localization is the direct result of an irregular geometry of lateral walls of room because in a rectangular room all eigenmodes are delocalized [10].

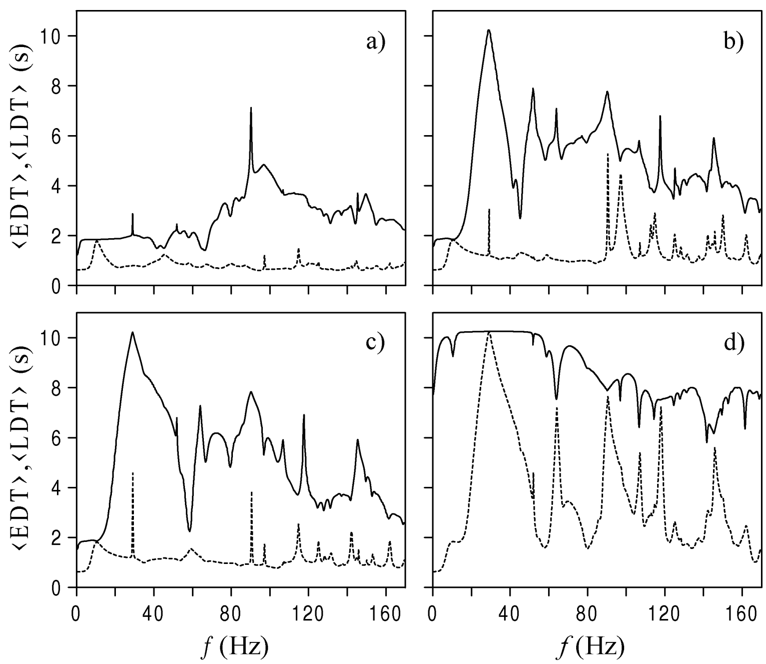

Fig. 4. Frequency dependences of average values of early decay time (dashed lines) and late decay time (solid lines) in subroom A (a, c) and subroom B (b, d) for absorption coefficients: $\alpha_a = 0.24$, $\alpha_b = 0.016$. (a, b) sound source located in subroom A, (c, d) sound source located in subroom B.

As may be seen in Fig. 4, frequency dependences of <EDT> and <LDT> are also hardly influenced by a source position. When it is located in subroom A that walls are covered by a material with a great absorption, high-value peaks of <LDT> are observed in subroom B because of small damping of acoustic energy for eigenmodes localized in this subroom (Fig. 4b). On the other hand, if a source position is in subroom B, the peaks of <LDT> occur in subroom A (Fig. 4c), however in subroom B the peaks of <EDT> and high values of <LDT> are noted (Fig. 4d). The lack of peaks in a frequency dependence of <LDT> may be explained by the fact that eigenmodes localized in subroom B have high amplitudes in a steady-state [8], thus it dominates the late sound decay in a wide frequency range.

For each eigenmode a modal reverberation time can be calculated. This reverberation time is defined as that for which the pressure decays in that mode by 60 dB, thus for modes with non-zero eigenfrequencies this time is given by [6]

$$T_{mn} = \frac{3\ln(10)}{r_{mn}}\,, \tag{1}$$

where r_{mn} is a damping factor. In Fig. 5 a frequency dependence of <LDT> from Fig. 4b is compared with values of T_{mn} calculated for modes localized in subroom B. As may be seen, frequencies of local maxima in <LDT> match eigenfrequencies for localized modes and moreover, in some cases the peak values of <LDT> correspond almost exactly to the values of modal reverberation time.

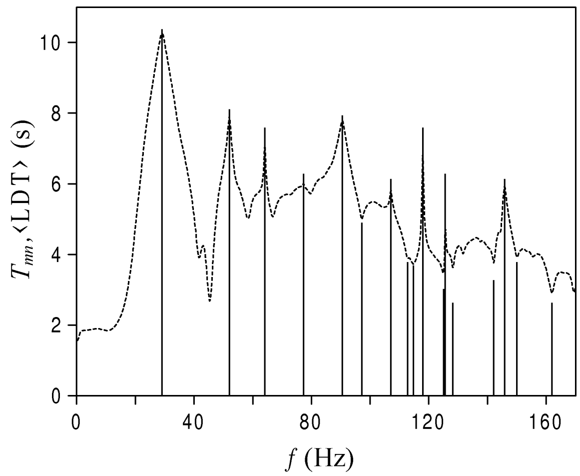

Fig. 5. Comparison between modal reverberation time for eigenmodes localized in subroom B (vertical solid lines) and frequency dependence of average value of LDT in this subroom (Fig. 4b).

3. Conclusions

Rooms which are coupled together by an open area are known to exhibit some interesting acoustical phenomena. In a system of two connected subrooms, a variation of sound absorption in one of the subrooms results in a change of decay curves' shape in both subrooms and can produce non-exponential sound decay in which a late slow decrease in a sound pressure is preceded by an initial sudden sound decay. The numerical method described in the papers [7–9] has been used to model a reverberant sound field in such a room system and to generate sound echograms from which the early decay time (EDT) and the late decay time (LDT) can be estimated. Calculation results have showed that LDT inside a subroom with a small sound damping can be several times greater than LDT in a subroom with a high sound absorption (Fig. 3) and large discrepancies between <EDT> and <LDT> (average values of EDT and LDT in subrooms) are a result of different damping of eigenmodes due to the effect of modal localization

(Fig. 4, Table 1). A nearly exponential sound decay was observed for frequencies corresponding to eigenfrequencies of localized modes when a sound source was located inside a subroom having walls with small absorption coefficient (Fig. 4d). It was found that peak values of <LDT> are very close to the values of modal reverberation time for localized modes (Fig. 5).

Appendix A. Frequencies of eigenmodes localized in subroom B

For coupled rooms having the same height and the horizontal section shown in Fig. 1 the normalized eigenfunctions Φ_{mn} can be determined by [8]

$$\Phi_{mn}(x,y,z) = \begin{cases} 1/V^{1/2} & m=0,\ n=0, \\ \Psi_n(x,y)\sqrt{h}, & m=0,\ n>0, \\ \sqrt{2/h}\,\cos(m\pi z/h)\,\Psi_n(x,y), & m>0,\ n>0, \end{cases} \tag{A1}$$

where V is a room volume, h is a room height and eigenfunctions Ψ_n are normalized over a horizontal section of the room. In this case eigenfrequencies are given by

$$f_{mn} = \frac{1}{2\pi}\sqrt{(m\pi c/h)^2 + \omega_n^2}\,, \tag{A2}$$

Table 1. Frequencies f_{mn} of eigenmodes localized in subroom B and parameters υ^a_{mn} and υ^b_{mn} together with corresponding combinations of subscripts m and n.

m	n	f_{mn} (Hz)	υ^a_{mn}	υ^b_{mn}
0	3	29.08	0.00007	0.98826
0	8	52.06	0.00014	0.97456
1	3	64.14	0.00007	0.98826
1	8	77.32	0.00014	0.97456
0	18	90.54	0.00116	0.97301
0	21	97.18	0.03029	0.92969
1	18	107.08	0.00116	0.97301
1	21	112.75	0.03029	0.92969
0	28	114.77	0.11575	0.87072
2	3	117.97	0.00007	0.98826
0	32	125.06	0.13664	0.81508
2	8	125.63	0.00014	0.97456
0	40	142.18	0.10520	0.86586
2	18	145.84	0.00116	0.97301
2	21	150.05	0.03029	0.92969
2	28	162.00	0.11575	0.87072

where c is a sound speed and $f_n = \omega_n/2\pi$ is an eigenfrequency for the function Ψ_n. An eigenmode is localized in one of the subrooms if the amplitude of Φ_{mn} in this subroom is high. Since eigenfunctions Φ_{mn} are normalized, the integral of Φ^2_{mn} over a room volume equals unity, thus to characterize mathematically the localization of modes one should compute two non-dimensional parameters

$$\upsilon^a_{mn} = \int\limits_{V_a} \Phi^2_{mn}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z, \qquad \upsilon^b_{mn} = \int\limits_{V_b} \Phi^2_{mn}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{A3}$$

where V_a and V_b are volumes of subrooms A and B. Thus, an eigenmode is localized in the subroom B when a value of υ^a_{mn} is much smaller than 1 or the parameter υ^b_{mn} is very close to unity. Frequencies f_{mn} of eigenmodes localized in subroom B together with parameters υ^a_{mn} and υ^a_{mn} are collected in Table 1. As may be seen, these parameters possess the same values for the same mode index n owing to the form of eigenfunctions Φ_{mn}.

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